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TM -Field Radiation of Rotationally Symmetric Inhomogeneous Structures

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Abstract
This paper presents a Finite Integration Technique (FIT) solution of electromagnetic radiation problems involving rotationally symmetric inhomogeneous domains. Analysis is restricted to rotationally symmetric TM fields. Discretized Maxwell's equations are given for a sample problem and matrix formulations deduced from the discretized equations. Using diagonal local impedance matrices, different boundary conditions are implemented by changing impedance matrix elements. Since this approach emphasizes the matrix nature of the discretized problem, MATLAB software was used for the FIT implementation. Results are presented for radiated fields of dielectric resonators excited by electrically small loops.

Introduction

Finite Integration Technique (FIT) [1.] is a dual technique to Finite Difference (FD) technique. FIT uses integral forms of Maxwell's equations whereas FD uses differential forms. Application of the FIT to electromagnetic field problems involving rotationally symmetric domains has been presented in [2.]. Whereas in [2.] inhomogeneous *interior* boundary value problems were solved using FIT, this paper addresses the solution of inhomogeneous *exterior* boundary value problems. Open boundary conditions are implemented in order to solve radiation of rotationaly symmetric sources using FIT. The paper starts with the matrix formulation of the discretized integral curl equations of Maxwell for rotationally symmetric electromagnetic fields. Matrix equations are formulated for TM-to-ϕ fields with components E_ϕ, H_r, and H_z. Rotationally symmetric sources are implemented as current loops whose currents do not vary with angle ϕ. All equations are derived for uniform square grids and rotationally symmetric domains with a rectangular cross-sections.

Sample FIT Equations for TM Fields

We start with TM field FIT equations for a 4 by 3 square ($\Delta r=\Delta z=\Delta l$) cell cross-section of a rotationally symmetric domain shown below. Note that, in principle, each cell can be of different permittivity.

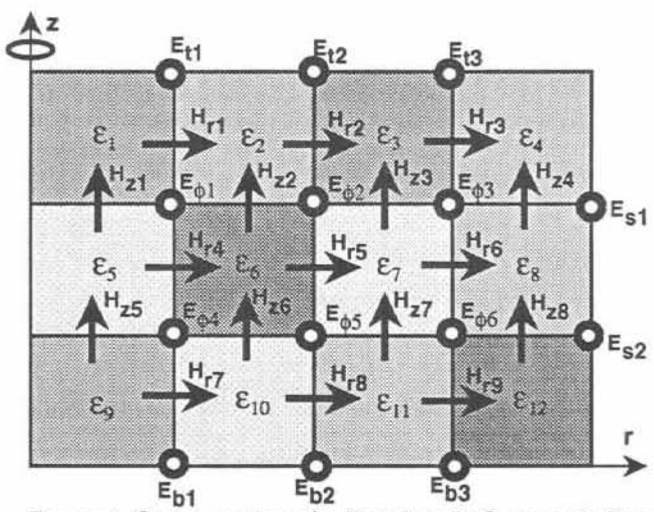

Figure 1: Cross-section of a Rotationally Symmetric Domain

The discretized (FIT) equations are written by approximating the line and surface integrals in the curl equations of Maxwell for each of the cells, the details of which are given in [2.]. Both curl equations (circulation of E and circulation of H) are used to write the FIT equations. Normalized frequency ω_n is introduced as $\omega_n=\omega\,(\Delta l)/c$, where Δl is the grid step and c is the speed of light in free space. The impedance and the admittance of free space are denoted Z_0 and Y_0, respectively. The number of cells in radial direction is denoted M, and the number of cells in the axial direction is denoted N.

Circulation of Electric Field

$E_{\phi1} - E_{t1} = -j\omega_n Z_0 H_{r1}$
$E_{\phi2} - E_{t2} = -j\omega_n Z_0 H_{r2}$
$E_{\phi3} - E_{t3} = -j\omega_n Z_0 H_{r3}$
$E_{\phi4} - E_{\phi1} = -j\omega_n Z_0 H_{r4}$
$E_{\phi5} - E_{\phi2} = -j\omega_n Z_0 H_{r5}$
$E_{\phi6} - E_{\phi3} = -j\omega_n Z_0 H_{r6}$
$E_{b1} - E_{\phi4} = -j\omega_n Z_0 H_{r7}$
$E_{b2} - E_{\phi5} = -j\omega_n Z_0 H_{r8}$
$E_{b3} - E_{\phi6} = -j\omega_n Z_0 H_{r9}$

$1E_{\phi1} = -j\omega_n Z_0 (1/2)H_{z1}$
$2E_{\phi2} - 1E_{\phi1} = -j\omega_n Z_0 (3/2)H_{z2}$
$3E_{\phi3} - 2E_{\phi2} = -j\omega_n Z_0 (5/2)H_{z3}$
$4E_{s1} - 3E_{\phi3} = -j\omega_n Z_0 (7/2)H_{z4}$
$1E_{\phi4} = -j\omega_n Z_0 (1/2)H_{z5}$
$2E_{\phi5} - 1E_{\phi4} = -j\omega_n Z_0 (3/2)H_{z6}$
$3E_{\phi6} - 2E_{\phi5} = -j\omega_n Z_0 (5/2)H_{z7}$
$4E_{s2} - 3E_{\phi6} = -j\omega_n Z_0 (7/2)H_{z8}$

Circulation of Magnetic Field

$H_{r1} - H_{r4} + H_{z1} - H_{z2} = j\omega_n Y_0 \varepsilon_{\phi1} E_{\phi1} + I_{\phi1}$
$H_{r2} - H_{r5} + H_{z2} - H_{z3} = j\omega_n Y_0 \varepsilon_{\phi2} E_{\phi2} + I_{\phi2}$
$H_{r3} - H_{r6} + H_{z3} - H_{z4} = j\omega_n Y_0 \varepsilon_{\phi3} E_{\phi3} + I_{\phi3}$
$H_{r4} - H_{r7} + H_{z5} - H_{z6} = j\omega_n Y_0 \varepsilon_{\phi4} E_{\phi4} + I_{\phi4}$
$H_{r5} - H_{r8} + H_{z6} - H_{z7} = j\omega_n Y_0 \varepsilon_{\phi5} E_{\phi5} + I_{\phi5}$
$H_{r6} - H_{r9} + H_{z7} - H_{z8} = j\omega_n Y_0 \varepsilon_{\phi6} E_{\phi6} + I_{\phi6}$

where the following averaged permittivities have been introduced:

$\varepsilon_{\phi1} = (\varepsilon_1 + \varepsilon_2 + \varepsilon_5 + \varepsilon_6)/4$ $\quad \varepsilon_{\phi2} = (\varepsilon_2 + \varepsilon_3 + \varepsilon_6 + \varepsilon_7)/4$ $\quad \varepsilon_{\phi3} = (\varepsilon_3 + \varepsilon_4 + \varepsilon_7 + \varepsilon_8)/4$
$\varepsilon_{\phi4} = (\varepsilon_5 + \varepsilon_6 + \varepsilon_9 + \varepsilon_{10})/4$ $\quad \varepsilon_{\phi5} = (\varepsilon_6 + \varepsilon_7 + \varepsilon_{10} + \varepsilon_{11})/4$ $\quad \varepsilon_{\phi6} = (\varepsilon_7 + \varepsilon_8 + \varepsilon_{11} + \varepsilon_{12})/4$

and the source current densities (in A/m) have been co-located with the electric fields. These equations can be written in a matrix form as well. The matrices for a general case of an M by N cross-section can be deduced from the equations above and matrix equations written as:

$$\mathbf{C}_{\phi r}\,\mathbf{e}_\phi = -j\omega_n Z_0\,\mathbf{h}_r - \mathbf{D}_{bt}\,\mathbf{e}_{bt} \qquad (1)$$
$$\mathbf{C}_{\phi z}\,\mathbf{R}_{e\phi}\,\mathbf{e}_\phi = -j\omega_n Z_0\,\mathbf{R}_{hz}\,\mathbf{h}_z - \mathbf{D}_S\,\mathbf{e}_s \qquad (2)$$
$$(\mathbf{C}_{\phi r})^T\mathbf{h}_r + (\mathbf{C}_{\phi z})^T\mathbf{h}_z = j\omega_n Y_0\,\mathbf{D}_\varepsilon\,\mathbf{e}_\phi + \mathbf{D}_i\,\mathbf{i}_\phi \qquad (3)$$

where $\mathbf{C}_{\phi r}$ and $\mathbf{C}_{\phi z}$ are "connection" matrices of sizes (M-1)N by (M-1)(N-1) and M(N-1) by (M-1)(N-1), respectively; $\mathbf{e}_\phi$ denotes a column vector of ϕ –directed electric fields E_ϕ; $(\ )^T$ denotes transpose of the matrix within parenthesis; $\mathbf{h}_r$ denotes a column vector of r-directed magnetic fields H_r, length (M-1)N; $\mathbf{h}_z$ denotes a column vector of z-directed magnetic fields H_z, length M(N-1); $\mathbf{R}_{e\phi}$ and $\mathbf{R}_{hz}$ are diagonal matrices of radial distances for electric and axial magnetic field nodes, respectively; $\mathbf{D}_{bt}$ denotes a diagonal matrix of coefficients for electric fields at the bottom and the top boundary size (M-1) by (M-1); $\mathbf{e}_{bt}$ denotes a column vector of ϕ–directed electric fields E_ϕ at the bottom and the top boundary, of length (M-1); $\mathbf{D}_s$ denotes a diagonal matrix of coefficients associated with electric fields at the side of the rotationally symmetric domain, size (N-1) by (N-1); $\mathbf{e}_S$ denotes a column vector of ϕ–directed electric fields E_ϕ at the domain side, length (N-1); and $\mathbf{D}_\varepsilon$ denotes a diagonal matrix of averaged relative dielectric constants, size (M-1)(N-1) by (M-1)(N-1). We implement impedance-type *local* boundary conditions by expressing the ϕ–directed electric fields on the boundaries in terms of the magnetic fields just inside the grid. For example, we may write: $E_{b1} = Z_{b1}H_{r1}$, $E_{t1} = Z_{t1}H_{r5}$, $E_{s1} = Z_{s1}H_{z3}$, and so on. Note that the local impedances Z_{**} are, in principle, all different. The local impedance boundary conditions can be represented in a matrix form using diagonal impedance matrices, one for the bottom and the top boundary and one for the side boundary:

$$\mathbf{e}_{bt} = \mathbf{Z}_{bt}\mathbf{h}_r \qquad (4)$$
$$\mathbf{e}_S = \mathbf{Z}_s\,\mathbf{h}_z\,. \qquad (5)$$

Replacing the electric fields on the boundaries with the local impedance boundary conditions we get:

$$\mathbf{C}_{\phi r}\mathbf{e}_\phi = -j\omega_n Z_0\,\mathbf{h}_r - \mathbf{D}_{bt}\,\mathbf{Z}_{bt}\,\mathbf{h}_r \qquad (6)$$
$$\mathbf{C}_{\phi z}\mathbf{e}_\phi = -j\omega_n Z_0\,\mathbf{h}_z - \mathbf{D}_S\,\mathbf{Z}_s\,\mathbf{h}_z \qquad (7)$$

We can express the magnetic fields from the above equations, denoting diagonal unit matrix as $\mathbf{I}$:

$$\mathbf{h}_r = (-1)[j\omega_n Z_0\mathbf{I} + \mathbf{D}_{bt}\,\mathbf{Z}_{bt}]^{-1}\,\mathbf{C}_{\phi r}\mathbf{e}_\phi \qquad (8)$$
$$\mathbf{h}_z = (-1)[j\omega_n Z_0\mathbf{I} + \mathbf{D}_s\,\mathbf{Z}_s]^{-1}\,\mathbf{C}_{\phi z}\mathbf{e}_\phi \qquad (9)$$

Note that the matrices get inverted are all diagonal whose inversion is simple. Substituting (8) and (9) into (3) we get:

$$\{ (\mathbf{C}_{\phi r})^T [j\omega_n Z_0 \mathbf{I} + \mathbf{D}_{bt} \mathbf{Z}_{bt}]^{-1} \mathbf{C}_{\phi r} + (\mathbf{C}_{\phi z})^T [j\omega_n Z_0 \mathbf{I} + \mathbf{D}_s \mathbf{Z}_s]^{-1} \mathbf{C}_{\phi z} + j\omega_n Y_0 \mathbf{D}_\varepsilon \} \mathbf{e}_\phi = (-1) \mathbf{D}_i \mathbf{i}_\phi \quad (10)$$

This system of equations can be solved for the electric field $\mathbf{e}_\phi$ (from which the magnetic fields are then calculated), subject to the known sources $\mathbf{i}_\phi$ and boundary conditions specified by matrices $\mathbf{Z}_{bt}$ and $\mathbf{Z}_s$. Once the system of equations has been solved, the equivalent electric and magnetic currents can be calculated for a cylindrical surface defined within the grid and the far-fields calculated using free-space radiation integrals (surface equivalence principle).

Iterative Implementation of Impedance Boundary Conditions

The local boundary impedances Z_{**} are ratios of local electric and magnetic fields. These fields are unknown (we want to solve for them) and thus the impedances can be found once the fields have been calculated. One way to resolve this is to *assume* some approximate values for the boundary impedances, calculate the fields for the assumed boundary impedances, and then find *new* boundary impedances from the calculated fields. This can be repeated until the relative change in the impedances between the two successive iterations is less than a user defined value. The initial "guess" for the boundary impedances can be the purely real impedance of free space, Z_0, which is equivalent to assuming that the wave incident on the boundary is a normally incident plane wave. Another possibility, implemented in our programs, is to multiply Z_0 by coefficients dependent on the assumed angles of incidence at the different nodes on the boundary. In both cases the initially assumed boundary impedances are purely real (which is true if the boundaries are in the far field zone) but the iterative solution introduces (via calculated electric and magnetic fields) the imaginary parts as well. Note that a local angle of incidence needs to be calculated (the tangent of this angle is the ratio of the magnitudes of the transverse magnetic field components) at all points on the boundary. The local boundary impedances are then calculated using the electric field, the angle of incidence, and the appropriate (axial or radial) component of the magnetic field. For example, as shown in Fig. 2, the local boundary impedance on the side boundary is calculated as:

$$Z_{S*} = (E_{S*} / H_{Z*}) \cos\varphi = (E_{S*} / H_{Z*}) \, |H_{Z*}| / (|H_{Z*}|^2 + |H_{r*}|^2)^{1/2} \quad (11)$$

where | | denotes absolute value. Note that the first term in (14) is complex and the second term is real.

Implementation and Sample Results

MATLAB software has been selected to implement the above because MATLAB provides a programming environment with built-in matrix functions and algorithms based on the EISPACK and LINPACK algebra packages. Since MATLAB is available for a variety of platforms, from PC's to workstations to mainframes, the programs are easily "ported" and exchanged. Results are presented for the radiation of a cylindrical dielectric resonator (DR) with the radius of 6mm and the height of 6mm. The relative permittivity of the DR is 38. The resonator is mounted on a PEC plane (impedances for the bottom boundary are all set to 0), as shown in Fig. 3. The computational domain is a uniform grid of 12 by 12 square cells with Δl = 1mm. The resonator is excited by a constant 1A current loop of the same radius as the resonator and positioned at 2/3 of the resonator height. The frequency of the current in the loop is "swept" from 4GHz to 9GHz and the maximum radiated field on a 1m circle plotted vs. frequency (Fig.4). The same is repeated for the loop without the DR (Fig.5), for comparison purposes. Notice that the radiated power for the same loop without the resonator is much smaller in the same frequency range, which indicates that the combination of a small loop and a dielectric resonator can be used as a radiating element. The radiation efficiency and coupling to the source depend on the field structures of the resonant modes of the DR. The magnetic vector plot (Fig.6) and the radiation pattern (Fig.7) are shown at the frequency of the first peak of the radiated field, corresponding to the first TM resonant mode of the open dielectric resonator. These results were obtained using MATLAB on a 25MHz, 386-class PC.

Summary

Finite Integration Technique has been applied to solve for radiation of rotationally symmetric dielectric bodies excited by a current loop with no angular variation of current. Open boundary condition has been implemented using local boundary impedances which are iteratively improved. The radiated fields are calculated using surface equivalence principle. Results show that DR's enhance radiation of small loops.

References:

[1.] M. Albani and P. Bernardi, "A numerical method based on the discretization of Maxwell's equations in integral form," *IEEE Trans. Microwave Theory Tech.*,vol. MTT-24, pp. 446-449, Apr. 1974.

[2.] J. Lebaric and D. Kajfez, "Analysis of dielectric resonator cavities using the Finite Integration Technique," *IEEE Trans. Microwave Theory Tech.*,vol. MTT-37, pp. 1740-1748, Nov. 1989.

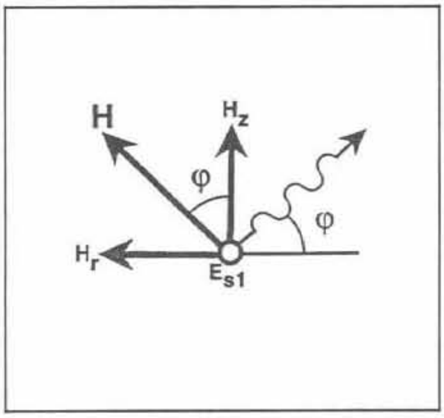

Fig.2: Fields at a boundary node

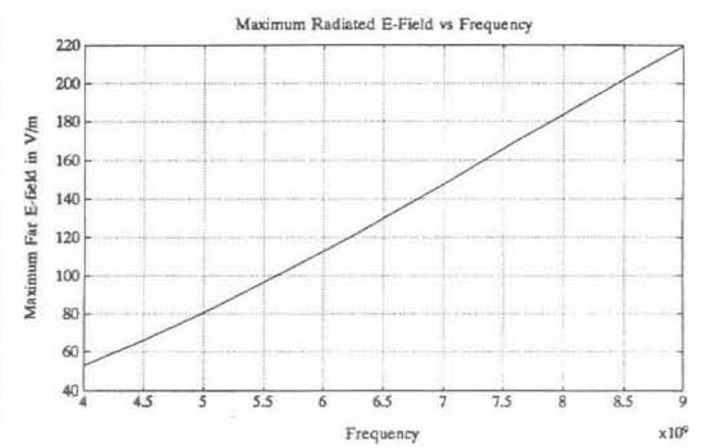

Fig.4: Max radiated field for a loop

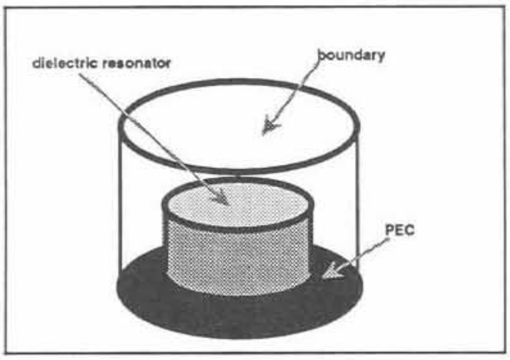

Fig.3: DR mounted on PEC

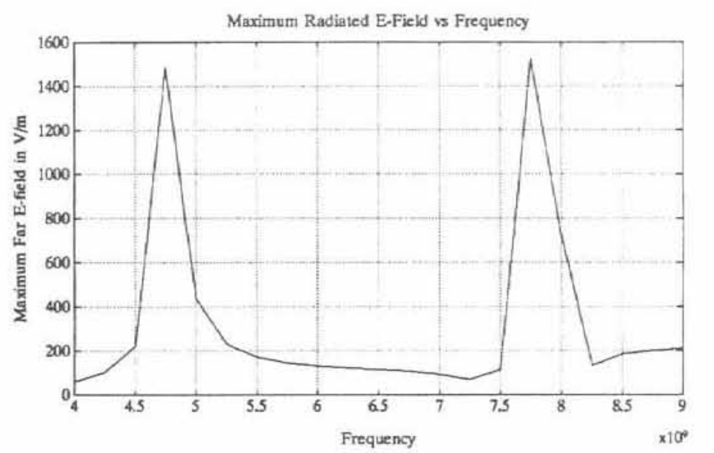

Fig.5: Max radiated field for a loop and DR

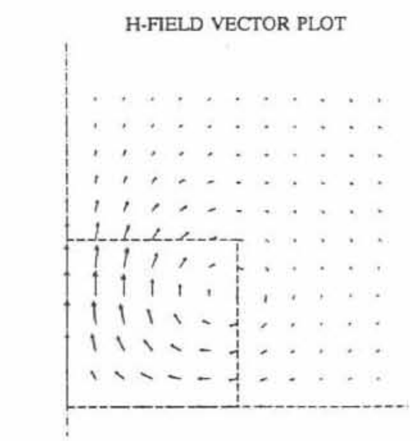

Fig.6: H-field plot for the first TM DR resonance

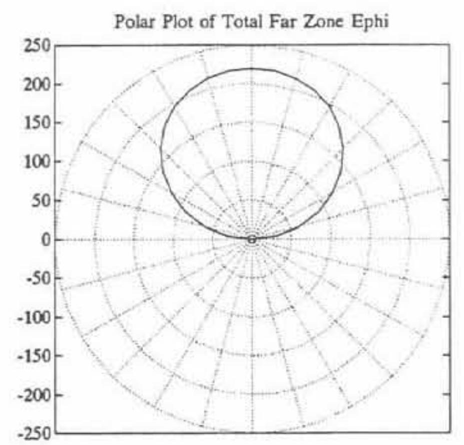

Fig.7: Far-field radiation pattern