

DOA Estimation with Uniform Circular Array Using Gauss-Newton Method Based on MUSIC and MODE Algorithms

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1 Introduction

Sensor array processing techniques have attracted considerable interest in the signal processing society. These techniques have focused mainly on high-resolution direction-of-arrival (DOA) estimation. Generally, the choice of DOA estimator is made adequately in accordance with the array geometries used. Root-MUSIC[1] and MODE[2] are well-known algorithms that take advantage of the structure of uniform linear array (ULA) to obtain DOA estimates via polynomial rooting approach. These algorithms can also be applied to the uniform circular array (UCA) using phase mode excitation[3]. In this case, the azimuth angles of multiple incident waves are estimated which lie in the plane where the UCA is placed. In the process of DOA estimation, the original DOA information in the element-space of the UCA is transformed into the beamspace manifold similar to ULA-type one. However, this transformation causes some degradation in performance of the algorithms. To remove the drawback, we present in the paper two iterative techniques using Gauss-Newton method based on MUSIC and MODE. The key idea of our techniques is to use the DOA estimates of conventional method as the initial values of the iterative DOA search for improved estimator performance. Although these techniques are somewhat referred in [3], detailed characteristics are not examined. In this paper, therefore, we formulate these techniques, and analyze their characteristics through computer simulation.

2 Data Formulation for DOA Estimation

A. The Data Model

Assume that an array is composed of K sensors that receive the signals from L ($L < K$) sources. The array output is denoted by the $K \times 1$ vector as

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where $\boldsymbol{\theta} = [\theta_1, \dots, \theta_L]^T$ is the DOA vector to be estimated, $\mathbf{s}(t)$ is the $L \times 1$ vector of signal waveforms, $\mathbf{n}(t)$ is the vector of internal noise, and $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1) \dots \mathbf{a}(\theta_L)]$ is the $K \times L$ matrix whose columns $\mathbf{a}(\theta_l)$ ($l = 1, 2, \dots, L$) are the array response vectors for L sources. Then, the array covariance matrix \mathbf{R}_{xx} is given by

$$\mathbf{R}_{xx} = E[\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{A}(\boldsymbol{\theta})\mathbf{S}\mathbf{A}^H(\boldsymbol{\theta}) + \sigma^2\mathbf{I} \quad (2)$$

where $E[\cdot]$ and $(\cdot)^H$ denote the expectation operator and the complex conjugate transpose, respectively. Also, $\mathbf{S} = E[\mathbf{s}(t)\mathbf{s}^H(t)]$ is a covariance matrix of signal waveforms, \mathbf{I} is the identity matrix, and σ^2 is the noise power. The eigendecomposition of (2) is given by

$$\mathbf{R}_{xx} = \mathbf{E}_S \mathbf{\Lambda}_S \mathbf{E}_S^H + \sigma^2 \mathbf{E}_N \mathbf{E}_N^H \quad (3)$$

where \mathbf{E}_S is the $K \times L$ matrix whose columns are the eigenvectors of \mathbf{R}_{xx} associated with the L largest eigenvalues, $\mathbf{\Lambda}_S$ is a diagonal matrix whose elements are the L largest eigenvalues, and \mathbf{E}_N is the $K \times (K - L)$ matrix containing the remaining $K - L$ eigenvectors corresponding to eigenvalues equal to σ^2 .

B. Phase Mode Excitation of UCA

The K -element omnidirectional UCA geometry is depicted in Fig.1, where $\gamma_k = 2\pi(k - 1)/K$. Taking the center of the UCA as reference, the array response vector is expressed as

$$\mathbf{a}(\theta) = [e^{j\zeta \cos(\theta - \gamma_1)}, e^{j\zeta \cos(\theta - \gamma_2)}, \dots, e^{j\zeta \cos(\theta - \gamma_K)}]^T \quad (4)$$

where $\zeta = 2\pi r/\lambda$, r is the UCA radius, and λ is the wavelength. The UCA beamforming weight vector \mathbf{w}_m^H that generates phase mode m is given by [3]

$$\mathbf{w}_m^H = \frac{1}{K} [e^{jm\gamma_1}, e^{jm\gamma_2}, \dots, e^{jm\gamma_K}] \quad (5)$$

Using a set of the weight vectors, the array response vector can be transformed into the phase mode space by

$$\mathbf{C}\mathbf{V}^H \mathbf{a}(\theta) = \sqrt{K} \mathbf{J}_\zeta \mathbf{v}(\theta) \quad (6)$$

$$\mathbf{C} = \text{diag}\{j^{-M}, \dots, j^{-1}, j^0, j^1, \dots, j^M\} \quad (7)$$

$$\mathbf{V} = \sqrt{K} [\mathbf{w}_{-M} \dots \mathbf{w}_0 \dots \mathbf{w}_M] \quad (8)$$

$$\mathbf{J}_\zeta = \text{diag}\{J_M(\zeta), \dots, J_1(\zeta), J_0(\zeta), J_1(\zeta), \dots, J_M(\zeta)\} \quad (9)$$

$$\mathbf{v}(\theta) = [e^{-jM\theta}, \dots, e^{-j\theta}, e^{j0}, e^{j\theta}, \dots, e^{jM\theta}] \quad (10)$$

where $J_m(\zeta)$ is the Bessel function of the first kind of order m and M denotes the highest order mode that can be excited by the aperture at a reasonable strength. The phase mode excitation transforms the UCA response vector $\mathbf{a}(\theta)$ into the ULA-like response vector $\mathbf{v}(\theta)$. Thus, most of the normal ULA signal processing methods can be applied to the UCA in the phase mode domain. The so-called UCA-ESPRIT algorithm[3] is included in this category. Therefore, we try to apply UCA-Root-MUSIC and UCA-MODE to $\mathbf{v}(\theta)$ in the same manner as UCA-ESPRIT.

3 Gauss-Newton method

A. Iterative MUSIC

For the MUSIC algorithm[1], minimization of the following cost function can be performed source by source

$$Q = \|\mathbf{E}_N^H \mathbf{a}(\theta)\|^2 = \|\mathbf{q}(\theta)\|^2 \quad (11)$$

where \mathbf{E}_N and $\mathbf{a}(\theta)$ are defined in (3) and (4), respectively. Each DOA estimate is calculated iteratively as

$$\theta_{l,n+1} = \theta_{l,n} - \left[\text{Re} \left\{ \mathbf{u}^H(\theta_{l,n}) \mathbf{u}(\theta_{l,n}) \right\} \right]^{-1} \text{Re} \left\{ \mathbf{u}^H(\theta_{l,n}) \mathbf{q}(\theta_{l,n}) \right\} \quad (12)$$

$$\mathbf{u}(\theta_{l,n}) = \frac{\partial \mathbf{q}(\theta_{l,n})}{\partial \theta} = \mathbf{E}_N^H \frac{\partial \mathbf{a}(\theta_{l,n})}{\partial \theta} \quad (13)$$

where $\theta_{l,n}$ is the DOA estimate of the l -th source at iteration n . Since each DOA is calculated source by source in (12), their initial values must be different from each other.

B. Iterative MODE

The cost function for the MODE algorithm[2] that is minimized can be described in the following form

$$F = \left\| \left\{ \mathbf{I} - \mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \right\} \mathbf{E}_S \mathbf{W}^{1/2} \right\|_F^2 = \|\mathbf{P} \mathbf{E}_W\|_F^2 = \|\mathbf{r}(\boldsymbol{\theta})\|^2 \quad (14)$$

$$\mathbf{W} = (\boldsymbol{\Lambda}_S - \sigma^2 \mathbf{I})^2 \boldsymbol{\Lambda}_S^{-1} \quad (15)$$

$$\mathbf{P} = \mathbf{I} - \mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \quad (16)$$

$$\mathbf{E}_W = \mathbf{E}_S \mathbf{W}^{1/2} \quad (17)$$

$$\mathbf{r}(\boldsymbol{\theta}) = \text{vec}\{\mathbf{P} \mathbf{E}_W\} \quad (18)$$

where the matrices \mathbf{E}_S and $\boldsymbol{\Lambda}_S$ are defined in (2), and $\|\cdot\|_F$ and $\text{vec}\{\cdot\}$ denote the Frobenius norm and operator stacking the columns of a matrix on top of each other, respectively. The expressions for Gauss-Newton iterative approach can be written as

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n - \left[\text{Re}\{\mathbf{G}^H(\boldsymbol{\theta}_n) \mathbf{G}(\boldsymbol{\theta}_n)\} \right]^{-1} \text{Re}\{\mathbf{G}^H(\boldsymbol{\theta}_n) \mathbf{r}(\boldsymbol{\theta}_n)\} \quad (19)$$

$$\mathbf{G}(\boldsymbol{\theta}_n) = [\mathbf{g}_1(\boldsymbol{\theta}_n) \ \mathbf{g}_2(\boldsymbol{\theta}_n) \ \dots \ \mathbf{g}_L(\boldsymbol{\theta}_n)] \quad (20)$$

$$\mathbf{g}_l(\boldsymbol{\theta}_n) = \frac{\partial \mathbf{r}(\boldsymbol{\theta}_n)}{\partial \theta_l} = -\text{vec}\{(\mathbf{P} \mathbf{D}_l \mathbf{A}^\dagger + \mathbf{A}^{\dagger H} \mathbf{D}_l^H \mathbf{P}) \mathbf{E}_W\} \quad (21)$$

$$\mathbf{D}_l = \frac{\partial \mathbf{A}(\boldsymbol{\theta}_n)}{\partial \theta_l} \quad (22)$$

$$\mathbf{A}^\dagger = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \quad (23)$$

where $\boldsymbol{\theta}_n$ is the estimate at iteration n . The steps involved in this calculation are similar to those in [4].

4 Computer Simulations

A UCA of radius $r = \lambda/2$ with $M = 4$ being the maximum phase mode excited, was employed for the simulations. The number of array elements was chosen to be $K = 12$. In each of the simulation examples outlined below, two equipowered sources were located at 0° and 20° , 100 samples of data were taken from the array, and algorithm performance for each case was analyzed based on an average over 1000 independent trials. Gauss-Newton method in the iterative MUSIC and MODE requires initial values for the DOA searches, and these values were obtained from the estimates of UCA-MODE.

The first example is for the uncorrelated sources with SNR = 20 dB. The root mean square errors (RMSE) of the estimates versus the iteration number are plotted in Fig.2, together with the Cramer-Rao bound (CRB). The results for iteration 0 are the statistics of the initial estimates by UCA-MODE. In the second example, we used the same scenario as in the first one but the sources are correlated with each other (a correlation coefficient of 0.95). Fig.3 displays the convergence property of the iterative algorithms. Figs.2 and 3 clearly demonstrate an improved performance of our techniques, except for the iterative MUSIC method in the case of the highly correlated sources. The Gauss-Newton search methods based on MUSIC and MODE fully converge in only 1 or 2 iterations.

Next, the performance of the algorithms was discussed at various SNRs and the results are plotted in Figs.4 and 5 for uncorrelated sources and correlated sources with a correlation coefficient of 0.95, respectively. The stopping criteria for iterative MUSIC and MODE were $|\text{Re}\{\mathbf{u}^H(\theta_{l,n}) \mathbf{q}(\theta_{l,n})\}| < 10^{-5}$

and $\|\text{Re}\{\mathbf{G}^H(\boldsymbol{\theta}_n)\mathbf{r}(\boldsymbol{\theta}_n)\}\| < 10^{-5}$, respectively. The iteration numbers to stop the Gauss-Newton methods are depicted in Fig.6. In Fig.4, the iterative MODE achieves the CRB in all cases of SNR, while the iterative MUSIC is unsuccessfully above the CRB for lower SNRs. From Fig.5, it turns out that the performance of iterative MODE is insensitive to correlation between sources, and achieves the CRB for SNR beyond about 5 dB. In the case of the sources being low SNR or highly correlated, iterative MUSIC is unable to resolve the DOAs, so that the Gauss-Newton search tends to fail, even if the initial values are close to the optimum points.

5 Conclusion

We have presented the iterative MUSIC and MODE algorithms using the Gauss-Newton method. It is clarified the iterative algorithms have reasonable properties in terms of estimation accuracy and computational cost. Especially, the iterative MODE has excellent performance for highly correlated sources.

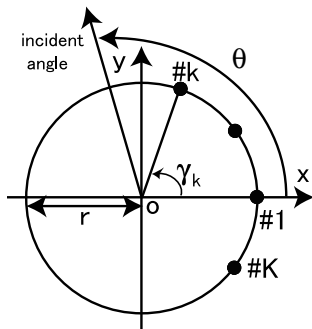


Figure 1: Uniform circular array geometry.

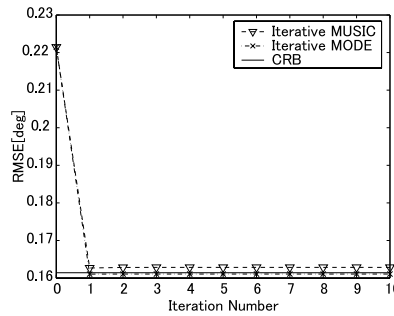


Figure 2: RMSE vs. iteration number for uncorrelated sources.

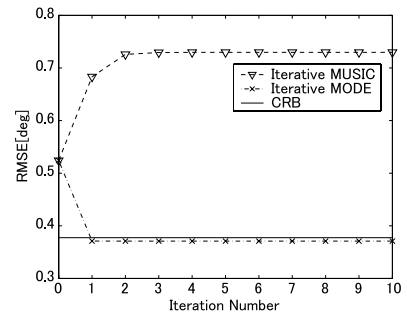


Figure 3: RMSE vs. iteration number for correlated sources.

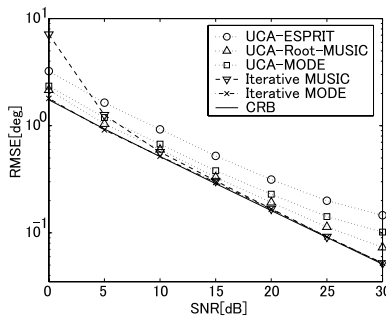


Figure 4: RMSE vs. SNR for uncorrelated sources.

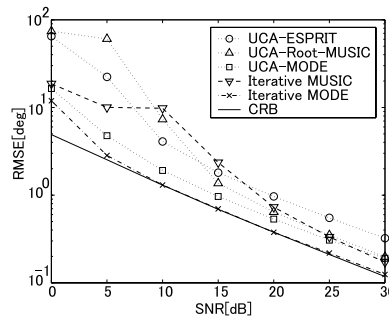


Figure 5: RMSE vs. SNR for correlated sources.

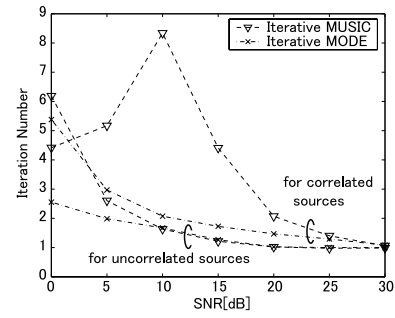


Figure 6: Iteration number required for convergence vs. SNR.

References

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