

THE METHOD OF MULTILEVEL MOMENTS FOR ELECTROMAGNETIC SCATTERING PROBLEMS

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INTRODUCTION

The method of moments is called as a lower frequency method because N^2 storage space, which maybe exceed the storage of a computer, is required for N expansion coefficients of the unknown function, thus it is not suitable for higher frequency problems. On the other hand, N^3 order of the multiplicative operations, for example $N^3/8$ for Gaussian elimination, must be taken for these coefficients. Recently, some iterative methods, such as the conjugate gradient method (CGM) with fast Fourier transform (CG-FFT) and the spatial decomposition technique (SDT)¹⁻³, are developed. But some scores of or hundreds of iterations must be done, and the iterative procedure is unstable for the round-off error in the CGM; several or some scores of iterations are required and the convergent rate depends on the coupling between the substructures in the SDT.

The method of multilevel moments (MoMM) is also an iterative method. Expanding the unknown function by some different sets of bases, then a set of linear equations are formulated. The relationships between the basis functions of two different sets should be determined by a given transform. Then the residual vector of each level can be transformed into the next lower level. After that the error function of an approximate solution may be calculated and then be transformed into the higher levels as an initial solution for an iterative procedure until the error of the highest level is obtained, which corresponds to a more accurate solution. The numerical results show that only several iterative cycles are necessary for getting a relatively accurate solution by using this method. For which only n^2 storage space is required where n is number of bases in the lowest level; the order of multiplicative operations is N^2 in general, or $N \log_2 N$ if FFT is applied.

FORMULATION

The electric field integral equation (EFIE) of a conducting body is simply written as

$$\bar{G} * \bar{J} = -\bar{E}_t^{i^n} \quad (1)$$

where $\bar{E}_t^{i^n}$ is the tangential components of an incident wave, $\bar{G}*$ means a convolution operator including the dyadic Green's function, and \bar{J} is the unknown current on the surface. Denote that \bar{J}_a is an approximate solution and \bar{J}_e is the exact one of the EFIE(1), then the residual of the equation is $\bar{R} = -\bar{E}_t^{i^n} - \bar{G} * \bar{J}_a$ and the error of \bar{J}_a is $\bar{F} = \bar{J}_e - \bar{J}_a$. Thus the error-residual equation with the same form of eq.(1) is formulated as

$$\bar{G} * \bar{F} = \bar{R} \quad (2)$$

Once \bar{F} is solved from eq.(2), then \bar{J}_e can be determined by $\bar{J}_e = \bar{J}_a + \bar{F}$. The method of multilevel moments can be used to find an approximate \bar{F} .

Let the bases and weighting functions of the m -th level are $\{\bar{x}_i^{(m)}, i = 1, 2, \dots, N_m\}$ and $\{\bar{w}_j^{(m)}, j = 1, 2, \dots, N_m\}$ respectively, where $m = 1, 2, \dots, M$; $N_m = n \cdot l^{M-m}$, both n and l are integers. $m = 1$ and $m = M$ correspond to the highest (1-st) and the lowest (M -th) level respectively. By expanding F in terms of $\{\bar{x}_i^{(m)}\}$ as follows

$$\bar{F}^{(m)} = \sum_{i=1}^{N_m} a_i^{(m)} \bar{x}_i^{(m)} \quad m = 1, 2, \dots, M \quad (3)$$

and substituting it into eq.(2), and then making the inner products with $\{\bar{w}_j^{(m)}\}$, M matrix equations are obtained

$$\mathbf{Z}^{(m)} \mathbf{A}^{(m)} = \mathbf{B}^{(m)} \quad m = 1, 2, \dots, M \quad (4)$$

where $\mathbf{Z}^{(m)}$ is a $N_m \times N_m$ matrix, $\mathbf{A}^{(m)}$ and $\mathbf{B}^{(m)}$ are $N_m \times 1$ vectors. Their elements are

$$z_{ji}^{(m)} = \langle \bar{w}_j^{(m)}, \bar{G} * \bar{x}_i^{(m)} \rangle, \quad i, j = 1, 2, \dots, N_m \quad (5)$$

$$b_j^{(m)} = \langle \bar{w}_j^{(m)}, R \rangle, \quad j = 1, 2, \dots, N_m \quad (6)$$

and the undetermined coefficients $a_i^{(m)}$, respectively.

The relationships between $\bar{x}_i^{(m)}$, $\bar{w}_j^{(m)}$ and $\bar{x}_i^{(m+1)}$, $\bar{w}_j^{(m+1)}$ can be determined by a transform selected in advance as follows

$$\bar{x}_i^{(m+1)} = \sum_{k=1}^l \alpha_k \bar{x}_{(i-1)l+k}^{(m)} \quad (7)$$

$$\bar{w}_j^{(m+1)} = \sum_{k=1}^l \beta_k \bar{w}_{(j-1)l+k}^{(m)} \quad (8)$$

where α_k and β_k are chosen as positive real constants. Because N_m is l times the N_{m+1} , eqs.(7) and (8) represent that $\bar{x}_i^{(m)}$ and $\bar{w}_j^{(m)}$ are divided into N_{m+1} groups and $\bar{x}_i^{(m+1)}$ and $\bar{w}_j^{(m+1)}$ are combined by each groups. Then the relationships between the elements of matrices and vectors are

$$\begin{cases} z_{j'i'}^{(m+1)} = \sum_{k_1=1}^l \sum_{k_2=1}^l \alpha_{k_1} \beta_{k_2} z_{j'i'}^{(m)} & i, j = 1, 2, \dots, N_{m+1} \\ i' = (i-1)l + k_1, \quad j' = (j-1)l + k_2 \end{cases} \quad (9)$$

$$b_{j'}^{(m+1)} = \sum_{k=1}^l \beta_k b_{(j-1)l+k}^{(m)} \quad (10)$$

In the case of electrically large scatterer, N_1 is a relatively large number, a block iterative method is generally used for $\mathbf{A}^{(1)}$. In order to accelerate the convergence, $\mathbf{A}^{(2)}$ is calculated at the second level with less execution time by use of the transform of eq.(7) and eq.(8) to provide a good initial vector of $\mathbf{A}^{(1)}$, and the initial solution of $\mathbf{A}^{(2)}$ is supplied by $\mathbf{A}^{(3)}$, etc., $\mathbf{A}^{(M)}$ can be computed using a direct method because of the lowest dimension n of $\mathbf{Z}^{(M)}$. It should be noted that the dimension of $\mathbf{A}^{(m+1)}$ and $\mathbf{A}^{(m)}$ are different, only the N_m -dimensional expansion of $\mathbf{A}^{(m+1)}$ which denoted by $\hat{\mathbf{A}}^{(m)}$ can acts as an initial vector of $\mathbf{A}^{(m)}$. According to eq.(7), a transform of the expansion is

$$\hat{a}_{(i-1)l+k}^{(m)} = \alpha_k a_i^{(m+1)} \quad k = 1, 2, \dots, l; \quad i = 1, 2, \dots, N_{m+1} \quad (11)$$

Then the procedure of the MoMM is shown as follows: Firstly, compute the residual \bar{R} and $\mathbf{B}^{(1)}$ from an approximate solution \bar{J}_s of eq.(1), and evaluate

$B^{(m)}$ using eq.(10) for $m = 2, 3, \dots, M$. Secondly, solve $A^{(M)}$ using a direct or iterative method, calculate $\hat{A}^{(m)}$ from $A^{(m+1)}$ as an initial vector of $A^{(m)}$ by using eq.(11) and improve it by means of a block iterative method for $m = M-1, M-2, \dots, 1$. The orders of all submatrices in the block iterations should be $n \times n$. Finally, compute $\bar{F}^{(1)}$ from the sets $\{a_i^{(1)}\}$ and $\{\bar{x}_i^{(1)}\}$, and add it to \bar{J}_e to construct a new solution of \bar{J} . The procedure can be repeated until $\|\bar{R}\|$ is small enough. Since the function of $A^{(m+1)}$ ($m \geq 2$) is to provide an initial $\hat{A}^{(m)}$, so an excessive accuracy of $A^{(m+1)}$ is not essential. Generally, only one or two iterations are made at the m -th level. A very accurate \bar{J} can be obtained after only several iterative cycles. It can be known easily that the number of multiplicative operations is about C_1KN^2 with a common program or $C_2KN \log_2 N$ with FFT, where $N = N_1$; K is the number of iterative cycles; C_1 and C_2 are constants.

NUMERICAL RESULTS

Choosing $n = 16$, $l = 2$, $\alpha_1 = \alpha_2 = 1$, $\beta_1 = \beta_2 = 1/2$. The current distributions induced by a normal incident plane wave of some conducting thin wire arrays are analyzed. The incident electric field is parallel with the wires. The length of wires are 1λ , they are spaced in 0.2λ . The current of each wire is expanded using 16 piecewise sinusoidal functions at the first level. When the element number of array is 4, 8, 16 or 32, then $M=3, 4, 5$ or 6, and $N_1=64, 128, 256$ or 512, respectively. Because the execution time per iterative cycle of MoMM and SDT is almost the same, their iteration numbers for a given accuracy 10^{-4} are compared in Fig.1 and Fig.2. As shown in the figures, the iteration number of MoMM is only 3 or 4 for the accuracy, and it is almost not increased with the element number. But that of the spatial decomposition technique (SDT) is much more than MoMM, and the residual is even divergent for the 32-element array. The CPU time on DPS-8 computer is compared in Fig.3 for SDT, CG-FFT, MoMM and the MoMM with FFT (MoMM-FFT). It can be seen that the growth rate of CPU time for the MoMM is less than that of SDT, and the CPU time of MoMM-FFT is less than CG-FFT obviously. The CPU time of MoMM-FFT is almost proportional to the element number. As shown in these results, MoMM is an efficient numerical method for analyzing a large scatterer. The computational efficiency will be more obvious for plate and finite periodic structures, by means of the combination with FFT.

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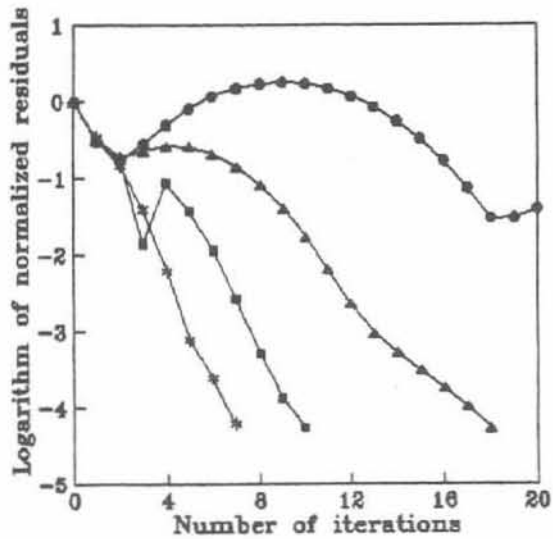


Fig.1 The convergence of SDT for wire arrays

***** 4-element
 ***** 8-element
 ***** 16-element
 ***** 32-element

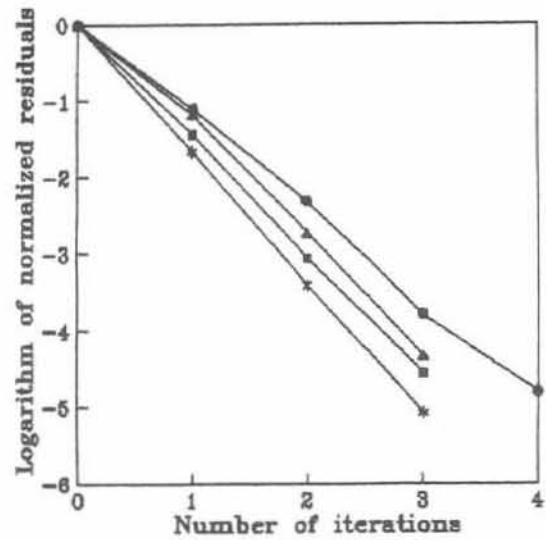


Fig.2 The convergence of MoMM for wire arrays

***** 4-element
 ***** 8-element
 ***** 16-element
 ***** 32-element

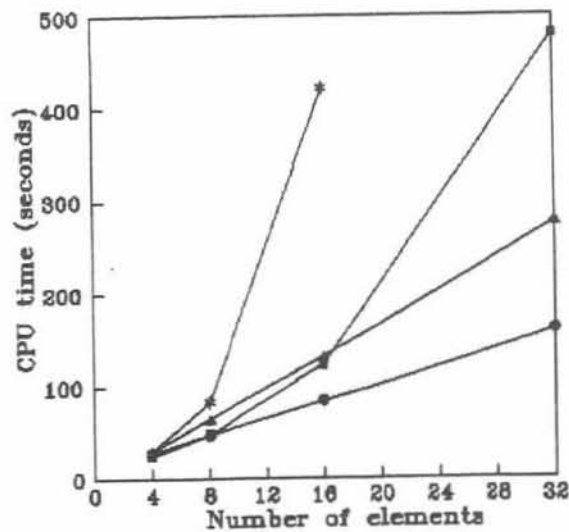


Fig.3 Comparison of CPU time

***** SDT
 ***** CG-FFT
 ***** MoMM
 ***** MoMM-FFT