Studying The Effects of High Order Roots of Array Factor Polynomial On Phased Array's Beamwidth

Ehsan Azordegan[1], Mahmoud Kamarei[2]
[1]*Electrical and computer engineering department, University of Tehran/Iran*
P.o. Box 14515/554, Tehran, Iran, Azordegan@gmail.com
[2] *Electrical and computer engineering department, University of Tehran/Iran*
Phone :(+9821)8800-9354, kamarei@ut.ac.ir

Abstract

Phased array antennas must scan a wide range of space and find the proper direction to send/receive the allocated data. It is desired to find the user directions and also interferer directions to aim the main beam of the array into the desired user and place nulls along interferer directions. There is always a trade off between the number and size of elements and the pattern beamwidth. This article shows how high order roots of array factor can assist in narrowing the overall beamwidth.
For this purpose a computer program was written in MATLAB and the simulation results is included.

1. INTRODUCTION

In studying the pattern of arrays, generally the far field pattern is interested. So, to design a planar array, the far-field pattern and formulation should be summarized, According to [1] the pattern of an array can be expressed as:

$$E_{(total)} = [E_{(single\ \ element\ \ at\ reference\ \ point)}] \times [Array\ \ Factor] \quad (1)$$

This is a basic rule in array patterns. According to this rule, very simple dipoles placed in some geometry, can form an arbitrary pattern. Therefore a desired pattern can be achieved by placing these elements in a proper geometry and feed them with required current. The array factor depends on the current feed of elements and also the way these dipoles have been placed together. To achieve desired pattern with a phased array, the array factor should be configured. The relation between the array factor and designing parameters like spacing, geometry and … is discussed in the following sections and it will be shown how these parameters can affect the entire pattern.

2. FORMULATION

2.1. Uniform Linear array

Referring to geometry of Figure (1) let us assume that all the elements have identical amplitudes but each succeeding element has a β progressive phase lead current excitation relative to the preceding one (β represent the phase by which the current in each element leads the current of the preceding element). The elements are positioned along the z-axis.

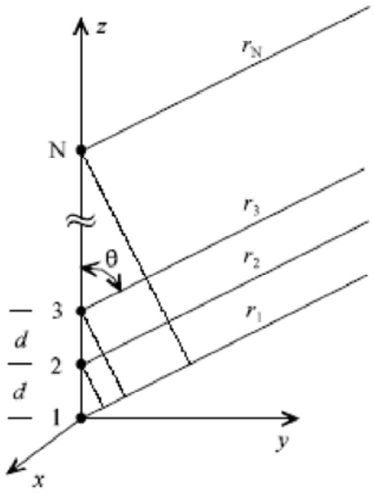

Fig. 1: Uniform Linear array geometry

An array shown in Figure (1) is called "uniform array" as it is equally-spaced and has identical elements with identical amplitudes and equal progressive phase. For this array, the array factor is given by:

$$AF = 1 + e^{+j(kd\cos\theta+\beta)} + e^{+j2(kd\cos\theta+\beta)} + ...+ e^{j(N-1)(kd\cos\theta+\beta)} \quad (2)$$

$$AF = \sum_{n=1}^{N} e^{j(n-1)(kd\cos\theta+\beta)} \quad (3)$$

$$AF = \sum_{n=1}^{N} e^{j(n-1)\psi} \quad (4)$$

Where:
$\psi = kd \cos\theta + \beta$ and k is the wavenumber.

2.2 Planar array

In addition to placing elements along a line (to form a linear array), individual radiators can be positioned along a rectangular grid to form a rectangular or planar array. Planar arrays provide additional variables which can be used to control and shape the pattern of array. One of the most precious features that this new array type provides is the ability to scan the main beam of antenna toward any points of space.

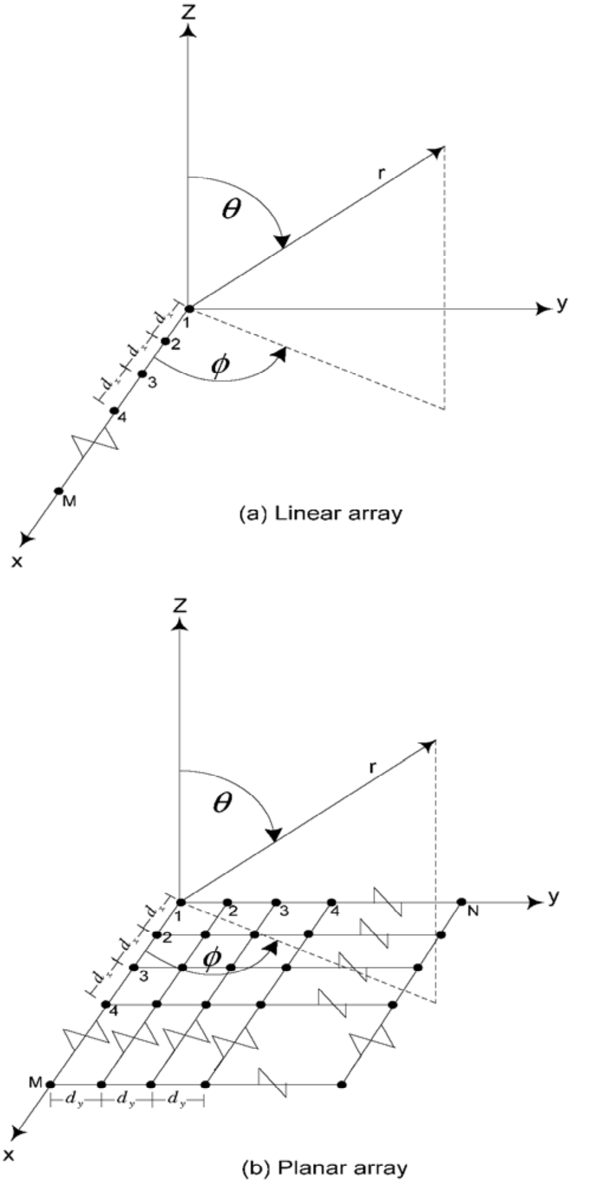

Fig. 2 : Linear and Planar array geometry

To derive the array factor for a planar array, refer to Figure (2), when M elements are initially placed along the x-axis, as shown in figure (2.a), its array factor can be written according to equation (3):

$$AF = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta\cos\phi + \beta_x)} \tag{4}$$

Where I_{m1}, is the excitation coefficient of each element. The spacing and progressive phase shift between the elements along the x-axis are represented respectively, by d_x and β_x. If N such arrays are placed next to each other in the y-direction, a distance d_y apart and with progressive phase β_y, a rectangular array will be formed as shown in figure (2.b) and so the entire array factor can be written as:

$$AF = \sum_{n=1}^{N} I_{1n} \left[\sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta\cos\phi + \beta_x)} \right] e^{j(n-1)(kd_y \sin\theta\sin\phi + \beta_y)} \tag{5}$$

Or:

$$AF = S_{xm} S_{yn} \tag{6}$$

Where:

$$S_{xn} = \sum_{n=1}^{M} I_{m1}\ e^{j(m-1)(kd_x \sin\theta\cos\phi + \beta_x)} \tag{7}$$

$$S_{yn} = \sum_{n=1}^{N} I_{1n}\ e^{j(n-1)(kd_y \sin\theta\sin\phi + \beta_y)} \tag{8}$$

3. SCHELKUNOV METHOD FOR ARRAY DESIGN

3.1 Uniform linear arrays:

One of the methods that can be exploited to design a phased array is Schelkunov circle [2], [4]. A synthesis procedure developed by Schelkunov makes use of the polynomial form of the array factor and presents an insightful technique for pencil-beam pattern synthesis. The array factor of equation (3) for a one-dimensional array can be written in the form of a polynomial in the complex variable z, where

$$F(z) = \sum_{n=1}^{N} a_n z^{n-1} \tag{9}$$

Where:

$$z = e^{j(kd\cos\theta)}$$

With excitation coefficients a_n at each element.

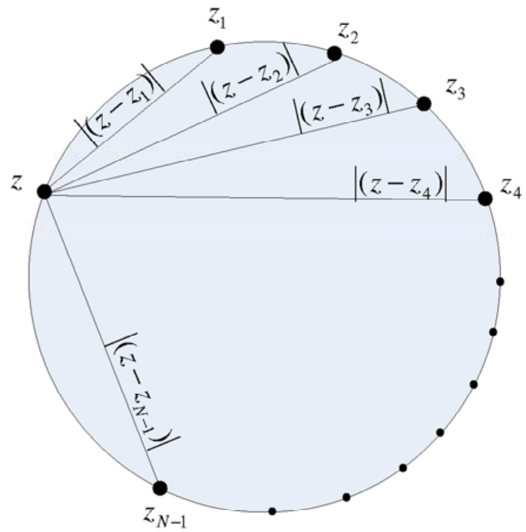

Fig. 3: Geometric representation of the roots of the Schelkunov polynomial

This form is a polynomial of degree $N-1$, where N is the number of elements in the array. Since the polynomial is of degree $(N-1)$, it has $(N-1)$ zeros and may be factored as:

$$F(z) = a_{N-1}(z-z_1)(z-z_2)\cdots(z-z_{N-1}) \quad (10)$$

Where the terms z_n, are the complex roots of the polynomial (as yet unspecified).
The magnitude of the array factor is thus:

$$|F(z)| = |a_{N-1}||(z-z_1)||(z-z_2)|\cdots|(z-z_{N-1})| \quad (11)$$

In this application the roots have the form of $z_i = e^{j(kd\cos\theta_i)}$, so they all occur on the unit circle shown in Figure (3). The magnitude of the array factor, as observed from any point on the unit circle, is the product of the lengths of the straight segments joining that point to the zeros of the array factor.

3.2 Planar arrays:

For planar arrays the procedure is the same. When there are M elements along the x-axis and N elements along the y-axis, we can set (M-1) nulls in x-direction and (N-1) nulls in y-direction. The array factor of equation (4) splits into the form of equation (5) and then the nulls will be dedicated to each of these linear arrays. After dedicating the nulls to the arrays, the roots can be simply calculated via following equations:

For roots #m from (M-1) roots of x-directed array:

$$z_{xm} = e^{j(kd_x \sin\theta_m \cos\phi_m)} \quad (12)$$

For roots #n from (N-1) roots of y-directed array:

$$z_{yn} = e^{j(kd_y \sin\theta_n \sin\phi_n)} \quad (13)$$

So the array factor polynomial can be written as:

$$F(z) = a_{MN} \sum_{m=1}^{M} (z - z_{xm})^{m-1} \sum_{n=1}^{N} (z - z_{yn})^{n-1} \quad (14)$$

It should be noted that the roots might be complex numbers. So the coefficients of the AF polynomial might also be complex.

The array factor of x and y directed arrays can be written as:

$$AF_x = \sum_{m=1}^{M} a_{xm} e^{j(m-1)(kd_x \sin\theta\cos\phi)} \quad (15)$$

$$AF_y = \sum_{m=1}^{M} a_{yn} e^{j(m-1)(kd_x \sin\theta\cos\phi)} \quad (16)$$

And the entire array factor is:

$$AF = AF_x \times AF_y \quad (17)$$

By comparing equations (5) and (14) the current of each element can be written as:

$$I_{mn} = a_{mn} e^{j((m-1)\beta_x + (n-1)\beta_y)} \quad (18)$$

4. DESIGNING PROCEDURE:

In smart antennas and phase array applications it is interested to steer the main beam of the antenna into a desirable location and place nulls in directions of interferes. For that, designing parameters should first be listed:

1- Nulls location
2- Direction of desired user
3- Number (and size) of elements
4- Frequency

After retrieving locations of all the required nulls, they should be divided into (M-1) nulls for x-directed and (N-1) nulls for y-directed arrays. From equations (12) and (13) the roots will be calculated and inserted into equation (14). Then according to (18) the elements' currents will be calculated.
Steering the main beam into the desired angle is the only thing that has left in this design. To maximize the AF in a desired direction, we should derive equations (7), (8) with respect to θ and ϕ. When I_{m1} and I_{1n} are real, the entire array will maximize when these conditions are satisfied simultaneously:

$$\begin{cases} kd_x \sin\theta\cos\phi + \beta_x = 0 \\ kd_y \sin\theta\sin\phi + \beta_y = 0 \end{cases} \quad (19)$$

That results:

$$\begin{cases} \beta_x = -kd_x \sin\theta\cos\phi \\ \beta_y = -kd_y \sin\theta\sin\phi \end{cases} \quad (20)$$

One of the big concerns in phased arrays designing is the beamwidth and side lobe level of designed antenna that usually exceeds the required threshold. As mentioned in [1] it is useful to define a Ω_A angle that shows the beamwidth in the 3D space. The Ω_A is defined as:

$$\Omega_A = \Theta_h \times \Psi_h \quad (21)$$

Where:

Θ_h is the elevation plane half-power beamwidth that can be approximately expressed with:

$$\Theta_h = \sqrt{\frac{1}{\cos^2\theta_0\left[\Theta_{x0}^{-2}\cos^2\phi_0 + \Theta_{y0}^{-2}\sin^2\phi_0\right]}} \quad (22)$$

Where Θ_{x0}, represents the half power beamwidth of a broadside linear array of M elements. Similarly, Θ_{y0} represents the half power beamwidth of a broadside array of N elements. And for square arrays as $\Theta_{y0} = \Theta_{x0}$, the above equation can be simplified to:

$$\Theta_h = \Theta_{x0}\sec\theta_0 = \Theta_{y0}\sec\theta_0 \quad (23)$$

Similarly the half power beamwidth Ψ_h, in the plane that is perpendicular to the $\phi = \phi_0$ elevation is given by:

$$\Psi_h = \sqrt{\frac{1}{\Theta_{x0}^{-2}\sin^2\phi_0 + \Theta_{y0}^{-2}\cos^2\phi_0}} \quad (24)$$

And for square arrays, this will be reduced to:

$$\Psi_h = \Theta_{y0} = \Theta_{x0} \quad (25)$$

So for a square array the Ω_A can be expressed with:

$$\Omega_A = \Theta_{x0}^2 \times \sec\theta_0 \quad (26)$$

Where:

$$\Theta_{x0} = \cos^{-1}\left(\cos\theta_0 - \frac{2.782}{Nkd}\right) - \cos^{-1}\left(\cos\theta_0 + \frac{2.782}{Nkd}\right) \quad (27)$$

In this new method it has been shown that using a typical NxN array with (N-1) second order roots comparing to a NxN array with 2N-2 single roots has much better beamwidth. The results are compared with computed results of the written MATLAB program in the following section.

5. EXAMPLES AND RESULTS

For better understanding of this method a numerical example is included. Designing parameters are:

1- Square array with 5x5 elements

2- Element spacing: $d_x = d_y = 0.65\lambda$

3- Frequency: 2.5 GHz

4-desired direction: $\begin{cases} \theta = 30^\circ \\ \phi = 60^\circ \end{cases}$

5-null locations (in degree) $(\theta^\circ, \phi^\circ)$:

(-20,0),(-45,0),(20,0),(45,0),(-20,70),(-45,70),(20,70),(45,70)

Using the MATLAB program and entering the above data calculates the coefficients of x-directed array as:

a_{x1}=0.6290 a_{x2}=1.0000 a_{x3}=0.8365 a_{x4}=1.0000

a_{x5} =0.6290

And for y-directed array:

a_{1y}=0.7639 a_{2y}=1.0000 a_{3y}=0.8180 a_{4y}=1.0000

a_{5y}=0.7639

Figure (4) shows the 3D array factor. The 3D power gain plot is also represented in Figure (5).the 2D polar plot of array factor in planes $\theta = 30^\circ$ and $\phi = 60^\circ$ is shown in Figures (6) and (7).

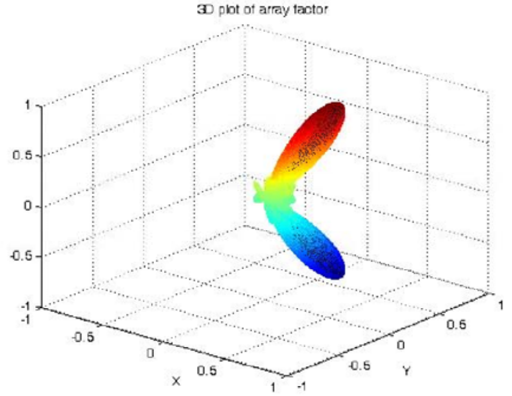

Fig. 4: 3D array factor for single roots

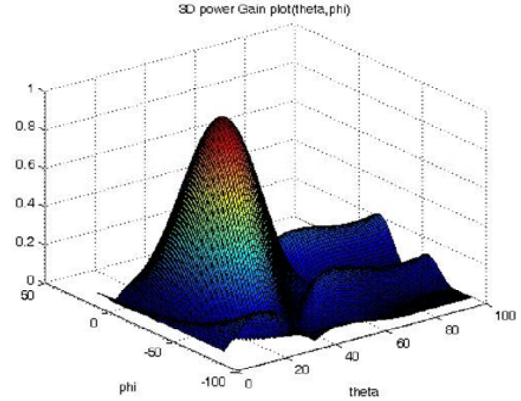

Fig. 5: Power Gain Plot

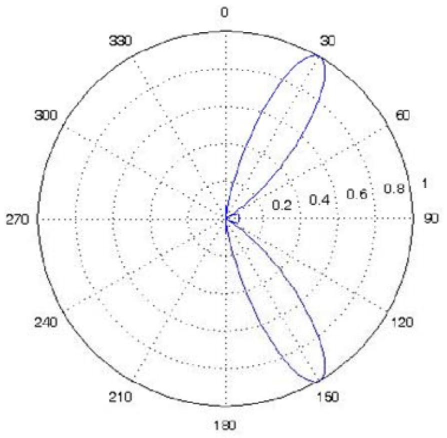

Fig. 6: polar plot of AF in the phi=60 plane

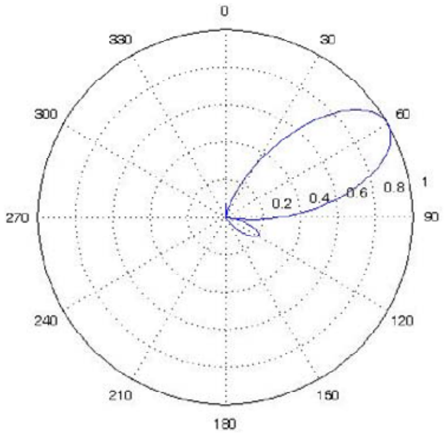

Fig. 7: polar plot of AF in the theta=30 plane

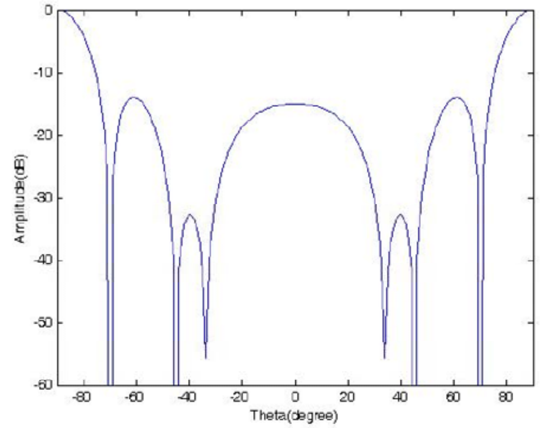

Fig. 8: nulls position of designed AF in phi=0 plane

Analyzing the array factor data show that:

$$\begin{cases} \Theta_h = 20^\circ \\ \Psi_h = 34^\circ \end{cases}$$

Referring to Figure (8), in the designed array the nulls are exactly positioned in their proper place.

Repeating above parameters with a 10x10 array that has the same nulls but with second order roots, results:

$$\begin{cases} \Theta_h = 13^\circ \\ \Psi_h = 25^\circ \end{cases}$$

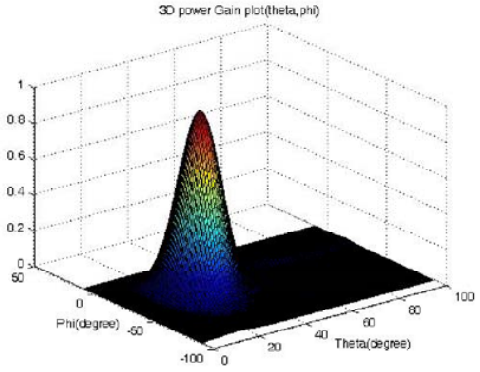

Fig. 9: Power Gain Plot for AF with second order roots

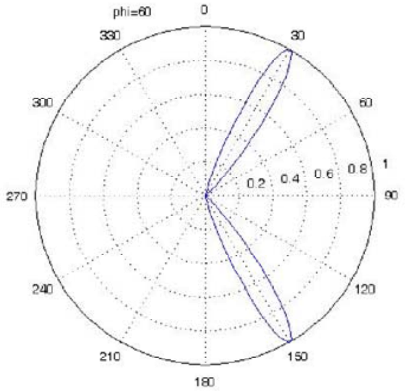

Fig. 10: polar plot of AF with second order roots in the phi=60 plane

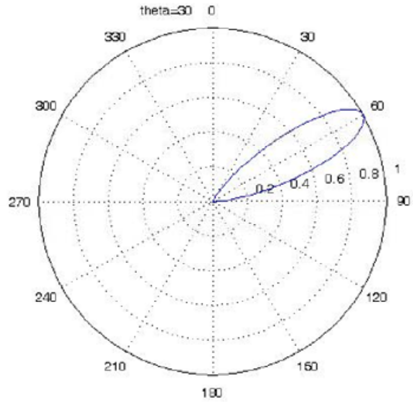

Fig. 11: polar plot of AF with second order roots in the theta=30 plane

The Figures (9), (10), (11) show different plots of the array factor with second order roots.
Repeating these parameters with a 15x15 array that has the same nulls but with third order roots, results:

$$\begin{cases} \Theta_h = 9^\circ \\ \Psi_h = 19^\circ \end{cases}$$

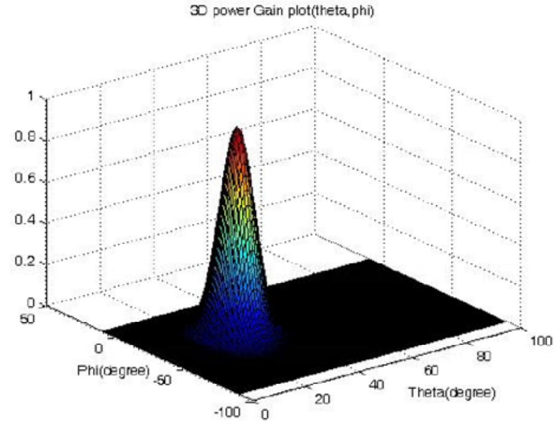

Fig. 12: Power Gain Plot for AF with third order roots

The power gain plot is presented in Figure (12), in addition to narrowing the main beamwidth it is obvious that the side lob levels have been also decreased significantly. (Compared to Figure (5))

6. CONCLUSION

Using high order roots in synthesizing array factor results in better beamwidth and side lob levels. In order to prove this hypothesis, numerical results are presented. For that, merging close nulls and making second (or higher) order roots for AF polynomial is suggested. Given results for synthesized AF have at least 10% better beamwidth (Ω_A) compared to graphs of the same array (in number of elements, frequency and spacing) depicted in [3]. On the other hand, it should be noted that this method is less effective in narrowing the main beamwidth near $\theta = \pi/2$, due to the fact that the phased arrays have an inherent characteristic that its beamwidth is proportional to $\sec\theta$ (see equation (22)) which makes the beamwidth to be widen when θ gets close to $\pi/2$. It seems that other geometries (like spherical) provides better results near $\theta = \pi/2$.

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