Performance Analysis of Unitary Capon Method for DOA Estimation with High Computational Efficiency

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1. Introduction

Recent development of wireless communications is remarkable as observed in the increased users of cellular phones, and at the same time, various kinds of radio waves make the radio environments much complicated. Therefore, it is important to understand the radio wave propagation structures in detail. For the purpose, it is most effective to estimate the signal parameters (e.g., DOA: directions of arrival) of individual incoming waves in the wireless systems. As one of the high-resolution DOA estimators, Capon method [1], [2] has the most excellent characteristics of the beam scanning schemes. Furthermore, it does not require eigendecomposition of the covariance matrix, which leads to the low computational load of the method.

In this paper, we present Unitary Capon estimator which has the real-valued formulation via Unitary transformation exploiting the centro-Hermitian property of the array [3]. Using this transformation will enable us to expect not only the higher computation efficiency but also the higher estimation accuracy of the estimator. In addition, the performance analysis of the presented method in DOA estimation is carried out in comparison with standard Capon estimator (conventional method) through computer simulation.

2. Principle of Unitary Capon Method

2.1 Data Formulation for DOA Estimation

Figure 1 depicts the K-element uniform linear array (ULA) with an antenna spacing of Δ. Assume that L waves with the complex envelopes $s_1(t)$, $s_2(t)$,..., $s_L(t)$ are incident on the array along the angles θ_1, θ_2,..., θ_L, respectively. Then, the array input vector $\boldsymbol{x}(t) = [x_1(t), x_2(t), \cdots, x_K(t)]^T$ is expressed as

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_L)] \tag{2}$$

$$\boldsymbol{a}(\theta_l) = \left[\exp\left\{-j\frac{2\pi}{\lambda}\Delta\left(1-\frac{K+1}{2}\right)\sin\theta_l\right\}, \cdots, \exp\left\{-j\frac{2\pi}{\lambda}\Delta\left(K-\frac{K+1}{2}\right)\sin\theta_l\right\}\right]^T \quad (l = 1, 2, \cdots, L) \tag{3}$$

$$\boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_L(t)]^T \tag{4}$$

where $\boldsymbol{a}(\theta_l)$ is the array response vector of the lth wave with the phase reference at the center of the array, $\boldsymbol{A}$ is the array response matrix, λ is the wavelength of the carrier, and $\boldsymbol{n}(t)$ is the internal noise vector. Here, it is assumed that each antenna element is isotropic and there is no mutual coupling effect among antenna elements.

Using the weight vector $\boldsymbol{w} = [w_1, w_2, \cdots, w_K]^T$, the array output $y(t)$ and the array output power P_{out} are given by

ISBN: 89-86522-77-2 94460©KEES

$$y(t) = \boldsymbol{w}^H \boldsymbol{x}(t) \tag{5}$$

$$P_{out} = \frac{1}{2}E\left[|y(t)|^2\right] = \frac{1}{2}\boldsymbol{w}^H \boldsymbol{R}_{xx}\boldsymbol{w} \tag{6}$$

$$\boldsymbol{R}_{xx} = E\left[\boldsymbol{x}(t)\boldsymbol{x}^H(t)\right] = \boldsymbol{ASA}^H + \sigma^2 \boldsymbol{I} \tag{7}$$

$$\boldsymbol{S} = E\left[\boldsymbol{s}(t)\boldsymbol{s}^H(t)\right] \tag{8}$$

where $\boldsymbol{R}_{xx}$ is the correlation (covariance) matrix of the array input, $\boldsymbol{S}$ is the correlation matrix of L sources, σ^2 is the noise power, $\boldsymbol{I}$ is the identity matrix, and $E[\cdot]$ denotes expectation operator.

2.2 Angular Spectrum of Unitary Capon Method

The array output power contains contributions from the desired signal along the look direction as well as the undesired ones along other directions of arrival. To minimize the contributions of the undesired signals, the array output power is minimized while maintaining the gain along the look direction to be constant. This is the principle of Capon method and it is written in the following form.

$$\min_{\boldsymbol{w}} \left(P_{out} = \frac{1}{2}\boldsymbol{w}^H \boldsymbol{R}_{xx}\boldsymbol{w} \right) \quad \text{subject to} \quad \boldsymbol{w}^H \boldsymbol{a}(\theta) = 1 \tag{9}$$

Thus, the angular spectrum of Capon method is expressed as the output power by the optimum weight vector, which is given by [1], [2]

$$P_{CP}(\theta) = \frac{1}{\boldsymbol{a}^H(\theta)\boldsymbol{R}_{xx}^{-1}\boldsymbol{a}(\theta)} \tag{10}$$

Using an appropriate Unitary matrix $\boldsymbol{Q}_K$ (i.e., $\boldsymbol{Q}_K\boldsymbol{Q}_K^H = \boldsymbol{I}$) like the following matrix for an example of $K = 2M$(even), the array response vector $\boldsymbol{a}(\theta)$ is transformed into the real-valued vector $\boldsymbol{d}(\theta) = \boldsymbol{Q}_K^H\boldsymbol{a}(\theta)$ [3].

$$\boldsymbol{Q}_K = \frac{1}{\sqrt{2}}\begin{bmatrix} \boldsymbol{I}_M & j\boldsymbol{I}_M \\ \boldsymbol{\Pi}_M & -j\boldsymbol{\Pi}_M \end{bmatrix}, \qquad \boldsymbol{\Pi}_M = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ 0 & \cdots & 1 & 0 \\ \vdots & . & \vdots & \vdots \\ 1 & \cdots & 0 & 0 \end{bmatrix} \in R^{M\times M} \tag{11}$$

where $\boldsymbol{I}_M$ denotes the identity matrix of dimension M. Via this transformation, the problem formulation of Unitary Capon method is derived as follows.

$$\min_{\boldsymbol{w}} \left(P_{out} = \frac{1}{2}\boldsymbol{w}^H \boldsymbol{Q}_K\boldsymbol{Q}_K^H \boldsymbol{R}_{xx}\boldsymbol{Q}_K\boldsymbol{Q}_K^H\boldsymbol{w} \right) \quad \text{subject to} \quad \boldsymbol{w}^H \boldsymbol{Q}_K\boldsymbol{Q}_K^H\boldsymbol{a}(\theta) = 1 \tag{12}$$

$$\Downarrow$$

$$\min_{\boldsymbol{v}} \left(P_{out} = \frac{1}{2}\boldsymbol{v}^T \boldsymbol{R}_{yy}\boldsymbol{v} \right) \quad \text{subject to} \quad \boldsymbol{v}^T \boldsymbol{d}(\theta) = 1 \tag{13}$$

Here, $\boldsymbol{R}_{yy}$ is the real-valued matrix defined by $\boldsymbol{R}_{yy} = \text{Re}\left[\boldsymbol{Q}_K^H\boldsymbol{R}_{xx}\boldsymbol{Q}_K\right]$, and $\boldsymbol{v}$ is a real-valued vector generated from $\boldsymbol{Q}_K^H\boldsymbol{w}$. Then, the angular spectrum of Unitary Capon method is given by

$$P_{UC}(\theta) = \frac{1}{\boldsymbol{d}^T(\theta)\boldsymbol{R}_{yy}^{-1}\boldsymbol{d}(\theta)} \tag{14}$$

2.3 Successive DOA Estimation

As found from (14), Unitary Capon method requires the inverse matrix of $\boldsymbol{R}_{yy}$. Normally, $\boldsymbol{R}_{yy}$ is calculated using N_s snapshots: $\boldsymbol{x}(1), \cdots, \boldsymbol{x}(N_s)$, which is given by

$$\boldsymbol{R}_{yy} = \frac{1}{N_s}\sum_{m=1}^{N_s} \text{Re}\left[\boldsymbol{Q}_K^H\boldsymbol{x}(m)\boldsymbol{x}^H(m)\boldsymbol{Q}_K\right] \tag{15}$$

On the other hand, we also try to utilize the following update expression of $\boldsymbol{R}_{yy}^{-1}$ derived from $\boldsymbol{R}_{yy}(m) = \beta\boldsymbol{R}_{yy}(m-1) + (1-\beta)\left[\boldsymbol{y}_r(m)\boldsymbol{y}_r^T(m) + \boldsymbol{y}_j(m)\boldsymbol{y}_j^T(m)\right]$ using $\boldsymbol{y}_r(m) = \mathrm{Re}\left[\boldsymbol{Q}_K^H\boldsymbol{x}(m)\right]$ and $\boldsymbol{y}_j(m) = \mathrm{Im}\left[\boldsymbol{Q}_K^H\boldsymbol{x}(m)\right]$.

$$\begin{cases} \boldsymbol{R}_{yy}^{-1}(m) = \boldsymbol{R}_r^{-1}(m) - \dfrac{(1-\beta)\boldsymbol{R}_r^{-1}(m)\boldsymbol{y}_j(m)\boldsymbol{y}_j^T(m)\boldsymbol{R}_r^{-1}(m)}{1+(1-\beta)\boldsymbol{y}_j^T(m)\boldsymbol{R}_r^{-1}(m)\boldsymbol{y}_j(m)} \\ \boldsymbol{R}_r^{-1}(m) = \dfrac{1}{\beta}\boldsymbol{R}_{yy}^{-1}(m-1) - \dfrac{(1-\beta)\boldsymbol{R}_{yy}^{-1}(m-1)\boldsymbol{y}_r(m)\boldsymbol{y}_r^T(m)\boldsymbol{R}_{yy}^{-1}(m-1)}{\beta^2+\beta(1-\beta)\boldsymbol{y}_r^T(m)\boldsymbol{R}_{yy}^{-1}(m-1)\boldsymbol{y}_r(m)} \end{cases} \tag{16}$$

where β is the forgetting factor ($0 < \beta < 1$), and m means here the iteration number for update. In this way, we can perform the successive DOA estimation using Unitary Capon method. The angular spectrum of standard Capon method can also be computed from $\boldsymbol{R}_{xx}$ in the same way as the above.

3. Computer Simulation

The simulation conditions are listed in Table 1 and the radio environments are given in Tables 2–4. First, Unitary Capon method is compared in the radio environment (1) with standard Capon method. Figure 2 shows the averaged angular spectrums over 100 independent trials in the case of the number of snapshots $N_s = 3$. Moreover, Fig. 3 shows the relations between the root-mean-square errors (RMSE) of all DOA estimates computed from 100 trials and the number of snapshots. It is verified that Unitary Capon method displays a marked tendency to give better performance than standard Capon method when the number of snapshots is less than about 10. Figure 4 shows RMSE of DOA estimates when the DOA of the first incoming wave is changed from 5° to 20° as in the radio environment (2) for $N_s = 6$. It can be confirmed that Unitary Capon method has a resolution capability that is higher than standard Capon method.

Table 1: Simulation conditions

array geometry	6-element uniform linear array
antenna spacing	half wavelength
number of waves	1 to 3
SNR	20 [dB] (for the 1st wave)
number of trials	100

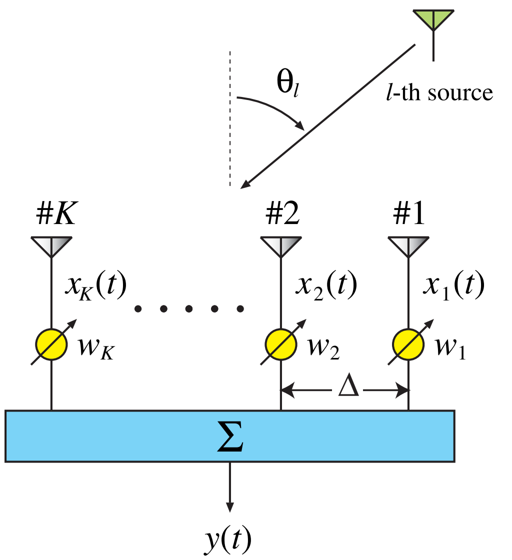

Figure 1: K-element uniform linear array with an antenna spacing of Δ.

Table 2: Radio environment (1)

	DOA	power
1st wave	−20°	1.0
2nd wave	0°	1.0
3rd wave	60°	0.5

Table 3: Radio environment (2)

	DOA	power
1st wave	5° to 20°	1.0
2nd wave	0°	1.0

Table 4: Radio environment (3)

	DOA	power
1st wave	$0° + 0.1°(m-1)$	1.0

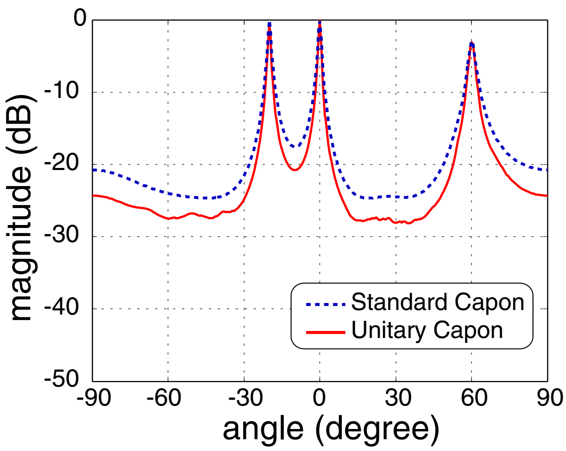

Figure 2: Averaged angular spectrums ($N_s = 3$).

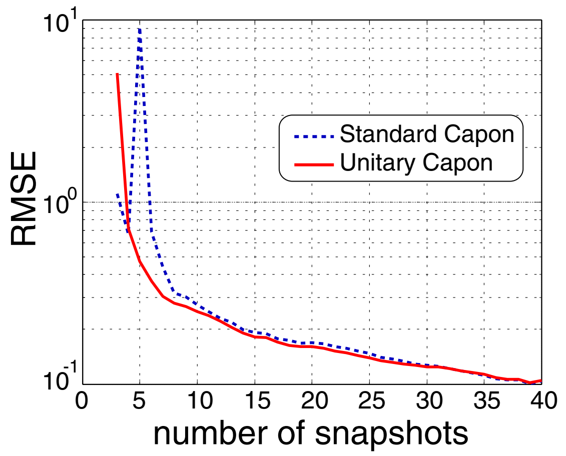

Figure 3: The relation between RMSE and number of snapshots N_s.

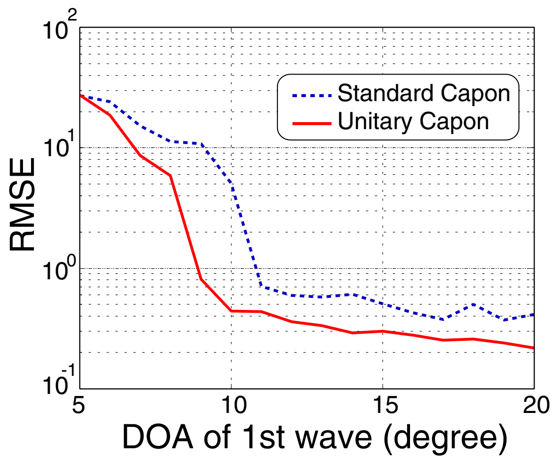

Figure 4: The relation between RMSE and DOA of the 1st wave ($N_s = 6$).

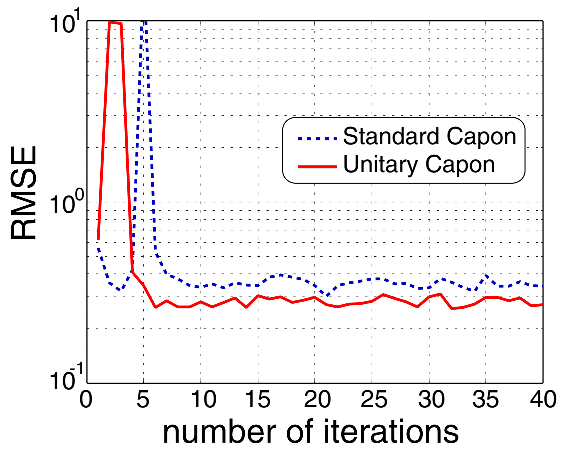

Figure 5: The relation between RMSE and number of iterations in the case of successive DOA estimation ($\beta = 0.5$).

The estimation results when the successive DOA estimation is performed for the radio environment (3) are given by Fig. 5. The relations between RMSE and the number of iterations are shown in the figure. The forgetting factor is set to be $\beta = 0.5$. To calculate the inverse of initial correlation matrices $\boldsymbol{R}_{yy}(1)$ and $\boldsymbol{R}_{xx}(1)$, we added mathematically 10^{-8} to their diagonal components. From the figure, it is understood that Unitary Capon method gives a better estimation performance than standard Capon method even if the DOA of incoming wave changes iteration by iteration as in this simulation.

4. Conclusion

This paper has presented Unitary Capon method for efficient DOA estimation. As a result of performance analysis via computer simulation, it is clarified that Unitary Capon method gives higher estimation accuracy and higher computation efficiency than standard Capon method. As the future works, comparative study with other DOA estimators will be given.

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