

## ROBUST BEAMFORMING FOR QUADRATIC ANTENNA ARRAYS IN PICOCELLULAR MOBILE ENVIRONMENTS

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### 1 Introduction

In future mobile communication systems the usage of smart antennas is aimed in order to increase the system capacity and system performance. Smart antennas provides a technique for increasing the system capacity in different ways. Spatial separation of different users can be used to support an increased number of subscribers within the same frequency band by space division multiple access (SDMA). Furthermore spatial filtering (SFIR, spatial filtering for interference reduction) effectively reduces interferences introduced by multipath propagation, multi-user access and other mobile communication systems within the same frequency band. By decreasing the influences of multipath propagation, spatial filtering also reduces delay spread and doppler spread which improves the overall channel characteristics and results in more benign communication channels. The performance increase to be expected when using smart antennas highly depends on how accurate the beampattern can be adapted to the actual interference pattern. In a macrocellular environment, where the distance between the base station and the mobile unit is sufficient large, all signals arrives at the base station only with different azimuth-angles. The differences referable to the elevation-angle are very small and can be neglected. In this case it is sufficient to use a one-dimensional linear antenna array to control the azimuth-angle of the beampattern. Using smart antennas in a pico-cellular environment the directions of arrival differs not only in the azimuth-angle but also in the elevation-angle. In order to achieve a maximum performance increase, the beampattern has to be controlled in both directions, the azimuth- and elevation-angle, by using a two dimensional linear antenna array. Thereby the beampattern has to steer the main lobe in direction of signal of interest and has to steer nulls in directions of the main disturbing signals. Two dimensional antenna arrays shall be used for base station antennas as well as for mobile station antennas. In contrast to the one-dimensional case, where the construction of an optimum beampattern is always possible, the construction of two-dimensional beams is quit difficult and not always possible.

In this paper we present a new blind beamforming algorithm where the azimuth is constructed by using the one-dimensional directivity controlled constrained beamformer from [1] and the elevation is constructed by using a phased array. This leads to a beamforming algorithm which solution is guaranteed. Furthermore the constructed beampattern provides a maximum directivity which leads to a minimum bit error rate in asynchronous CDMA systems (CDMA, Code Division Multiple Access), according to the model of Rappaport and Liberti [2, 3].

### 2 Array Signal Model

The derivation of the array signal model is made from the signal processing point of view. Consider a set of  $L$  waveforms

$$\begin{aligned} u_l(t) &= a_l(t) \cdot e^{j(2\pi f_c t + \phi_l(t))} \\ &= s_l(t) \cdot e^{j2\pi f_c t}, \quad 1 \leq l \leq L \end{aligned} \quad (1)$$

with carrier  $e^{j2\pi f_c t}$  and complex baseband signal  $s_l(t) = a_l(t) \cdot e^{j\phi_l(t)}$ , impinging on an array of  $M \times N$  spatially distributed identical sensors as shown in Fig. 1. It is commonly assumed

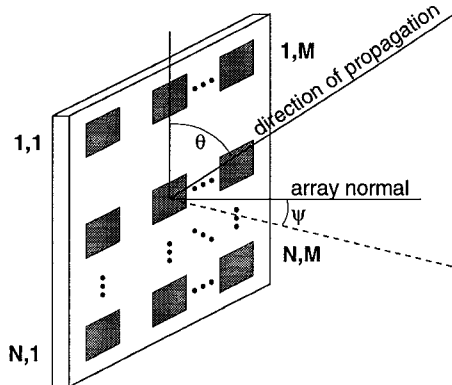


Figure 1: Arriving signal at a  $M \times N$  linear antenna array

that the narrowband signal variations can be neglected as the wavefront passes along the array:  $s(t - \Delta\tau_{max}) \approx s(t)$ . The output signal of the  $(m, n)$ -th sensor is then given as

$$x_{m,n}(t) = \sum_{l=1}^L s_l(t) \cdot e^{j2\pi f_c(t - \tau_{m,n}(\psi_l, \theta_l))} + r_{m,n}(t),$$

$$1 \leq m \leq M, 1 \leq n \leq N \quad (2)$$

with  $r_{m,n}(t)$  being a noise component of the  $(m, n)$ -th sensor with zero mean and variance  $\sigma^2$ . The signals are weighted and combined in order to produce the output signal

$$y(t) = \sum_{m=1}^M \sum_{n=1}^N w_{m,n} \cdot x_{m,n}(t), \quad (3)$$

where  $w_{m,n}$  are the element weights. Clearly, for a fixed set of weights,  $y(t)$  is a function of the incident angles  $\psi$  and  $\theta$ . In order to describe the array response in terms of a pattern multiplication the coordinates are transformed with  $\sin u = \sin \psi$  and  $\sin v = \cos \psi \cos \theta$ . If the transmitter is placed far enough from the array, the impinging waveforms can be considered as plane wavefronts. Using an uniform linear array (ULA) with  $M \times N$  elements with spacing  $d = \lambda/2$  the transfer function or beampattern of this array is defined in terms of the normalized wavenumbers  $\Omega_a = -\pi \sin u$  and  $\Omega_e = -\pi \sin v$ :

$$H(e^{j\Omega_a}, e^{j\Omega_e}) = \sum_{m=1}^M \sum_{n=1}^N w_{m,n} \cdot e^{j(m-1)\Omega_a} \cdot e^{j(n-1)\Omega_e}, \quad (4)$$

Now the beamformer has to calculate the antenna weights, that the beampattern will provide a maximum performance increase. The directivity of the beampattern is then defined as

$$D(H) = \frac{4\pi^2}{\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |H(e^{j\Omega_a}, e^{j\Omega_e})|^2 d\Omega_e d\Omega_a}. \quad (5)$$

In section 3 it is shown that the directivity  $D(H)$  is very important for the noise reduction at the output of an antenna array (see eq. (10)).

### 3 Robust Blind Beamforming

In [4] we have presented a constrained beamforming algorithm for two-dimensional linear antenna arrays where the direction of the main beam as well as the directions of nulls can be controlled for both, the azimuth and the elevation. Using an antenna array consisting of  $N \times N$  antenna elements, the number  $l$  of controllable nulls can be varied in the range  $0 \leq l \leq N^2 - 1$ . We found out that the maximum number  $N^2 - 1$  of controllable nulls cannot be guaranteed for arbitrary positions of the nulls. Placing all nulls along a well defined curve, the maximum number of nulls can be reduced rapidly. If all nulls are placed along the diagonal, the maximum number of nulls is reduced to  $2N - 2$ . In case of placing all nulls along a line with constant azimuth- or elevation-angle, this maximum number is reduced to  $N - 1$ . Thus, the behavior of this approach is quite difficult and consequently not well suited for SDMA and SFIR applications.

In order to solve this problems we present a new blind beamforming algorithm where the beam-pattern is constructed with a product of two basic beampattern which controls the azimuth and elevation separately

$$H(e^{j\Omega_a}, e^{j\Omega_e}) = \sum_{m=1}^M w_{a,m} \cdot e^{j(m-1)\Omega_a} \cdot \sum_{n=1}^N w_{e,n} \cdot e^{j(n-1)\Omega_e} = H_a(e^{j\Omega_a}) \cdot H_e(e^{j\Omega_e}) . \quad (6)$$

The construction of these basic pattern is performed by using different methods for each plane. In order to place nulls in the beampattern the azimuth is constructed with a maximum-directivity-beamformer from [1]. The elevation is constructed only by steering the main beam direction with a phased array. The constraints of the beamformer are  $H_a(e^{j\Omega_{a,l}}) = 1$  for  $l = L$  and  $H_a(e^{j\Omega_{a,l}}) = 0$  for  $l = 1, \dots, L - 1$ , where the nulls are only steered in the corresponding azimuth angle. According to the structure of the beampattern from eq. (6) it is also possible to control the number  $L$  of nulls. The basic pattern can be formulated as

$$H_e(e^{j\Omega_e}) = \frac{1}{N} \sum_{n=1}^N e^{j(n-1)(\Omega_e - \Omega_{e,L})} \quad (7)$$

for the elevation, and

$$H_a(e^{j\Omega_a}) = \sum_{l=1}^L b_l \cdot \sum_{m=1}^M e^{j(m-1)(\Omega_a - \Omega_{a,l})} \quad (8)$$

for the azimuth. Using the constraints the coefficients  $b_l$  can be calculated from a linear set of equations. The weights of the beamformer are calculated from

$$w_{m,n} = \frac{1}{N} \sum_{l=1}^L b_l \cdot e^{-j(m-1)\Omega_{a,l}} e^{-j(n-1)\Omega_{e,L}} . \quad (9)$$

Figure 2 shows the beampattern of the robust beamformer, where the main beam direction is steered to  $\Omega_a = \pi/8$ ,  $\Omega_e = 0$ . In figure 3 the contour-plot of the beampattern is shown in order to demonstrate the correct placement of the constraints, where the main beam direction is marked with  $\diamond$  and the nulls are marked with  $\circ$ . It can be shown that the constructed beampattern provides a maximum directivity in comparison to all other beampattern which are based on this approach. According to the model of Rappaport and Liberti, this maximum directivity  $D$  leads to a minimum bit error rate in an asynchronous CDMA system. Maximizing the directivity leads also to a reduced noise variance at the output of the array, where the noise variance at the output of the array is given by

$$\sigma_{out}^2 = \sigma^2 / D(H) . \quad (10)$$

Note that the suppression of disturbing signals take place only by placing a null at the corresponding azimuth-angle. A reduced calculation effort can be achieved by a previous analysis

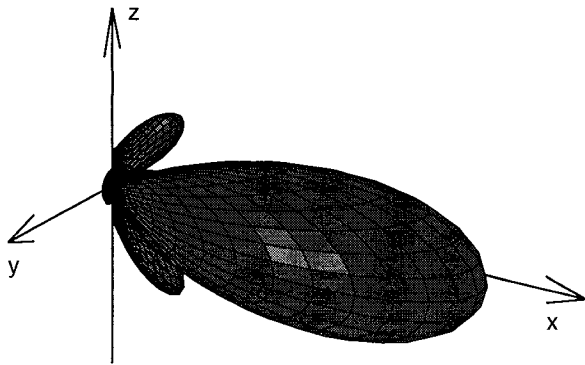


Figure 2: Beampattern of the robust beamformer in azimuth and elevation.

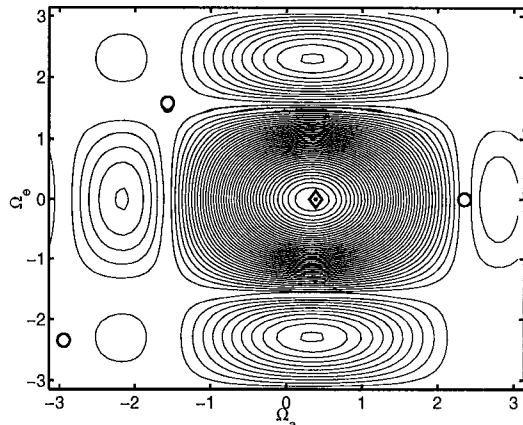


Figure 3: Contour-plot of the beam-pattern in respect of the normalized wavenumbers.

of all elevation-angles of the constraints. Using the special properties of the phased-array, a given attenuation  $a_{\min}$  can be achieved from the elevation pattern. With the estimation  $|\sin \frac{1}{2}(\Omega_e - \Omega_{e,L})| < \frac{1}{2} |(\Omega_e - \Omega_{e,L})|$  this attenuation is guaranteed if the distance between the main beam direction and the desired null is

$$|(\Omega_e - \Omega_{e,L})| > \frac{2}{N \cdot a_{\min}} . \quad (11)$$

That means, if the difference between the elevation-angles of the main beam direction and a disturbing signal are large enough, the direction of this disturbing signal has not to be considered in the list of constraints, which leads to a lower calculation effort, and also to a higher directivity of the resulting beampattern.

#### 4 Conclusions

In this paper we have proposed a new robust constrained beamforming algorithm for two-dimensional linear antenna arrays. The algorithm is based on a product of two basic pattern, which controls the azimuth- and elevation-plane separately. In contrast to two dimensional constrained beamforming algorithms, where the existence of a beampattern is not always guaranteed, this approach leads to a optimum solution for  $N - 1$  arbitrary positions of the nulls. Furthermore the constructed beampattern provides a maximum directivity, which leads to a minimum bit error rate in asynchronous CDMA systems.

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