# Radiation Analysis of Parabolic Cylindrical Reflector Antenna With Horn Array As Feed 

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#### Abstract

A parabolic cylindrical reflector antenna with a horn array as feed has advantages of high power transmission, great gain and good directivity and has a good prospect of application in satellite communications and radar. In this paper, a novel method combining HFSS simulation is proposed to analyze the reflector antenna, the expressions of radiation characteristics is given. A good agreement between results of calculation and measurement demonstrates validness of this method.


Keyword: horn array, parabolic cylindrical reflector antenna, radiation characteristics.

## 1. INTRODUCTION

Reflector antennas have been used widely in communication, satellite-signal receiving, radar and radio astronomy. Especially a parabolic cylindrical reflector antenna, if fed with linear feed or array which is approximate to linear feed, can transmit high power and have a great gain, thus it can be well used as a emitting antenna. Characteristics of a EISCAT VHF parabolic cylinder antenna was studied roundly in [1][2], radiation characteristics, effect of braces to radiation patterns, the line feed phase center and aperture efficiency, including incidence efficiency, spillover efficiency and phase error losses were given. In [3], radiation characteristics of a parabolic cylindrical reflector antenna with a rectangular micro strip patch array as feed is analyzed and calculated with numerical method. In this paper, a parabolic cylindrical reflector antenna fed by a horn array is analyzed, which has larger gain, higher power transmission
and well directivity. A novel method combing HFSS simulation is used, fields on the aperture of the horn array is more actual and the mutual coupling is taken into account , so the result is more accurate.

## 2. ANALYSIS

The proposed antenna consists of a cylindrical reflector with parabolic cross section and a horn array as feed, as shown in Fig.1. The coordinates system is also shown in Fig. 1, the geometric relations are as follows:

$$
\begin{align*}
& y(x)=\frac{x^{2}}{4 f}-f  \tag{1}\\
& \hat{n}=(\hat{x} x-\hat{y} 2 f) / \sqrt{x^{2}+4 f^{2}} \tag{2}
\end{align*}
$$

where $\vec{r}=\hat{x} x+\hat{y} y+\hat{z} z$ is the vector of an arbitrary point on the reflect surface and $\hat{n}$ is the normal vector of the reflector surface. $f$ is the focal length of the parabolic cross section. The feed of the reflector is an array which is composed of some E-plane horn with identical amplitude and phase located separately parallel to the focal line of the parabolic cylindrical reflector with identical distance. Using the theory of array, given the electric field on the aperture surface as follows:

$$
\begin{equation*}
\stackrel{\rightharpoonup}{E}_{S}=\hat{x} E_{x}+\hat{y} E_{y}+\hat{z} E_{z} \tag{3}
\end{equation*}
$$

The radiation electric field is then obtained by

$$
\hat{E}_{P}=j \frac{e^{-j k r}}{2 \lambda r}\left\{\begin{array}{l}
\hat{\theta}\left[\begin{array}{l}
N_{x} \cos \theta \cos \phi+N_{y} \cos \theta \sin \phi+ \\
N_{z}(\sin \phi-\sin \theta)
\end{array}\right]+  \tag{4}\\
\hat{\phi}\left[\begin{array}{l}
N_{x}(\sin \theta-\sin \phi)+N_{y} \cos \phi+ \\
N_{z} \cos \theta \cos \phi
\end{array}\right]
\end{array}\right\}
$$

where $N_{i}=\int_{S} E_{i} e^{j k(x \sin \theta \cos \phi+z \cos \theta)} d s$
$(i=x, y, z)$
The aperture surface of the horn feed of the antenna is the XOZ plane, the electric field on which is given by:

$$
\begin{align*}
& E_{x}=E_{0} \cos \left(\frac{\pi z}{A}\right) \exp \left(-j \frac{\pi x^{2}}{\lambda_{g} R}\right) \\
& E_{y}=0, E_{z}=0 \tag{6}
\end{align*}
$$

substituting equations(5) and (6)into equation (4), the radiation field expressions of horn feed can be obtained by

$$
\begin{align*}
& \stackrel{\rightharpoonup}{E}_{P}=j E_{0} \frac{\exp (-j k r)}{\lambda r} \frac{A}{\pi} \sqrt{\frac{\lambda_{g} R}{2}} \frac{\cos (k A \cos \theta / 2)}{1-(2 A \cos \theta / \lambda)^{2}} \\
& \exp \left(j \frac{\pi \lambda_{g} R}{\lambda^{2}}(\sin \theta \cos \phi)^{2}\right) \\
& \left\{\left[C\left(v_{1}\right)-C\left(v_{2}\right)\right]-j\left[S\left(v_{1}\right)-S\left(v_{2}\right)\right]\right\} \\
& {\left[\hat{\theta}_{0} \cos \theta \cos \phi+\hat{\phi}_{0}(\sin \theta-\sin \phi)\right]} \tag{7}
\end{align*}
$$

in which $A$ and $B$ are the length and width of aperture ( $B$ is the size of splay direction of horn) R is the distance between the line of intersection of splay side and aperture surface, and

$$
\lambda_{g}=\frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{2 A}\right)^{2}}}
$$

$$
v=\frac{B}{\sqrt{2 \lambda_{g} R}}
$$

$$
\begin{aligned}
& v_{1}=\frac{1}{\sqrt{2}}\left(\sqrt{\lambda_{g} R} \frac{2 \sin \theta \cos \phi}{\lambda}+\frac{B}{\sqrt{\lambda_{g} R}}\right) \\
& v_{2}=\frac{1}{\sqrt{2}}\left(\sqrt{\lambda_{g} R} \frac{2 \sin \theta \cos \phi}{\lambda}-\frac{B}{\sqrt{\lambda_{g} R}}\right),
\end{aligned}
$$

where $C(x)=\int_{0}^{x} \cos \left(\frac{\pi}{2} t^{2}\right) d t \quad S(x)=\int_{0}^{x} \sin \left(\frac{\pi}{2} t^{2}\right) d t$
are Fresnel integral.
supposing the horn in the array is 2 N , according to the theory of linear array, we can get the expression of radiation field as follows:

$$
\begin{align*}
& \stackrel{\rightharpoonup}{E}_{A}=2 \stackrel{\rightharpoonup}{E}_{P} \sum_{n=1}^{N} \cos \left[(2 n-1) \frac{\psi}{2}\right]  \tag{8}\\
& \psi=k d \cos \theta
\end{align*}
$$

$$
\begin{equation*}
\vec{H}_{A}=\sqrt{\frac{\varepsilon_{0}}{u_{0}}} \hat{r}_{0} \times \vec{E}_{A} \tag{9}
\end{equation*}
$$

in which $d$ is the distance between the aperture centre of adjacent horn, and For the feed array, the parabolic cylindrical reflector is in radiation area, and between them is vacuum, using the Physical optics method, the PO current on the reflect surface induced by radiation magnetic field $\vec{H}_{A}$ can be expressed as

$$
\begin{equation*}
\vec{J}_{s}=2 \hat{n} \times \vec{H}_{A} \tag{10}
\end{equation*}
$$

also using the PO method and Referring to reference[3],the radiation electric field of the parabolic cylindrical reflector antenna can be obtained by:

$$
\begin{equation*}
\vec{E}=-j \frac{\eta e^{-j k R}}{2 \lambda R} \int_{\sigma} \sqrt{\left\{\vec{J}_{S}(\stackrel{\rightharpoonup}{r})-\left[\vec{J}_{S}(\vec{r}) \cdot \hat{a}_{R}\right] \hat{a}_{R}\right\}} 1 \tag{11}
\end{equation*}
$$

where $\sigma$ is the area of the aperture, and

$$
\begin{aligned}
& \hat{a}_{R}=\hat{x} \sin \theta \cos \phi+\hat{y} \sin \theta \sin \phi+\hat{z} \cos \theta \\
& \stackrel{\rightharpoonup}{r} \cdot \hat{a}_{R}=x \sin \theta \cos \phi+y \sin \theta \sin \phi+z \cos \theta
\end{aligned}
$$

All the edge diffractions are neglected in this study. The directivity and the gain of the antenna can be expressed separately as

$$
\begin{align*}
& D=\frac{4 \pi}{\lambda^{2}} A \eta_{a}  \tag{12}\\
& G=\frac{4 \pi}{\lambda^{2}} A \eta_{a} \eta_{1}==\frac{4 \pi}{\lambda^{2}} A g \tag{13}
\end{align*}
$$

where $\eta_{a}$ is called aperture efficiency, and $\eta_{1}$ incidence efficiency, which is defined as the ratio of the whole power radiated by the feed to the incident power of the reflect surface. $g=\eta_{a} \eta_{1}$ is defined as gain coefficient of the antenna. According to reference [2],we can induce the expression of $g$ as follows:

$$
\begin{gather*}
g=\eta_{a} \eta_{1} \\
=\frac{c \tan \left(\frac{g}{2}\right)}{4 H} \frac{\left|\int_{-\frac{H}{2}}^{\frac{H}{2}} \int_{-g}^{g} \sqrt{G_{f}(\phi, z)} \sec \left(\frac{\phi}{2}\right) d \phi d z\right|^{2}}{\int_{-\frac{H}{2}}^{\frac{H}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} G_{f}(\phi, z) d \phi d z} \tag{14}
\end{gather*}
$$

in which $G_{f}(\phi, z)$ is the radiation pattern function of the feed array of variable $\phi$ and $z$.

## 3. NUMERICAL RESULTS

The radiation field expression of feed array includes two Fresnel integrals, so in the practical operation, we should have the $\theta$ and $\phi$ in the $v_{1}$ and $v_{2}$ dispersed. By using the method introduced in reference[3],for reducing computation time, when calculating the $\vec{J}_{s}$ and $\vec{E}$, we disperse the aperture surface of the reflector into many symmetrical grid , the PO current on which is equal approximatively.
What is more important is that the method of together using of HFSS simulation and program calculation is used. First, using HFSS, we gain the aperture fields, then we disperse the aperture surface of the horn into many symmetrical grid, regarding the fields on each grid as equal. Second, we pick up and save the fields as the data format, including amplitude and phase of electric field and magnetic field, and substitute these quantities into the program of equation(4)for numerical calculation. Finally, with vector addition, we can get the radiation fields of the horn array, which is more accurate than the results calculated directly by program. The designed antenna has 16 horns in the array, which Is fed by $T E_{10}$ mode with identical amplitude and phase, besides, offset-feed is used in the system, and the cross section of the reflector is a unsymmetrical part of parabola, shown in fig.2. Comparing the curves shown in fig. 3 and fig.4,one can see that the calculation and measurement is coincident in main lobe, because of the low radiation power and interference in measurement in microwave darkroom, so in side lobe, they warped a little. The gain of calculate is 32.1 dB , a little more than the measurement of 31.3 dB ,which is led by measurement error. In a word, a conclusion can be drew that the analysis and expressions in this paper is correct.

## REFERENCES

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Fg. 1 The structure of the parabolic cylindrical reflector antenna with a horn array as feed


Fig. 2 The cross section of the parabolic cylindrical reflector


Fig. 3 Comparison of E-plane gain patterns between calculation and measurement


Fig. 4 Comparison of H-plane gain patterns between calculation and measurement

