1E2-5

Proceedings of ISAP'04, Sendai, JAPAN

ULF ELECTROMAGNETIC WAVY STRUCTURES IN F-REGION OF THE SPHERICAL IONOSPHERE CAUSED FROM INHOMOGENEITY OF THE GEOMAGNETIC FIELD

George ABURJANIA, Khatuna CHARGAZIA, George JANDIERI, Oleg KHARSHILADZE
Laboratory of investigation of extraordinary phenomena
I. Vekua Institute of Applied Mathematics, Tbilisi State University
2 University str., 0143 Tbilisi, Georgia
E-mail: aburj@mymail.ge, aburj@viam.hepi.edu.ge

1. Introduction

Large-scale wave structures play an important role in the energy balance and in circulations of the atmosphere and oceans. Numerous observations show that planetary-scale perturbations of an electromagnetic nature are always present in the ionosphere in the form of background wave perturbations [1-3]. Of particular interest among these perturbations are so-called large-scale ultra-low-frequency (ULF) ionospheric perturbations propagating around the Earth along the parallel at fixed latitude. They are especially pronounced during geomagnetic storms and substorms [4], earthquakes [5], major artificial explosions, military operations [6], etc.

In nature, these perturbations manifest themselves as background oscillations. Observations showed that forced oscillations of this type occur in the ionosphere under the pulsed action from above (geomagnetic storms [4]) or from below (earthquakes, volcanic eruptions, and major artificial explosions [5,6]). In the latter case, the perturbations exist in the form of localized solitary wave structures.

2. Model equations

We investigate the possible generation of the planetary-scale ULF electromagnetic wave structures in F-layer of the spherical ionosphere by permanently acting fundamental factors for the planetary – scale processes - the latitude gradient of the geomagnetic field and angular velocity of the Earth rotation. We use magnetohydrodynamic equations for the ionosphere taking into account the fact, that large-scale flows don't perturb density and concentration of the medium particles and excepting acoustic-gravity waves. We consider horizontal incompressible flow in a spherical coordinate system bounded to the rotating Earth. Let θ-denotes an addition to the latitude $\varphi'(\theta = \pi/2 - \varphi')$, λ is a longitude; r -distance from a center of the Earth. It is considered that velocity component on r axis is equal to zero $V_r = 0$; $V_\theta(\theta, \lambda, t)$ - velocity component is directed along meridian (it is positive if velocity is directed to the north); $V_\lambda(\theta, \lambda, t)$- velocity component along the parallels (it is positive if velocity is directed to the east). For simplicity we assume, that the geomagnetic field has only vertical component $H_{0r} = -H_p \cos\theta$, i.e. we shall consider moderate and high latitudes; where $H_p = 5 \cdot 10^4\,\text{nT}$ is the strength of geomagnetic field near Pole. Consequently, perturbation of geomagnetic field has only vertical component $h_r(\theta, \lambda, t)$. Normal, to the Earth surface, component of the angular rotational velocity is important for the dynamic of motion for our case $\Omega_r = \Omega_0 \cos\theta$.

Further, we would accept that the zonal wind determined by the experimental expression $\overline{V}_\lambda = \alpha r \sin\theta$, where α is the constant angular velocity of the atmosphere zonal circulation (the so-called index of the circulation). The value of α varies seasonably: $\alpha = 0.05\Omega_0$ in winter, $\alpha = 0.025\Omega_0$ in summer. Identically, the stream function is $\Psi = \overline{\Psi}(\theta) + \Psi'(\theta, \lambda, t)$. Then, by ignoring V_r in the continuity equation along with the term of non-compressibility, we can determine the component of velocity through stream function:

$$V_\theta = -\frac{1}{r\sin\theta}\frac{\partial\Psi}{\partial\lambda}, \qquad V_\lambda = \frac{1}{r}\frac{\partial\Psi}{\partial\theta} \ . \tag{1}$$

$$\overline{\Psi}(\theta) = -\alpha r^2\cos\theta \ . \tag{2}$$

In these conditions the close system of dynamical equations for the large-scale wave structures – the equation of motion of the medium particles and induction equation, for dissipative ionosphere can be reduced to the form:

$$\frac{\partial\Delta\Psi}{\partial t} + \alpha\frac{\partial\Delta\Psi}{\partial\lambda} + 2(\alpha+\Omega_0)\frac{\partial\Psi}{\partial\lambda} + \alpha_H\frac{\partial h}{\partial\lambda} + \Lambda\Delta\Psi = -\frac{1}{R^2\sin\theta}J(\Psi,\Delta\Psi), \tag{3}$$

$$\frac{\partial h}{\partial t} - \Omega_H\frac{\partial\Psi}{\partial\lambda} + \alpha\frac{\partial h}{\partial\lambda} = -\frac{1}{R^2\sin\theta}J(\Psi,h). \tag{4}$$

Here, we introduce designations:

$$\alpha_H = \frac{C_H}{R\sin.\theta}, \quad C_H = -\frac{c}{4\pi eN}\frac{1}{R}\frac{\partial H_{0r}}{\partial\theta} = -\frac{cH_p\sin\theta}{4\pi eNR},$$

$$\Omega_H = \frac{N}{N_n}\frac{e}{Mc\sin\theta}\frac{\partial H_{0r}}{\partial\theta} = \frac{N}{N_n}\frac{eH_p}{Mc}, \quad h = \frac{N}{N_n}\frac{eR^2}{Mc}h_r, \tag{5}$$

$$\Delta = \frac{1}{\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2}{\partial\theta^2}\right], \quad J(a,b) = \frac{\partial a}{\partial\theta}\frac{\partial b}{\partial\lambda} - \frac{\partial a}{\partial\lambda}\frac{\partial b}{\partial\theta},$$

where N is a concentration of the charged particles; N_n - a neutral concentration; M - unit mass of ions and molecules; c is a light speed; R - the Earth radius; e - elementary charge. In our task r is a parameter and therefore we shall replace r by the Earth radius R in the (3)-(5) taking into account thickness of the atmospheric layer; Λ is constant coefficient of surface friction between atmospheric layers and is of the order of $10^{-5}s^{-1}$ for the ionospheric heights. Further for investigation dynamic of large-scale (planetary) zonal flows in F-region of the ionosphere we will use the system of nonlinear equations (3) and (4).

3. ULF electromagnetic planetary linear waves

We begin an investigation of (3) and (4) from the analyses of motion having small amplitude. It is expedient to analyze necessary group of the solutions of linear dynamic equations on the sphere at investigation of planetary waves having horizontal spatial scale of the order of Earth radius $L \sim R$. Therefore, we shall seek the solution of these equations in the form $\Psi, h \sim f(\theta)\exp(im\lambda - i\omega t)$ in linear approximation, where m is a whole number; ω - frequency of perturbations and $f(\theta)$ is an unknown function of θ.

For this kind of solution of eq. (3) (4) yields a new equation for f:

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \left[-\frac{m^2}{\sin^2\theta} + \frac{2(\alpha+\Omega_0)m}{\alpha m - \omega - i\Lambda} + \frac{\Omega_H m\cdot\alpha_H m}{(\alpha m-\omega)(\alpha m - \omega - i\Lambda)}\right]f = 0 \ . \tag{6}$$

The relation (6) presents the equation for the associated Legrangian pollinoms. This equation has unique bounded solution if the sum of the second and third terms in the brackets equal to $n(n+1)$ (n is a whole number). Corresponding condition give the dispersion equation:

$$\frac{2(\alpha+\Omega_0)}{\alpha m - \omega - i\Lambda} + \frac{\Omega_H m\cdot\alpha_H m}{(\alpha m-\omega)(\alpha m - \omega - i\Lambda)} = n(n+1). \tag{7}$$

Submitting eigen-frequency ω_0 and decrement of damping γ by formula $\omega = \omega_0 + i\gamma$, $|\gamma| << \omega_0$, from (6) we get the dispersion equation for eigen- frequencies:

$$\omega_0'^2 - \omega_{0p}\omega_0' - \omega_{0H}\alpha_H m = 0 \tag{8}$$

and expression for decrement:

$$\gamma = -\frac{\omega_0'}{2\omega_0' - \omega_{0p}}\Lambda, \tag{9}$$

where

$$\omega_0' = \alpha m - \omega_0, \qquad \omega_{0p} = \frac{(2\alpha + \Omega_0)m}{n(n+1)}, \qquad \omega_{0H} = \frac{\Omega_H m}{n(n+1)}.$$

Let us estimate the order of the values of coefficients in equation (8) for the characteristic values of the parameters in F-region of the ionosphere: $\Omega_0 = 7{,}3\times10^{-5}s^{-1}$, $\alpha = 0.04\Omega_0 \sim 3\times10^{-6}s^{-1}$, $H_p = 5\times10^{-5}T$, $N_n = 3\times10^{14}\,m^{-3}$, $N = 3\times10^{11}\,m^{-3}$, $R = 6{,}4\times10^6\,m$. Thus, $\alpha_H = 10^{-3} \div 10^{-4}s^{-1}$, $2\Omega_H \approx 10^{-4} \div 10^{-5}s^{-1}$, $\Lambda = 10^{-5}s^{-1}$. From there it follows, that $\alpha_H m >> \omega_{0p}, \Lambda, \omega_{0H}, \alpha m$, $\omega_0' \approx \omega_0$. taking all these into account, from (8), (9) we get a spectrum of the oscillations:

$$\omega_0 = \pm\frac{H_p m}{\sqrt{4\pi\rho n(n+1)}R}, \qquad \gamma = -\frac{1}{2}\Lambda. \tag{10}$$

We introduce the angular velocity of the wave propagation $d\sigma/dt = \omega_0/m$, using (10) and determine the linear phase velocity of the wave motion along the latitudinal circles (parallels) - $V_\lambda = V_{ph} = R\sin\theta\cdot d\sigma/dt$. Thus, we get the expression for the phase velocity of the waves:

$$V_{ph} = \pm\frac{H_p}{\sqrt{4\pi\rho n(n+1)}}\sin\theta. \tag{11}$$

This wave is the fast and represents itself a new mode of eigen-frequencies of the oscillations in F-region of the ionosphere. The wave (10), (11) is weakly damped with decrement: $\gamma \approx 10^{-6}s^{-1}$ and can propagate along the parallel to the west as well as to then east. Calculation show, that phase velocity of these waves are in the range (20-1400)km/s; the period of these waves is in the interval (1-105) s; the wavelength is 10^3 km or more; their frequencies are in the ULF range of $(10 - 10^{-3})$ s^{-1}. From induction equation (4) can be estimated amplitude of the geomagnetic pulsations generated by these waves, $h_r \approx H_{0r}\xi/R$, where ξ is a transversal displacement of charged particles. At displacement ξ equals 0.1 and 1 km for $\theta = 45^0$, h_r is equal to 8 and 80 nT correspondingly. These waves, apparently, were registered experimentally in midlatitude region of the ionosphere in [1-3].

4. Nonlinear ULF vortical structures

Now we begin solving system of the nonlinear equations (3),(4). These equations represent the partial differential equations with variable coefficients, analytical solution of which is very difficult. To simplify the problem we assume the motion at any fixed latitude $\varphi_0 = \pi/2 - \theta_0$. It is convenient to introduce the new latitudinal and longitudinal coordinates $x = \lambda R\sin\theta_0$, $y = -(\theta - \theta_0)R$, which allows us to freeze variable coefficients in the system of Eqs. (3),(4). Then, in mentioned system of equations the coefficients become constant and their stationary solution can be sought following the work [7].

Thus, we will seek the solution of the nonlinear equations (3), (4) in the non-dissipative approximation $(\Lambda = 0)$ in the form of the stationary regular waves $\Psi = \Psi(\eta, y)$, $h = h(\eta, y)$, propagating along the latitudes x with velocity $U = \mathrm{const}$, without changing the shape, $\eta = x - Ut$. It is not difficult to show the Eqs. with the η and y variables are equivalent to

$$\Delta_\perp \Psi - \beta_2 y = F(\Psi + \beta_1 y), \tag{11}$$

where F is an arbitrary chosen differential function of its argument,

$$\beta_1 = C_R - U, \qquad \beta_2 = \frac{2(\alpha + \Omega_0) + C_H \Omega_H}{R(C_R - U)} \sin\theta_0, \tag{12}$$

where $C_R = \alpha R \sin\theta_0$ is the characteristic phase velocity of ordinary Rossby waves.

Following Aburjania (1996) we use the polar coordinates along the Earth surface $r = (\eta^2 + y^2)^{1/2}$, $\mathrm{tg}\varphi = y/\eta$ and the circle with the radius a . Further, we would demand that $\Psi(r,\varphi)$ and $h(r,\varphi)$ be twice differentiable continuously (including the circle $r = a$) along its argument and vanishes exponentially when $r \to \infty$. Then (11) will have the following solutions:

$$\Psi(r,\varphi,t) = \frac{C_R - U}{\Omega_H R \sin\theta_0} h(r,\varphi,t) = a\beta_1 F(r) \sin\varphi, \tag{13}$$

where

$$F(r) = \begin{cases} (p/\chi)^2 J_1(\chi r)/J_1(\chi a) - (\chi^2 + p^2) r/(a\chi^2), \text{at } r < a, \\ -K_1(pr)/K_1(pr), \text{at } r \geq a, \end{cases} \tag{14}$$

where J_n Bessel's function of the n-th degree and K_n the Mcdonald function; p and χ parameters are related by the dispersion relation

$$\frac{J_2(\chi a)}{\chi J_1(\chi a)} = -\frac{K_2(pa)}{pK_1(pa)}, \qquad p^2 = -\frac{\beta_2}{\beta_1} > 0. \tag{15}$$

Taking into account the dispersive equation (15), solution (13) has two free parameters U and a . The disturbed solution obtained from (13) equals to $\Psi, h \sim r^{-1/2} \exp(-pr)$ when $r \to \infty$. So, the wave is localized on the Earth surface (η, y). The structures of this type represent a pair of oppositely circulating vortices (cyclone-anticyclone) of equal intensity, propagating along the parallels on the background of the pure zonal –mean west to east flow. The nonlinear vortex structures move with velocity of order of $U > C_R + C_H \Omega_H /(2\alpha + 2\Omega_0)$. The characteristic scale is of order of $d \sim a \approx [UR/(\alpha + \Omega_0)]^{1/2} \sim 10^4 \mathrm{km}$.

The properties of the wave structures under investigation are very similar to those of ULF perturbations observed experimentally in the ionosphere at middle latitudes [1-6].

Reference

1. Al'perovich, L.S., Drobgev, V.I., Sorokin, V.M. et al. Geomagn. aeron., 22, 797, 1982.
2. Sharadze, Z.S., Japaridze, G.A., Kikvilashvili, G.B. et al.. Geomagn. aeron., 28, 446, 1988.
3. Bauer, T.M., Baumjohann, W., Treumann R.A.et al. J. Geophys. Res., 100, 9605, 1995.
4. Haykowicz, L.A. **.** Planet. Space Sci**.** ,39, 583, 1991.
5. Hayakawa, M. (Edit). Atmospheric and Ionospheric Phenomena Associated with Earthquakes, Terra Sci. Publ. Comp. , Tokyo, 1999.
6. Drobzhev, V.I. et al. Ionosf. Issl., 39, 61, 1986.
7. Aburjania, G.D. Plasma physics reports. 22, 954, 1996.