METHOD OF MOMENTS COMPUTATIONAL STORAGE MINIMIZATION

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Abstract

In order to store the information of a Method of Moments (MoM) matrix one can save the maximum of computational memory by the application of a mapping procedure to the subsectional basis function approach. The so-called law of mapping is illustrated by using the subsectional basis function approach to the mixed potential integral equation (MPIE).

Summary

The numerical solution of electromagnetic field problems usually requires the solution of large systems of linear equations. Considering, for example, two dimensional conducting objects (patches) embedded in a stratified medium one characterizes them by certain boundary conditions. An integral equation can be derived with the sole unknown being the true electric surface current. Replacing these planar structures by an electric current sheet one is able to reduce the integral equation by the MoM to a matrix algebraic equation which is then solved on the computer. If one takes advantage of the rotational and translational symmetry of the Green's functions in the transverse directions, one can map the whole MoM-matrix onto the first row of the matrix. Thus the whole information of a $N \times N$ matrix can be stored in the computational memory of a $1 \times N$ matrix. Let us now

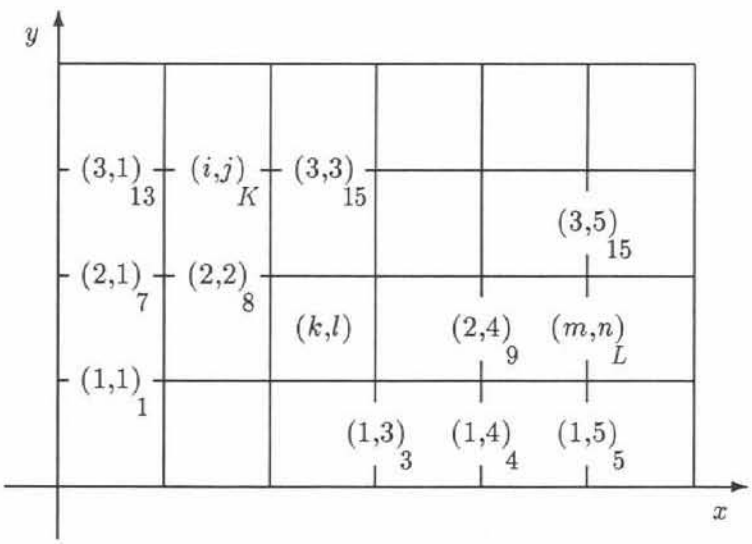

Figure 1: Geometry under investigation

apply the procedure of mapping to the subsectional basis function approach in the MoM. Following [1], there is an area A given by $k_y \times l_x$ equivalent subsections $A_{k,l}$. Taking two adjacent elementary cells $A_{k,l}$ that share a common border perpendicular to the x-direction, we introduce a new kind of subsections, the so-called x-directed subsections $A^x_{m,n}$. Taking two adjacent elementary cells $A_{k,l}$, that share a common border perpendicular to the y-direction, we generate the so-called y-directed subsections $A^y_{i,j}$. These directed subsections satisfy $A^x_{m,n} = A_{k,l} + A_{k,l+1}$ and $A^y_{i,j} = A_{k,l} + A_{k+1,l}$. The arrangement is classified in an example in Fig.(1). The indices (i,j) and (m,n) are used within the intervals $(i,j) \in [(1,1),(k_y-1,l_x)]$ and $(m,n) \in [(1,1),(k_y,l_x-1)]$ with $(i_y,j_x)=(k_y-1,l_x)$ and $(m_y,n_x)=(k_y,l_x-1)$, respectively . The area A is composed of 4×5 equivalent subsections $A^x_{m,n}$ in the x-direction or 3×6 equivalent subsections $A^y_{i,j}$ in the y-direction.

Let us now introduce correspondences c that depend on the mean value of the distance between two weighted subsections $A_{i,j}$ and $A_{m,n}$. In general we describe them by

$$c := c(A^p_{i,j}(s) \longleftrightarrow A^q_{m,n}(t)) \, , \, c \in \mathcal{C} \tag{1}$$

with $\mathcal{C}$ is the set of all correspondences c. The superscripts p and q represent the orientation of the specific subsection in p-direction and q-direction, in our case $p = y$ and $q = x$. The symmetric or antisymmetric functions which weight the subsections, are given by the indices s and t. Even functions are represented by (e), odd functions by (o). Fig.(2) illustrates the symbolic writing in Eq.(1) with $s = e$ and $t = o$. In connection with the application of the MoM to the MPIE the

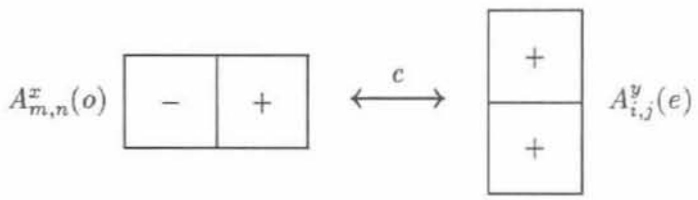

Figure 2: Correspondence c

function $A^x_{m,n}$ represents the cartesian component of the surface charge q generated by a x-directed surface current $\bar{I}_x$. The subsections $A^y_{i,j}$ refer to the so-called razor test functions.

Working with the surface charge q and a surface current distribution $\bar{I}^T = (I_x, I_y)n^T$ we choose two-dimensional pulse functions and rooftop-type functions for the basis functions as well as uni-dimensional pulse functions for the test functions as shown in Fig.(3).

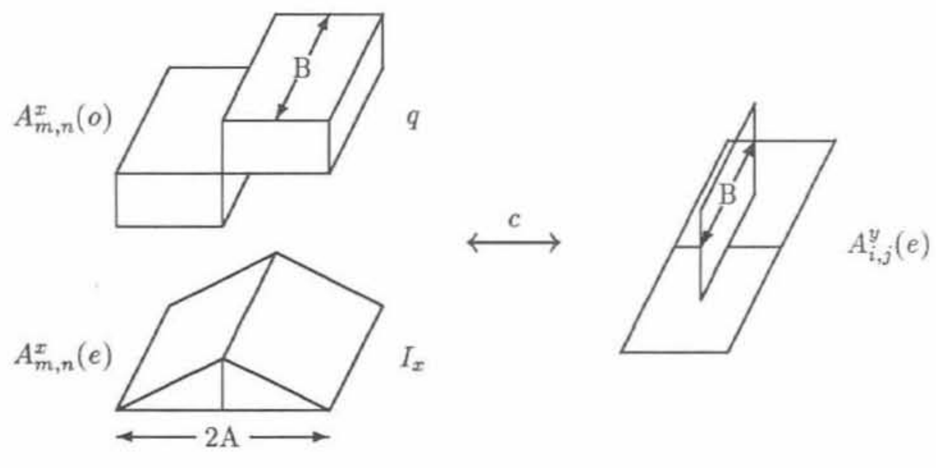

Figure 3: Basis and test functions

Following [1] we get the required **law of mapping**:

$$c(A^p_{i,j}(s) \longleftrightarrow A^q_{m,n}(t)) \longmapsto CoS * a((|m-i| + \delta_y) * n^q_x + |n-j| + 1 + \delta_x) \tag{2}$$

Eq.(2) maps all the correspondences c onto an one-dimensional array $\bar{a}^T$. Introducing a general correspondence matrix

$$\overline{\overline{C}}^{pq}(s,t) = \sum_K \sum_L \bar{e}_K C^{pq}_{K,L}\,(s,t)\,\bar{e}^T_L\ , \tag{3}$$

on account of Eq.(2) we can assign every element $C^{pq}_{K,L}\,(s,t)$ of the correspondence matrix $\overline{\overline{C}}^{pq}\,(s,t)$ to one element $a(L)$ of the array $\bar{a}^T$. It is obvious that the array $\bar{a}^T$ represents exactly the first row in our correspondence matrix $\overline{\overline{C}}^{pq}\,(s,t)$. We obtain

$$\bar{a}^T = \sum_L \bar{e}_1 C^{pq}_{1,L}\,(s,t)\,\bar{e}^T_L. \tag{4}$$

Depending on which symmetry relations $i \longleftrightarrow m$ and $j \longleftrightarrow n$ are used the additional terms $\delta_y = m^q_y - i^p_y$ and $\delta_x = n^q_x - j^p_x$ must be added to Eq.(2). In all, we have to distinguish between the four cases in Fig.(4). Considering the weigting functions one discovers that they directly interfere

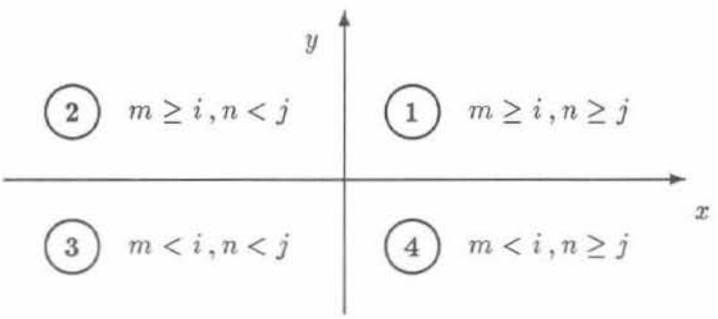

Figure 4: The four cases

with the four cases of Fig.(4), too. Defining the mean distance for correspondences c being always positive in the first quadrant the introduction of weighted subsections forces only a change of sign (CoS). Applying the procedure of mapping to the MoM-matrix equation for ideal conductivity [2],

$$\left(\begin{array}{c|c} \overline{\overline{C}}^{xx} & \overline{\overline{C}}^{xy} \\ \hline \overline{\overline{C}}^{yx} & \overline{\overline{C}}^{yy} \end{array}\right) \left(\begin{array}{c} \mathbf{I}_x \\ \hline \mathbf{I}_y \end{array}\right) = \frac{1}{jZ_0} \left(\begin{array}{c} \mathbf{V}^{(e)}_x \\ \hline \mathbf{V}^{(e)}_y \end{array}\right), \tag{5}$$

the contributions of the surface current I and the surface charge q in the system matrix $\overline{\overline{C}}$ can be written as

$$\overline{\overline{C}} = \left(\begin{array}{c|c} \overline{\overline{C}}^{xx\,(I)}(e,e) + \overline{\overline{C}}^{xx\,(q)}(e,o) & \overline{\overline{C}}^{xy\,(q)}(e,o) \\ \hline \overline{\overline{C}}^{yx\,(q)}(e,o) & \overline{\overline{C}}^{yy\,(I)}(e,e) + \overline{\overline{C}}^{yy\,(q)}(e,o) \end{array}\right). \tag{6}$$

Let us now demonstrate the law of mapping by an example. If we assume that there are basis functions $A^q_{m,n}(t)$ and test functions $A^p_{i,j}(s)$ defined over on area of 4×6 equivalent cells, the correspondence matrix $\overline{\overline{C}}^{yx\,(q)}(e,o)$, for example, can be easily computed. The additional terms $\delta_y = m^x_y - i^y_y = 4 - 3 = 1$ for $m < i$ (cases 3 and 4) and $\delta_x = n^x_x - j^y_x = 5 - 6 = -1$ for $n < j$ (cases 2 and 3) must be added to Eq.(2). Following [1] we get the change of sign matrix $\overline{\overline{F}}$, which means a negativ sign for the cases 2 and 3. With elementary correspondences

$c^* \in \mathcal{C}^*, \mathcal{C}^* := \{\mathbf{+1,+2,+3,+4,+5,+1,+2,+3,+4,+5,+6,+7,+8,+9,+10,+11,+12,+13,+14,+15}\}$, the matrix is explicitly given by

$$\left[\begin{array}{cccccccccccccccccccc}
\mathbf{+1} & \mathbf{+2} & \mathbf{+3} & \mathbf{+4} & \mathbf{+5} & \mathbf{+1} & \mathbf{+2} & \mathbf{+3} & \mathbf{+4} & \mathbf{+5} & \mathbf{+6} & \mathbf{+7} & \mathbf{+8} & \mathbf{+9} & \mathbf{+10} & \mathbf{+11} & \mathbf{+12} & \mathbf{+13} & \mathbf{+14} & \mathbf{+15} \\
-1 & +1 & +2 & +3 & +4 & -1 & +1 & +2 & +3 & +4 & -6 & +6 & +7 & +8 & +9 & -11 & +11 & +12 & +13 & +14 \\
-2 & -1 & +1 & +2 & +3 & -2 & -1 & +1 & +2 & +3 & -7 & -6 & +6 & +7 & +8 & -12 & -11 & +11 & +12 & +13 \\
-3 & -2 & -1 & +1 & +2 & -3 & -2 & -1 & +1 & +2 & -8 & -7 & -6 & +6 & +7 & -13 & -12 & -11 & +11 & +12 \\
-4 & -3 & -2 & -1 & +1 & -4 & -3 & -2 & -1 & +1 & -9 & -8 & -7 & -6 & +6 & -14 & -13 & -12 & -11 & +11 \\
-5 & -4 & -3 & -2 & -1 & -5 & -4 & -3 & -2 & -1 & -10 & -9 & -8 & -7 & -6 & -15 & -14 & -13 & -12 & -11 \\
+6 & +7 & +8 & +9 & +10 & +1 & +2 & +3 & +4 & +5 & +1 & +2 & +3 & +4 & +5 & +6 & +7 & +8 & +9 & +10 \\
-6 & +6 & +7 & +8 & +9 & -1 & +1 & +2 & +3 & +4 & -1 & +1 & +2 & +3 & +4 & -6 & +6 & +7 & +8 & +9 \\
-7 & -6 & +6 & +7 & +8 & -2 & -1 & +1 & +2 & +3 & -2 & -1 & +1 & +2 & +3 & -7 & -6 & +6 & +7 & +8 \\
-8 & -7 & -6 & +6 & +7 & -3 & -2 & -1 & +1 & +2 & -3 & -2 & -1 & +1 & +2 & -8 & -7 & -6 & +6 & +7 \\
-9 & -8 & -7 & -6 & +6 & -4 & -3 & -2 & -1 & +1 & -4 & -3 & -2 & -1 & +1 & -9 & -8 & -7 & -6 & +6 \\
-10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & -5 & -4 & -3 & -2 & -1 & -10 & -9 & -8 & -7 & -6 \\
+11 & +12 & +13 & +14 & +15 & +6 & +7 & +8 & +9 & +10 & +1 & +2 & +3 & +4 & +5 & +1 & +2 & +3 & +4 & +5 \\
-11 & +11 & +12 & +13 & +14 & -6 & +6 & +7 & +8 & +9 & -1 & +1 & +2 & +3 & +4 & -1 & +1 & +2 & +3 & +4 \\
-12 & -11 & +11 & +12 & +13 & -7 & -6 & +6 & +7 & +8 & -2 & -1 & +1 & +2 & +3 & -2 & -1 & +1 & +2 & +3 \\
-13 & -12 & -11 & +11 & +12 & -8 & -7 & -6 & +6 & +7 & -3 & -2 & -1 & +1 & +2 & -3 & -2 & -1 & +1 & +2 \\
-14 & -13 & -12 & -11 & +11 & +9 & -8 & -7 & -6 & +6 & -4 & -3 & -2 & -1 & +1 & -4 & -3 & -2 & -1 & +1 \\
-15 & -14 & -13 & -12 & -11 & -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & -5 & -4 & -3 & -2 & -1
\end{array}\right]$$

The first row of the matrix is written in bold type and the law of mapping (2) is verified for all matrix elements.

Conclusion

Consequent application of the subsectional basis function approach of the Method of Moments to the Mixed Potential Integral Equation yields a simple algorithm which reduces the computational storage of the MoM-matrix to a minimum. Applying a procedure of mapping (2) it is possible to set up a matrix containing only one row. Thus the information of a quadratic $N \times N$ matrix requires only the storage of a $1 \times N$ matrix. For a $10^3 \times 10^3$ matrix, for example, this represents a reduction in storage by three orders of magnitude. That is clearly the minimum storage to be needed.

References

[1] S. Sattler, P. Russer, "A Note on the Computational Storage in the Subsectional Basis Function Approach to the Method of Moments," **to be published in**: *International Journal of Numerical Modelling Electronic Networks, Devices and Fields*, 1992.

[2] J. R. Mosig and F. E. Gardiol, "General integral equation formulations for microstrip antennas and scatterers, "*IEE Proc., Part H: Microwaves, Opt. Antennas*, **Vol. 132**, Part H, no.7, pp. 424-432, 1985.