MUTUAL COUPLING COMPENSATION IN ARRAY ANTENNA FOR HIGH-RESOLUTION DOA ESTIMATION

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1 Introduction

Direction of arrival (DOA) estimation is an important feature of smart antenna arrays. Several algorithms have been proposed for high-resolution DOA estimation such as the MUSIC algorithm[1]. From a signal processing point of view, these so-called superresolution techniques have excellent resolution capability, however, array calibration is indispensable to realize the performance. In real arrays, calibration of mutual coupling among elements is a difficult task. Several calibration techniques have been proposed to obtain an accurate calibration matrix[2],[3]. These techniques require external reference waves or knowledge of current distributions on the elements, hence they sometimes become difficult to apply.

For arrays with single-mode elements such as half-wave length dipoles, the calibration matrix derived by the Thévenin equivalent circuit becomes a good approximation[4]. The calibration matrix of this method can be estimated by terminal current and voltage of each element. Modified method can be found in [5]. While this type of methods is preferable in practical array calibration, there still remain calibration errors that affect DOA estimation even in calibration of hale-wavelength dipole arrays.

In this report, we proposed a new calibration method that can be also derived by using terminal currents and voltages of the elements. Definition of self and mutual impedances in the equivalent circuit is modified in this derivation. Performance of the proposed technique is verified numerically by DOA estimation results of the MUSIC algorithm in coherent and incoherent signal environments.

2 Mutual Couping Conpensation in Receiving Array

We consider an array of single-mode elements, meaning that the element aperture currents may change in amplitude but not in shape as a function of DOA of incident waves. In this report, mutual coupling compensation problem of a dipole array shown in Fig.1 is considered. When d plane waves impinge on the array, the received data vector can be given by

$$\boldsymbol{r} = [r_1, r_2, \cdots, r_N]^T = \boldsymbol{CAs} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{A}$ and $\boldsymbol{s}$ denote the $N \times d$ mode-matrix including d mode-vectors and the source vector, respectively. $\boldsymbol{n}$ is the additive noise vector and ^{T} denotes transpose. The matrix C denotes a mutual coupling matrix whose element shows coupling coefficient between elements.

When $\boldsymbol{C}$ is known, the calibrated covariance can be obtained by

$$\boldsymbol{R}_{\text{cal}} = \boldsymbol{C}^{-1}(E[\boldsymbol{rr}^H] - \sigma^2\boldsymbol{I})(\boldsymbol{C}^H)^{-1} \tag{2}$$

where $E[\cdot]$ and σ^2 denote ensemble average and the noise power, respectively. This is the basic procedure of mutual coupling compensation, or calibration, for the DOA estimation. The problem is how to estimate the mutual coupling matrix $\boldsymbol{C}$.

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2.1 Conventional Calibration Matrix

Gupta *et. al.,* derived, by using Thévenin equivalent circuit, that the relation between received/terminal voltages $\boldsymbol{v}$ and open circuit voltages $\boldsymbol{v}_{\text{open}}$ of an array can be expressed by $\boldsymbol{Z}_c\boldsymbol{v} = \boldsymbol{v}_{\text{open}}$, where

$$\boldsymbol{Z}_c = \begin{bmatrix} 1+\frac{Z_{11}}{Z_L} & \frac{Z_{12}}{Z_L} & \cdots & \frac{Z_{1N}}{Z_L} \\ \frac{Z_{21}}{Z_L} & 1+\frac{Z_{22}}{Z_L} & & \frac{Z_{2N}}{Z_L} \\ \vdots & & \ddots & \vdots \\ \frac{Z_{N1}}{Z_L} & \frac{Z_{N2}}{Z_L} & \cdots & 1+\frac{Z_{NN}}{Z_L} \end{bmatrix}. \tag{3}$$

Z_{ii} and Z_{ij} is the self and the mutual impedance of equivalent circuit of the array. Z_L is the load impedance. Applying this result, we can obtain the calibration matrix by $\boldsymbol{C}^{-1} = \boldsymbol{Z}_c$. As several researchers reported, the open circuit voltages are not coupling free in real arrays and as a result the calibration matrix is biased. Hui proposed the modified method[5], however, the matrix is still slightly biased.

2.2 Proposed Calibration Matrix

Several reports on problem of equivalent circuits for a receiving antenna can be found recently[6]. These reports show that discrepancy of power dissipation in the network derived by the circuit, especially when there exist reradiated/scattering objects. This may also affects derivation of mutual coupling coefficient of a receiving array.

To evaluate the power dissipation correctly, we propose to use the following definition:

$$\begin{bmatrix} v'_1 \\ v'_2 \\ \vdots \\ v'_j \\ \vdots \\ v'_N \end{bmatrix} = \begin{bmatrix} -Z_L i_{1j} \\ -Z_L i_{2j} \\ \vdots \\ V_g - Z_L i_{jj} \\ \vdots \\ -Z_L i_{Nj} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z^s_{12} & \cdots & Z_{1j} & \cdots & Z^s_{1N} \\ Z^s_{21} & Z_{22} & \cdots & Z_{2j} & \cdots & Z^s_{2N} \\ \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ Z^s_{j1} & Z^s_{j2} & \cdots & Z_{jj} & & Z^s_{jN} \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ Z^s_{N1} & Z^s_{N2} & \cdots & Z_{Nj} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} i_{1j} \\ i_{2j} \\ \vdots \\ i_{jj} \\ \vdots \\ i_{Nj} \end{bmatrix} \tag{4}$$

The equation corresponds to the case when j-th element is excited by induced voltage V_g. v'_i and i_{ij} is the terminal voltage and current of i-th element in this excitation. In this definition, we allow to have different values in the mutual impedances to make power dissipation correct, that is $Z^s_{ij} \neq Z_{ij}$. Note that Z^s_{ij} and Z_{ij} relate to mutual impedance of the reradiate and the transmitted element, respectively, and the mutual impedances in j-th column of the matrix become Z_{ij}s.

By using the definition in (4), the calibration matrix of a receiving array can be derived by

$$\boldsymbol{C}^{-1} = \boldsymbol{Z}^s_c = \begin{bmatrix} 1+\frac{Z_{11}}{Z_L} & \frac{Z^s_{12}}{Z_L} & \cdots & \frac{Z^s_{1N}}{Z_L} \\ \frac{Z^s_{21}}{Z_L} & 1+\frac{Z_{22}}{Z_L} & & \frac{Z^s_{2N}}{Z_L} \\ \vdots & & \ddots & \vdots \\ \frac{Z^s_{N1}}{Z_L} & \frac{Z^s_{N2}}{Z_L} & \cdots & 1+\frac{Z_{NN}}{Z_L} \end{bmatrix} = \boldsymbol{I} + \frac{1}{Z_L}\boldsymbol{Z}_s. \tag{5}$$

This shows that the calibration matrix changes when $Z_{ij} \neq Z^s_{ij}$ and the reradiation mutual impedances should be used for the calibration. These impedances can be estimated by a receiving array alone. When each port is excited by V_g separately, the following equation can be obtained.

$$diag\{V_g, \cdots, V_g\} =$$

$$= \quad (\boldsymbol{Z}_m + Z_L\boldsymbol{I}) \begin{bmatrix} i_{11} & & \text{\huge 0} \\ & \ddots & \\ \text{\huge 0} & & i_{NN} \end{bmatrix} + (\boldsymbol{Z}_s + Z_L\boldsymbol{I}) \begin{bmatrix} 0 & i_{12} & \cdots & i_{1N} \\ i_{21} & 0 & & i_{2N} \\ \vdots & & \ddots & \vdots \\ i_{N1} & i_{N2} & \cdots & 0 \end{bmatrix} \tag{6}$$

where $\boldsymbol{Z}_m$ is the impedance matrix whose non-diagonal elements are the Z_{ij}, not Z^s_{ij}. This is the N^2 simultaneous equations having N^2 unknowns $(Z_{ii}, Z_{ij}, Z^s_{ij})$. For uniform linear arrays (ULA), only $\left\lfloor \frac{N^2}{2} \right\rfloor$ equations are independent because of array symmetry. Number of unknowns also becomes $2N-1$ since both $\boldsymbol{Z}_m$ and $\boldsymbol{Z}_s$ become Toeplitz matrices. Therefore, all impedances can be estimated when $N > 2$. Note that the self-impedances, Z_{ii}, are different from those derived by the conventional definition. Numerical results show that each self-impedance in this definition almost corresponds to the input impedance of the isolated element.

3 Simulation Results

Calibration performance of the proposed technique is verified numerically by using the Method of Moments. A 4-element ULA of dipoles is employed in these simulations. The array parameters are listed in Table.1. Here we apply the Hui's calibration method[5] as the conventional method and compare the calibration performance of the proposed and the conventional methods by MUSIC spectrums. We omit the noise component in (1) to remove the noise and snapshots effect.

Figure 2 shows the DOA estimation results of 2 data sets. The first set has incoherent two incident waves coming from -20° and 0°, and the second has the waves from 30° and 60°. All of the waves have the same power. As can be seen in this figure, the peaks of MUSIC spectrum become dull and biased. Furthermore, corresponding peak often disappears without the calibration. The Hui's calibration method works well at broadside direction, however, the detected peak for the wave from 60° becomes small and biased. On the other hand, the MUSIC spectrums obtained by the proposed calibration technique show sharp peaks and almost no bias.

The second example is the DOA estimation of coherent waves. The Spatial Smoothing Preprocessing[7] is applied to the calibrated covariance matrix in (2) before the MUSIC analysis. In each processing, number of the subarray is 2 (number of elements in each subarray is 3). The estimated MUSIC spectrums are shown in Fig.3. DOAs of the waves are the same in Fig.2. In the incoherent case, peaks of the dataset for broadside waves ($-20°\&0°$) can be almost correctly detected without calibration, however these waves cannot be resolved after the SSP in this coherent simulation. This figure also shows that the SSP with the Hui's calibration often deteriorates the DOA estimation accuracy of the MUSIC. Bias of the estimation results by the proposed method is acceptably small in these simulations. As shown in these examples, precise calibration is important especially for the coherent signal detection with the SSP.

4 Conclusions

In this report, we propose a new calibration method by using the modified mutual impedances. The numerical results show that accuracy of the calibration matrix estimated by the proposed definition can be improved. By using the method, mutual coupling effect of single-mode arrays can be calibrated effectively with measured parameters (voltages and currents or S parameters) of the receiving array without external reference plane waves. Therefore, the proposed method would be useful for high-resolution DOA estimation by arrays with single-mode elements.

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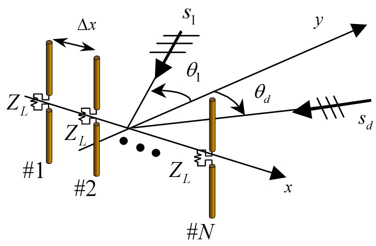

Fig.1: Configulation of the dipole array.

Table.1 Array Parameters

Frequency	2.4 GHz
Length of wire	5.8 cm (0.464 λ)
Radius of wire	0.5 mm
Load impedance	50 Ω
Element separation	6 cm (0.48 λ)

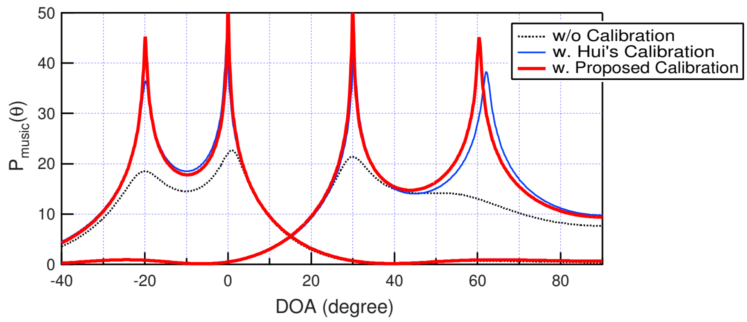

Fig.2: DOA Estimation Results of MUSIC algorithm for incoherent two wave incidence.

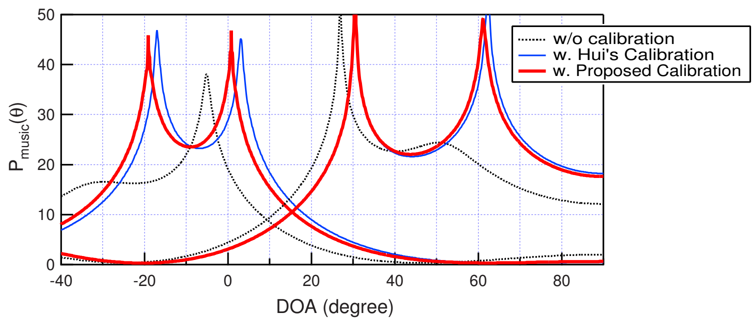

Fig.3: DOA Estimation Results of MUSIC algorithm for coherent two wave incidence.