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AN INVERSION TECHNIQUE OF THE PLASMASPHERIC ELECTRON DENSITY ESTIMATION BASED ON PROPAGATION CHARACTERISTICS OF WHISTLER MODE WAVE

Yoshitaka GOTO [1], Yoshiya KASAHARA [1] and Toru SATO [2]
[1] Dept. of Information and Systems Engineering, Kanazawa University
2-40-20, Kodatsuno, Kanazawa, 920-8667, JAPAN
[2] Dept. of Communications and Computer Engineering, Kyoto University
Yoshida, Sakyo-ku, Kyoto, 606-8501, JAPAN
E-mail: ygotou@is.t.kanazawa-u.ac.jp

1 Introduction

The electron density estimation in the earth's plasmasphere has been a critical issue not only for scientific interests but also for practical uses by artificial satellites. Radiowave propagations from satellites to the ground enable us to measure total electron contents along their paths. The total electron content, however, mainly covers ionospheric electrons. For the measurement of plasmaspheric electrons, the ionospheric influence should be isolated from propagation characteristics of the wave. It is known that propagations of VLF waves in the ionosphere and the plasmasphere are in whistler mode. The propagation path and the velocity are determined by the dispersion relation of plasma wave which is deduced from in-situ electron density and geomagnetic field. Thus, the characteristics of whistler mode waves, such as k-direction and delay time, contain the infomation of the medium in the plasmasphere well.

If the global electron density profile is known, the characteristics of the whistler mode wave can be theoretically calculated at any location in the plasmasphere, and these quantities can be compared with observed values. Thus, the estimation of a global plasmaspheric electron density profile from satellite observations of whistler mode waves is a kind of ill-posed inverse problems. *Kimura et al.* [1] demonstrated the feasibility of the reconstruction of global electron density profiles in the plasmasphere from Omega signals observed by the Akebono satellite (EXOS-D). Their method was based on a model-fitting method using ray tracing.

In the present study, we have improved the method in order to deal with nonducted whistlers which are one of the most typical VLF wave in the plasmasphere. The nonducted whistlers originate from atmospherics, and the occurrence propability at an observational satellite is higher than that of Omega signals. Although it is difficult to measure the propagation delay times of nonducted whistlers accurately, the frequency dependences of the arrival time can be utilized instead. The final purpose of the present study is to derive long-term variations and average features of the earth's plasmasphere by processing the huge dataset of the Akebono satellite.

2 Inversion method

2.1 Electron density model

The realization of the reconstruction algorithm requires an appropriate model to represent global plasmaspheric density model. A desirable model should satisfy the following two conditions; (i) the model is represented by a function which has as small number of parameters

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as possible, and (ii) any plasmaspheric density profile can be reproduced by choosing a set of appropriate paramters in the model. In the present study, the global electron density profile is reconstructed with a high-flexible model-fitting method. To represent the electron density distribution along magnetic field lines, we adopt a diffusive equilibrium (DE) model where gravity and diffusion to a plasma particle are balanced, as shown in Fig.1. We also assume that the electron density at an equatorial plane depends on L-value. Then, the global electron density profile $N(L,s)$ is represented by

$$N(L,s) = N_{\mathrm{eq}}(L) \cdot N_{\mathrm{de}}(L,s), \tag{1}$$

$N_{\mathrm{eq}}(L)$: electron density at an geomagnetic equatorial plane,
$N_{\mathrm{de}}(L,s)$: effect of DE,
L : L-value,
s : field-aligned distance from the equator.

We adopt a stochastic difference model for the representation of $N_{\mathrm{eq}}(L)$ instead of conventional parametric models. Namely, in order to represent the trend component of electron density distribution at the equatorial plane, we adopt a second order stochastic difference equation. This model is often used in time series analysis for the estimation of a trend component. Then, giving a smoothing parameter which determines the smoothness of a trend component of $N_{\mathrm{eq}}(L)$ and L-dependent DE parameters, we can calculate $N_{\mathrm{eq}}(L)$ from observed electron densities along satellite trajectory through an appropriate filtering process. In the model, the number of free parameters is up to six in the global electron density model.

2.2 Fitting method with ray tracing

Fig.2 shows the algorithm to obtain an evaluation index for a parameter set of the electron density model. When we give appropriate values to the model parameters, the wave normal direction and the spectrogram of a nonducted whistler can be theoretically calculated by ray tracing technique. In ray tracing, the IGRF (international geomagnetic reference field) model is used to represent a geomagnetic field. Then, the model parameters are evaluated by the degree of fitness between the theoretically calculated values and observed ones of wave normal direction and arrival time dependence of each frequency wave.

In nonlinear least squares, we can not deal with the observations of wave characteristics from multipaths since any probability density function of the error is characterized by a Gaussian distribution. To address this problem, it is effective to represent observed and theoretical wave charcteristics as stochastic variables which can have arbitrary probability distribution functions [2]. The optimum set of parameters is determined by a grid search method. It is important to verify the stability and the uniqueness of the obtained solution, and thus the grid search is the best way to derive the best solution though it is a very time-consuming procedure.

3 Computer simulation

3.1 Conditions of ray tracing

In the ray tracing, we calculated ray paths of whistler mode waves whose frequencies were from 1kHz to 10kHz, and we reconstructed the spectrograms of nonducted whistlers using the propagation delay times from the initial points to the observation point. We assumed the initial locations at an altitude of 500km, and plural initial wave normal directions were given in a certain range. The range of the initial directions were determined from the electron densities at an altitude of 500km and localtime. The delay time to 500km which was added to the result of ray tracing were also determined from the densities. In this simulation, we assumed the distributed wave sources located in the southern hemisphere, and the initial locations of ray tracing were assumed from L-value of 2 to 4 at geographic longitude of 130°E. The number of traced rays was totally up to 50,000 for one electron density profile.

Fig.3 shows an example of a theoretically calculated wave characteristics of nonducted whislters under the above-mentioned conditions of ray tracing. The right panel in the figure shows a given electron density profile. In this panel, "X" shows a supposed observation point. The left upper and lower panels show the probability distributions of wave normal direction and spectrogram at the observation point, respectively. Using this probability distribution, the theoretical spectrogram can be reconstructed by multiplying the initial electric or magnetic field strength and decrement ratio. Since all possible initial values are taken into account in the numerical calculation of ray tracing, any spectrogram at the observation point is a part of the distribution under the condition of a given electron density profile. The given profile is, therefore, evaluated by the fitness of the probability distributions and observations with observational errors.

3.2 Verification of the uniqueness of the solution

We verified the uniqueness of a solution of the developed method using a simulated electron density profile. First, using the density profile, we theoretically calculated the wave characteristics at eight hundred observation points arranged all over the plasmasphere. On the other hand, various types of independent density profiles are constructed by discretizing values of the parameters in the electron density model. The number of the profiles was up to two hundred fifty. Under the condition of each profiles, we derived distributions of the wave characteristics at the same observation points and calculated the quantified total fitness to the given wave characteristics. Then, we examined the relations between the error of the various types of electron density profiles from the simulated profile and the fitness of the wave characteristics. Fig.4 shows the scatter plot of the fitness of the wave characteristics for the errors of the profiles from the simulated one. In the abscissa, the fitness is represented, and the error of the electron density profile is shown in the ordinate. From the figure, the error of the density profile is found to be always small for good fitness, that is, the solution derived from the total fitness was guaranteed to be unique.

It is not realistic that we can get in-situ wave characteristics all over the plasmasphere simultaneously. Then, we examined the case that the observation points are limited along satellite trajectory. In the present study, we assumed a polar orbit at an altitude of 2.5 earth's radius and twelve observation points along the trajectory. Fig.5 shows the same kind of scatter plot in this case. The number of examined electron density profiles were about two hundreds. which have differences only at lower altitudes than the simulated trajectory. In the figure, same kind of trend as the previous case can be found though it is not very clear.

4 Conclusions

In the present study, we improved a reconstruction method of electron density profiles in the plasmasphere in order to use nonducted whislters. In the computer simulation, we confirmed the uniqueness of the soltion obtained by the developed mehtod when the wave characteristics were observed along an appropriate trajectory. In further study, it is necessary to overview more cases of observational conditions and reveal the reconstruction accuracy.

References

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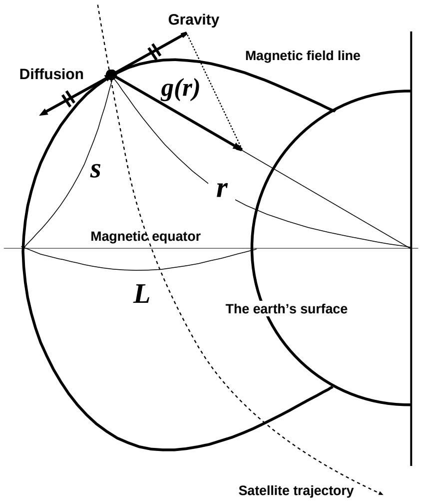

Figure 1: Diffusive equilibrium (DE) model

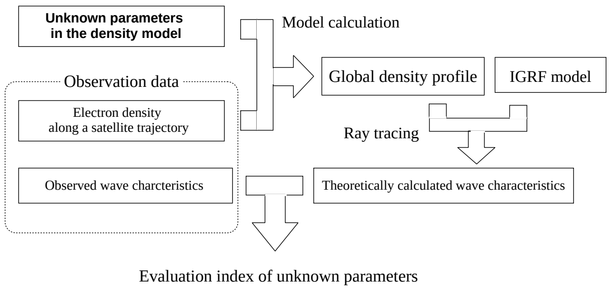

Figure 2: Flowchart of the fitting algorithm.

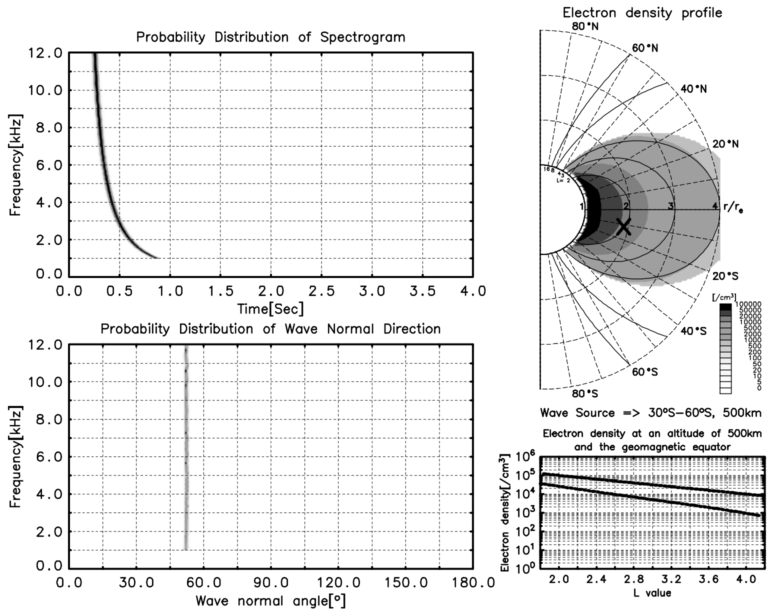

Figure 3: Probability distributions of wave characteristics by ray tracing.

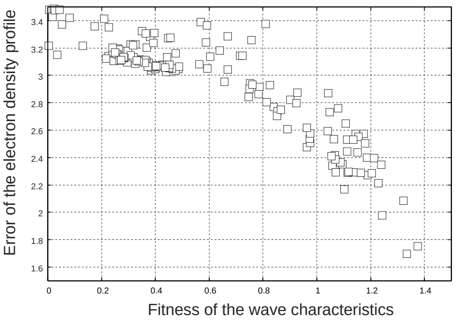

Figure 4: Case that fitness are derived from the wave characteristics all over the plasmasphere.

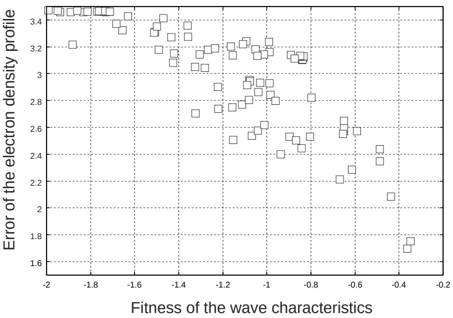

Figure 5: Case that Fitness are derived from the wave characteristics along one simulated trajectory.