

E-Polarized Diffraction by Dielectric Wedge

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1. Introduction

Geometrical optics (GO), the most widely used tool in analysis of high frequency electromagnetic problems, provides the first term of an asymptotic series solution to the Maxwell's equations [1]. It is implemented completely by employing the ordinary ray-tracing technique. To include the second asymptotic term systematically, Keller [2] suggested a generalization of the Fermat's principle on diffracted rays. It is called the geometrical theory of diffraction (GTD), where the initial value of diffracted rays should be determined from the exact diffraction coefficients of canonical structures, e.g., perfectly conducting half-plane, wedge, and cone. In spite of some improved GTD versions, those applications to penetrable objects have been hindered by the lack of rigorous diffraction coefficients of such canonical structures as penetrable wedges and cones.

Until now, there is no rigorous solution to the diffraction by a dielectric wedge [3]. Some heuristic modification of the exact diffraction coefficients of perfectly conducting wedge have been performed to account finite dielectric constant or conductivity. But their diffraction coefficients could not satisfy the edge condition at wedge tip [4]. Numerical calculations of diffraction coefficients of dielectric wedges have also been performed using the method of moment and the FDTD method. But numerical techniques could not provide comparable achievements in the physical understanding of edge diffraction.

In recent, an approximate but accurate analytical solution to the diffraction coefficients of composite wedge was constructed by employing the method of hidden rays [5]. The hidden rays obey the usual principle of geometrical optics (GO) but do not exist in the physical region. These rays can be traced only in the complementary region, in which original media of background and scatterer are exchanged each other. While the physical optics (PO) approximation of ordinary rays provides not only GO term but also diffracted field in physical region, the PO approximation of hidden rays contributes only to diffracted field in physical region. Hence the method is called the hidden rays of diffraction (HRD). In this paper, the HRD method is applied to E-polarized diffraction by a dielectric wedge.

2. Method of Hidden Rays

Fig. 1 shows the geometry of a dielectric wedge with relative dielectric constant ε in S_d for $0 \leq \theta_d \leq \pi$. When an E-polarized unit plane wave $u_i(\rho, \theta)$ with an arbitrary angle θ_i is incident on the dielectric wedge, the z -component of the total electric field $u(\rho, \theta)$ may be written into the dual integral equations [6]. At first, the uniform currents induced on the wedge interfaces were derived from GO field. The complete PO solution could be obtained analytically by applying the uniform currents into the dual integral equations approximately. It should be noted that the cotangent functions of PO diffraction coefficients correspond to the ordinary rays of GO field one by one. But the error of PO solution was interpreted by the non-zero of the artificial field emanating from the uniform currents in the complementary regions, in which the media inside and outside a dielectric wedge are exchanged each other. The concept of the complementary region may be considered as an extension of the extended boundary condition or the null-field method. Previously, the non-uniform currents were approximated by the multi-pole expansion at wedge tip or the Neumann's expansion along wedge interfaces [6]. Those expansion coefficients should be calculated numerically under the condition that those radiated field had to cancel out the artificial PO field in the complementary regions. Although the highly accurate solution to the diffraction by a

dielectric wedge was obtained, its numerical calculation suffered from instability. But there is no systematic way to solve the dual integral equations rigorously.

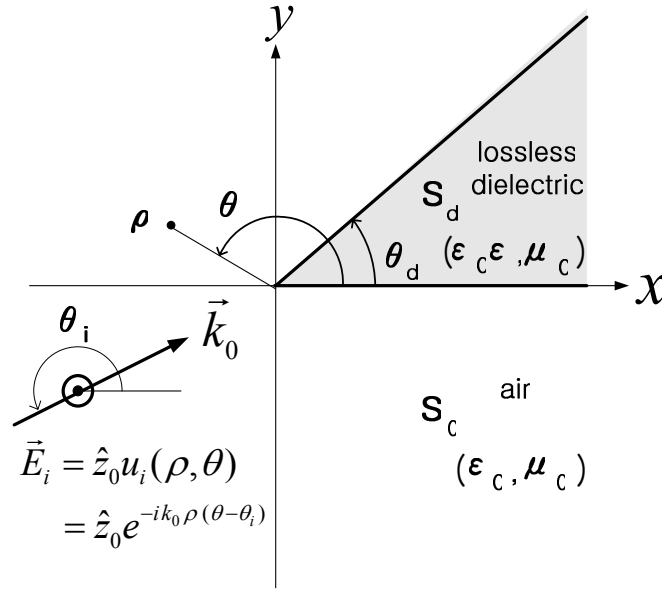


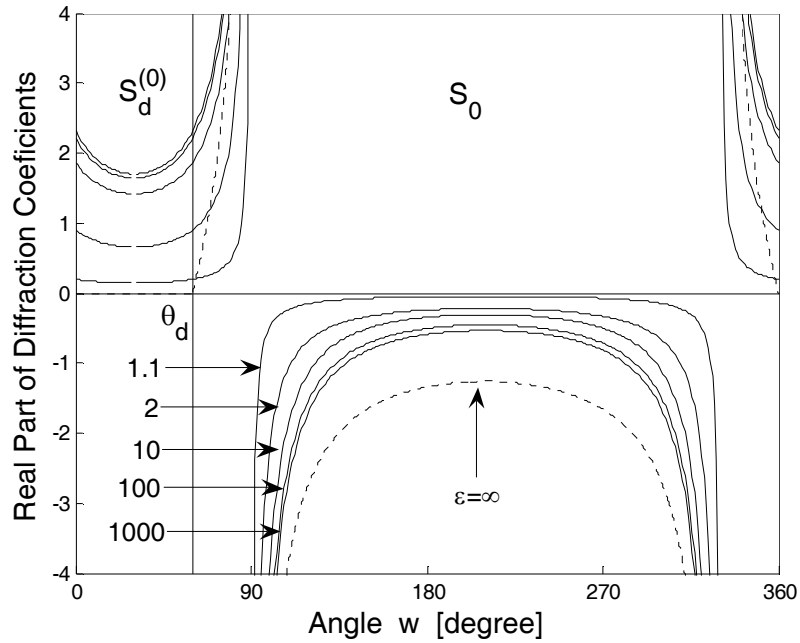
Figure 1: Geometry of a Dielectric Wedge Illuminated by an E-Polarized Plane Wave

Hence we have tried to develop an analytical method, even if it may be approximate. After the ordinary ray-tracing in physical region is terminated, the hidden rays can be traced only by extending the usual principle of GO in its complementary region. Multiple reflections inside the dielectric wedge should be accounted. The internal reflection continues before any hidden ray penetrates into the next periodic physical dielectric regions. If a hidden ray falls in those physical dielectric regions, the hidden ray may be changed into an ordinary ray. The amplitudes of hidden rays are obtained routinely by multiplication of the Fresnel's reflection coefficients. The diffraction coefficients of a dielectric wedge may be also constructed only by performing two additional treatments to the ray-tracing data. One is the correspondence between the rays of GO field and the cotangent functions of its PO diffraction coefficients one by one. In view of an extended concept, the diffraction coefficients may be expressed by finite series of cotangent functions, which correspond to not only the ordinary rays in the physical region but also hidden rays in the complementary region. The other is the relation between the singular behavior of edge-diffracted field and the cotangent functions of diffraction coefficients. The angular period of the cotangent functions is adjusted to satisfy the edge condition at wedge tip. Then the accuracy of the HRD diffraction coefficients in the physical region may be accounted by checking how closely the diffraction coefficients satisfy the null-field condition in the complementary region. It implies that the diffraction coefficients of penetrable wedges can be constructed in analytic form only by employing the extended ray-tracing data and the edge condition at the wedge tip.

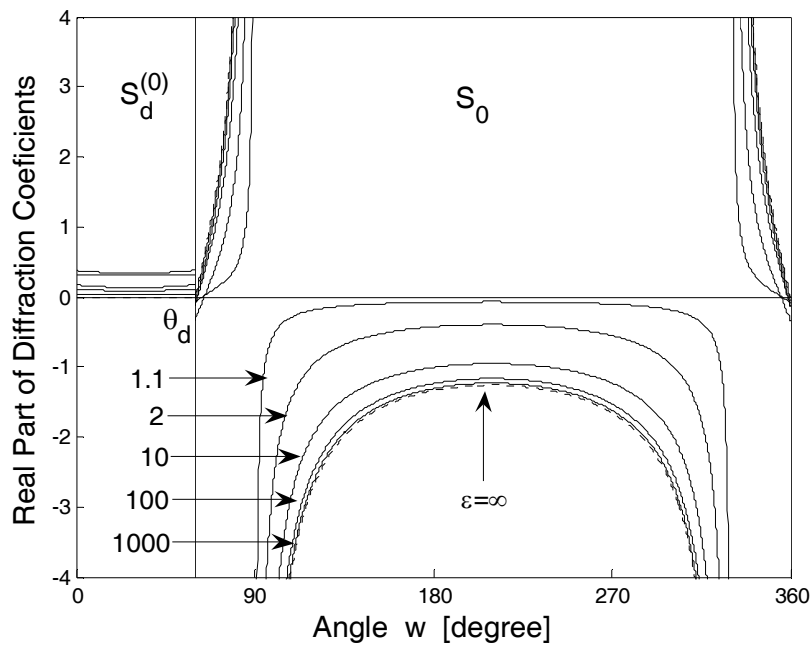
3. Diffraction Coefficients and Field Patterns

A typical example is considered in case that in Fig. 1, the dielectric wedge with $\theta_d = 60^\circ$ is illuminated by an E-polarized unit plane wave with $\theta_i = 210^\circ$. Fig. 2(a) shows the PO diffraction coefficients in the physical and complementary air regions for the relative dielectric constant $\epsilon = 1.1, 2, 10, 100,$ and 1000 . The dotted line marked by $\epsilon = \infty$ denotes the exact diffraction coefficients of the corresponding perfectly conducting wedge. As expected, the PO diffraction coefficients intersect the exact solution near the dielectric boundaries. Fig. 2(b) shows the HRD diffraction coefficients in the physical and complementary air regions. As ϵ increases to 1000 , the HRD curves approach the exact diffraction coefficients of $\epsilon = \infty$ monotonically. Based

on the formulation of the dual integral equations, the accuracy of the diffraction coefficients can be verified by showing the degree of deviation from zero in the complementary region. According to this criterion, Fig. 2(a) shows PO diffraction coefficients erroneous in the complementary air region $S_d^{(0)}$. In contrast, Fig. 2(b) assures that the HRD diffraction coefficients become nearly zero in $S_d^{(0)}$.



(a) PO diffraction coefficients



(b) HRD diffraction coefficients

Figure 2: Diffraction Coefficients in the Physical and Complementary Air Regions for $\theta_d = 60^\circ$, $\theta_i = 210^\circ$, and $\epsilon = 1.1$ to 1000. (Dotted Line: Exact for $\epsilon = \infty$)

The field patterns may be easily calculated by applying the uniform asymptotic integration to the diffraction coefficients. The PO (dotted line) and HRD (bold line) fields for $\theta_d = 60^\circ$, $\theta_i = 210^\circ$, and $\varepsilon = 1000$ are calculated at 5λ (wavelength) away from the wedge tip, and then compared to the exact field (broken line) of the corresponding perfectly conducting wedge. Figs. 3(a) and (b) shows the amplitude patterns of the edge-diffracted and total fields, respectively. While the PO fields violate the boundary condition and deviate from the broken lines significantly, the HRD fields are nearly overlapped to the exact field patterns of $\varepsilon = \infty$.

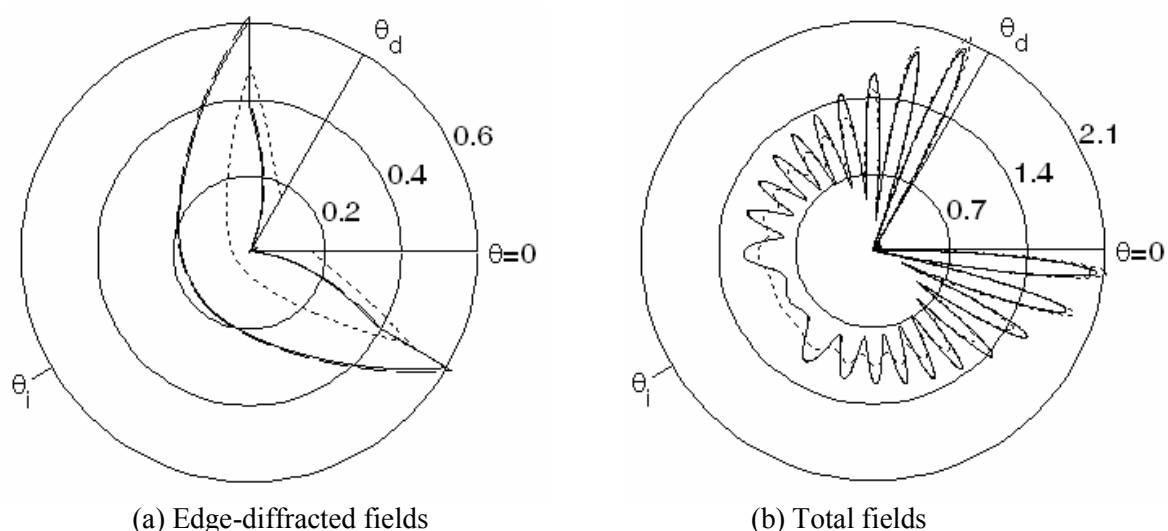


Figure 3: Field Patterns at 5λ away from the Wedge Tip for $\theta_d = 60^\circ$, $\theta_i = 210^\circ$, and $\varepsilon = 1000$.
(Dotted Line: PO, Bold Line: HRD, and Broken Line: Exact for $\varepsilon = \infty$)

4. Conclusion

The E-polarized diffraction coefficients of a dielectric wedge were represented in a simple analytic form only by employing the ray-tracing data including the hidden rays in the complementary region and the edge condition at wedge tip. In view of the dual integral equations, the accuracy of the HRD diffraction coefficients in the physical region may be accounted by checking how closely the diffraction coefficients satisfy the null-field condition in complementary region. The presented diffraction coefficients approach to zero more closely in the complementary regions than the PO solution. It leads us to conclude that the HRD method may be suitable to treat more complicated dielectric scatterers in the GTD format.

References

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