METHOD OF DISCRETE SINGULARITIES IN ACCURATE 2-D MODELING OF QUASIOPTICAL REFLECTOR ANTENNAS

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1. Abstract. We study the E-polarized beam wave scattering by finite-size cylindrical reflectors to model quasioptical antennas. The incident complex-source-point (CSP) field simulates a beam generated by a flat aperture feed. Numerical solution is obtained from the singular integral equation (SIE) discretized by using new quadrature formulas of interpolation type with nodes in the nulls of the Chebyshev polynomials.

2. Introduction

Curved metallic reflectors have been studied as quasioptical antennas able to provide very high directivity [1]. Here, basic shape is parabolic one due to the well-known property of collimating the incident beam of a small horn placed to the geometrical focus of parabola. Electromagnetic modeling of reflectors is usually done with "quasioptical" methods, the main being Geometrical and Physical Optics supplemented with Geometrical and Uniform Theories of Diffraction [2,3]. However, these methods are based on the ray tracing and fail to characterize wave effects and resonances. For a more accurate modeling full-wave methods are mandatory, however, popular today Finite-Element and Finite-Difference field solvers require prohibitively large computer resources when applied even to a single - larger than 10-wavelength - reflector in open domain.

All this shows that economic and reliable tools for the modeling of reflector-type scatterers are still in demand. A general way to build such tools is to use the IE approach. Here, the crucial point is development of an efficient discrete model, i.e. a fast and convergent numerical algorithm having controlled accuracy. Here, discrete models based on the analytical preconditioning, i.e. conversion of a singular IE for the surface current function to a Fredholm second kind matrix equation are known [4-5]. We present here the basic ideas of another efficient discrete model based on the Method of Discrete Singularities (MDS) [7-8], and review the results of accurate modeling of quasioptical reflector antennas and waveguides.

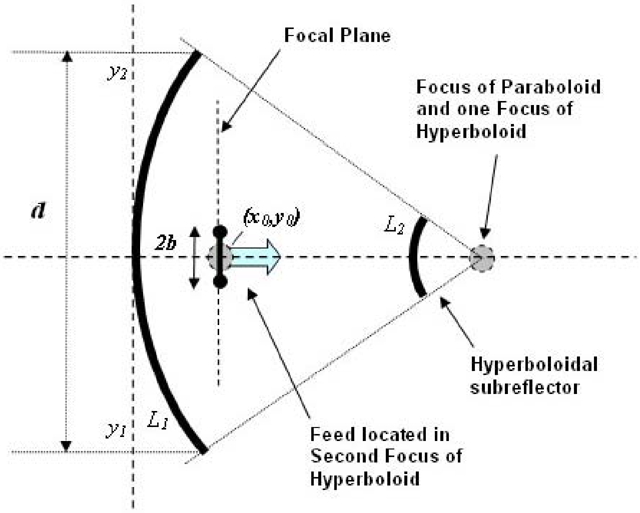

Fig.1 Geometry of a Cassegrain parabolic reflector with hyperbolic subreflector, front-fed by a CSP feed

3. Problem formulation

The geometry of a generic 2-D two-reflector structure can be seen in Fig. 1. Reflectors are assumed to be perfectly electrically conducting (PEC) and have zero thickness. The feed is a line current placed at the complex-valued source point and has time dependence given by $e^{-i\omega t}$. This factor will be omitted in the analysis. Then the field generated by such a CSP feed can be characterized by the z-component of the electric field, which is given by

$$U_0(\vec{r}) = H_0^{(1)}(k|\vec{r} - \vec{r}_c|), \quad \vec{r}_c = \vec{r}_0 + i\vec{b}, \ \vec{r}_0 = (x_0, y_0), \ \vec{b} = (b\cos\beta, b\sin\beta) \tag{1}$$

where $k = \omega / c = 2\pi / \lambda$. Function (1) is a rigorous solution to the Helmholtz equation. It can be shown that (1) simulates a beam wave [6], as it has the maximum oriented in the direction $\phi = \beta$ for any *r*. Besides, it has two branch points at $(x_0 \pm b\cos\beta,\ y_0 \mp b\sin\beta)$, thus to single out a unique value of $U_0(x, y)$ one has to join them with a branch cut *B* of the length 2*b* in the real space that does not intersect the contours of reflectors, $L_{1,2}$. This cut can be considered as a model of the aperture of a real horn feed, whose radiation field is simulated by the CSP field. Fig. 2 shows sample near fields of such feeds. Note that the greater *kb*, the narrower the CSP beam and the further the distance of significant field intensity.

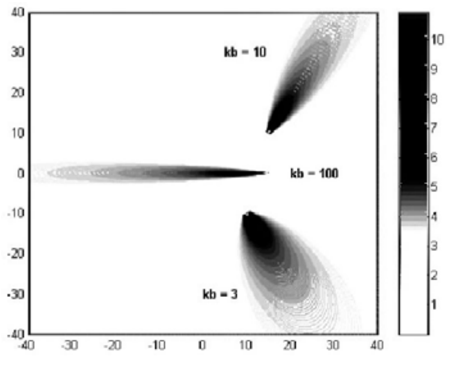

Fig.2 Near fields of CSP feeds having parameters $\beta = \pi / 3, \pi, 5\pi / 3$ and *kb* as indicated

The field (1) is taken as the incident one, thus the total filed is a sum, $U = U_{sc} + U_0$, where $U_{sc}(x, y)$ is the secondary field scattered by the reflector. At first we study the scattering of a 2-D beam-like E-polarized wave (1) by a single curved PEC strip as electromagnetic boundary-value problem. It requests U_{sc} to solve the Helmholtz equation off *L* and satisfy (a) the PEC boundary condition on *L*, (b) the edge condition at endpoints of *L* , and (c) the radiation condition. This problem is reduced to one singular IE of the first kind for the currents induced on the strip *j(s)*:

$$\int_L H_0^{(1)}\left(k\left|\vec{r}(s) - \vec{r}(s_0)\right|\right) j(s)ds = -U_0\left(\vec{r}(s_0)\right), \qquad \mathbf{(2)}$$

Applying contour parameterization *x(t)*, *y(t)* and transforming SIE (2) to another, Cauchy-singular IE with a supplementary condition brings us to

$$\int_{-1}^{1} \frac{v(t)}{t - t_0} \frac{dt}{\sqrt{1-t^2}} + \int_{-1}^{1} K(t, t_0) v(t) \frac{dt}{\sqrt{1-t^2}} = f(t_0) \ , \qquad \int_{-1}^{1} M(t) v(t) \frac{dt}{\sqrt{1-t^2}} = c \qquad \mathbf{(3)}$$

Here *K, M, f* are smooth functions, *c* is a known constant, and *v(t)* is the new smooth unknown function. IE (3) is further discretized by using the quadrature formulas of interpolation type with the nodes in the nulls of the Chebyshev polynomials of the first kind: $t_i^n = \cos\left[\pi\left(2i - 1\right)/2n\right]$, $T_n(t_i^n) = 0$, $i = 1, 2, \ldots n$, and the second kind $t_{0j}^n = \cos\left[\pi j / n\right]$, $U_{n-1}(t_{oj}^n) = 0$, where $j = 1, 2, \ldots n - 1$. As a result we get a set of linear equations

$$\frac{1}{n}\sum_{i=1}^{n} \frac{v(t_i^n)}{t_i^n - t_{0j}^n} + \frac{1}{n}\sum_{i=1}^{n} K(t_i^n, t_{0j}^n) v(t_i^n) = \frac{1}{\pi} f(t_{0j}^n), \quad (j = 1, 2, \ldots n-1) , \qquad \frac{1}{n}\sum_{i=1}^{n} M(t_i^n) v(t_i^n) = \frac{1}{\pi} c, \quad (j = n) \qquad \mathbf{(4)}$$

The solution of this set yields the desired surface-current function *v(t)*, which can be further used in numerical reflector modeling. This method of numerical solution is a new one, developed recently [7,8]. It enables us to study the wave propagation and scattering for reflector antennas and beam waveguides with high accuracy. Near and far field patterns, efficiency of power transmission, and losses can be readily computed for various reflectors, feed locations, and beam widths.

4. Numerical results. ***Far and near fields of symmetric parabolic reflector***

Consider a single front-fed parabolic reflector illuminated by a CSP feed. Here, reflector's contour is $L = \left\{(x, y) \in R^2 : x = 2py^2,\ y_1 < y < y_2\right\}$, and the feed aperture center is placed at the geometrical focus of parabola, i.e. at $(x_0, y_0) = (p / 2, 0)$, (Fig. 3)

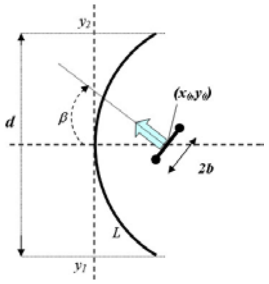

Fig.3 Geometry of a parabolic reflector front-fed by a CSP feed

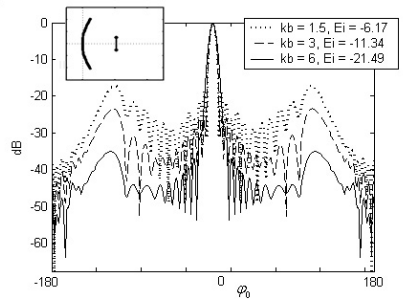

Fig.4 Far-field RPs of antenna shown in Fig. 3 with $\beta = \pi$

The radiation patterns (RP) for several different values of *kb* are shown in Fig. 4 for the reflector whose aperture is $d = 30\lambda$. One can witness the main reflected beam and two symmetric spillovers, which get lower when the edge illumination is reduced. Here, the width of the main beam stays the same as it is determined primarily by the electrical size of reflector. In Fig. 5, one can see the near-field portraits of reflectors fed by different-*kb* feeds. Here, edge illuminations are –6 dB and –21 dB, respectively.

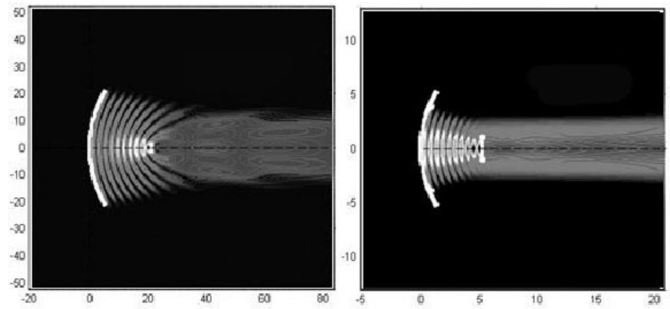

Fig. 5 Near fields for a $d = 10\lambda$ reflector fed by a CSP having $kb = 1.5$ (left) and $kb = 6$ (right), $\beta = \pi$.

Far and near fields of offset parabolic reflector

The following example is an offset reflector fed by a CSP feed (see Fig. 6). The radiation patterns (RP) for several different values of *kb* are shown in Fig. 7 for the reflector whose aperture is $d = 15\lambda$. Unlike a front illumination case, here one can see the main beam and two non-symmetric spillover lobes.

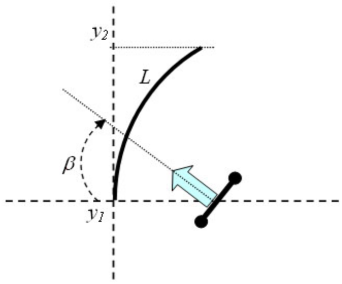

Fig. 6 Geometry of offset parabolic reflector antenna.

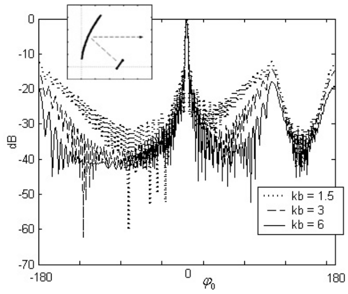

Fig.7 Far-field RPs of antenna shown in Fig. 6. $\beta = 7\pi/9$

Fig. 8 demonstrates the capabilities of the modeling of electrically large reflector antenna with MDS-based numerical algorithm. Here, offset reflector has the aperture of $d = 47\lambda$ that is clearly in the quasioptical domain. Accurate reconstruction of the near field reveals an amazing interference pattern of several types of waves. “Searchlight” main beam is well visible in the specular direction, while the other waves are apparently scattered by two differently illuminated edges.

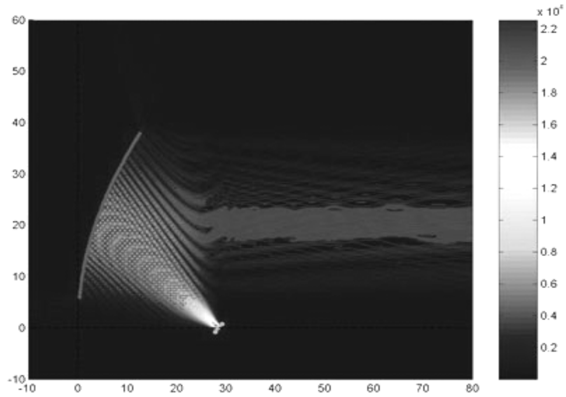

Fig.8 Near field for a quasioptical reflector antenna. $d = 47\lambda$, $kb = 11$, $\beta = 7\pi/9$

Near field focusing with elliptic reflectors

Consider now a single elliptic reflector, whose contour is $L = \left\{(x, y) \in R^2 : \ (x/a)^2 + (y/A)^2 = 1, \ \ y_1 < y < y_2\right\}$, and a CSP feed placed is in the geometrical focus F_1 (Fig. 9). This geometry is interesting for the near-field focusing applications such as mm-wave plasma heating and microwave laser pumping. Fig.10 illustrates how the reflected rays gather at the other focus F_2.

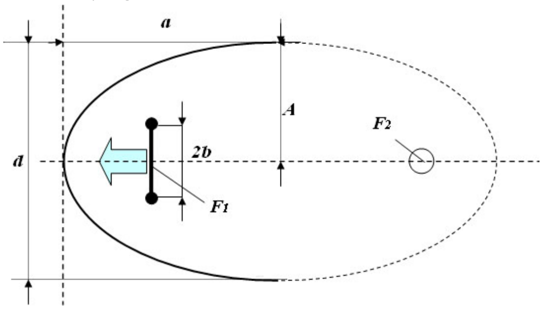

Fig. 9 Geometry of elliptic reflector fed by a CSP feed.

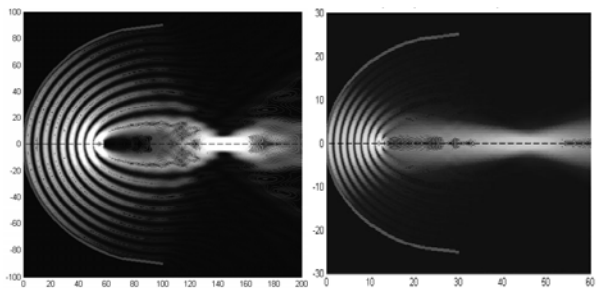

Fig.10 Near fields of half-ellipse focusers having d=14λ, kb=0.5 (left) and d=30λ, kb=2 (right).

Next are some offset elliptic reflectors showing examples of the focusing for the various d/λ and kb values. In each case, the focusing in the other focus is seen clearly (Figs. 11 and 12) and can be accurately quantified if a numerical optimization is desired.

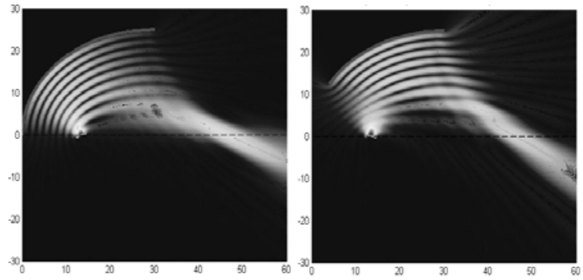

Fig. 11 Near fields of single offset focusers. Quarter-ellipse with d=12.5λ, kb=2 (left) and 1/6-ellipse with d=9λ, kb=2 (right).

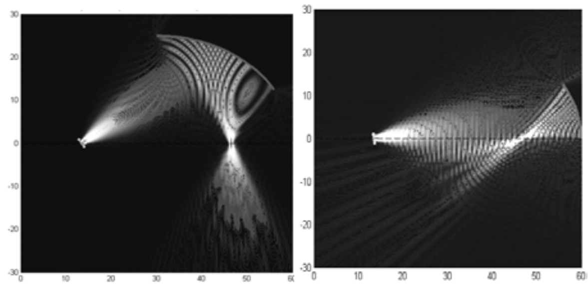

Fig. 12 Near fields of single offset focusers. 1/6-ellipse with d=46λ, kb=10 (left) and 1/12-ellipse with d=20λ, kb=10 (right).

Consider now a cassegrain reflector antenna case shown in Fig. 1. The simulation of two-reflector structure (parabolic L_1 and hyperbolic L_2) leads to the pair of coupled SIEs. After applying the MDS discretization, we get a block-matrix equation, which has the same properties as the matrix equation for a single reflector. Fig. 13 shows the near field for sample cassegrain geometry front-fed with CSP.

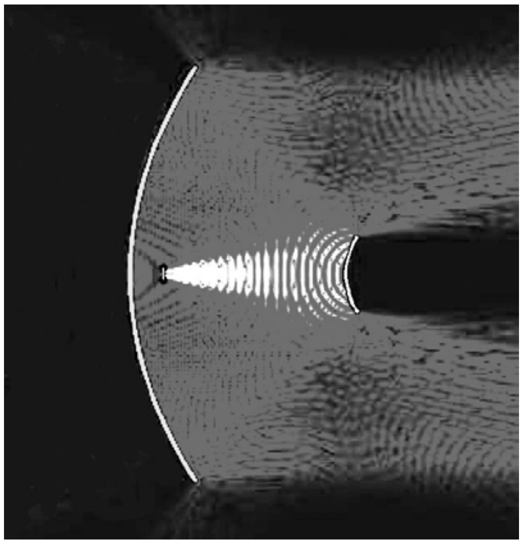

Fig. 13 Near field of a cassegrain antenna with kb=9, d_1/λ=100, d_2/λ=20, f_1/d_1=0.5, β=0.

5. Conclusions

The main advantage of the presented numerical method based on MDS is that it enables one to compute 2-D models of real-life reflector antennas of arbitrary shape mostly in minutes, with desired accuracy. For very moderate computing resources, even household, like for example 1.2GHz, AMD Athlon desktop computer with 256Mb RAM, the time for computing the near field of a d=100λ single-reflector is approximately 7-10 minutes and can be greatly reduced on more powerful modern multi-processor computer facilities. Thus, the electromagnetic performance of reflector antennas and beam waveguides can be controlled by drawing and analyzing the field patterns, directivity plots, etc., for a multitude of parameters like kb, d/λ , x_0, etc. Analogous method has been developed for the H-polarization case in 2-D as well. Besides, MDS has a potential to be extended to the 3-D reflectors, at least rotationally symmetric ones. Finally, it is relatively straightforward to modify it for imperfect reflectors, either impedance, dielectric, or resistive. Due to a lucky combination of high efficiency and controlled accuracy, the method can be used as a core of the computer optimization software based on the gradient-type approach or on the genetic-algorithm one.

6. References

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