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Analysis of reflector and horn antennas using adaptive integral method

Wei-Bin Ewe, Le-Wei Li, and Mook-Seng Leong

Department of Electrical & Computer Engineering
National University of Singapore, Kent Ridge, Singapore

Email: {engp1136,LWLi,elelms}@nus.edu.sg

Abstract— **This paper presents the adaptive integral method (AIM) to analyse electrically large antennas. The arbitrarily shaped perfectly conducting surfaces are modelled using triangular patches and use electric field integral equation to find the radiation pattern. Method of moments (MoM) will be used to discretize the integral equations and the resultant matrix system will be solved by iterative solver with AIM. Radiation pattern of parabolic reflectors and X-band horn are computed using the proposed method.**

Keywords— **Adaptive Integral Method, Electric Field Integral Equation, Method of Moments.**

I. Introduction

HIGH frequency methods such as Physical Optics and Physical Theory of Diffraction have been used to analyse the reflector antenna and horn antenna. However, the high frequency methods cannot give the accurate approximation to the entire radiation pattern. In order to predict the radiation pattern accurately, one can formulate the problem using integral equation and solve it using method of moment (MoM) [1]. MoM converts the integral equation to a system of linear equations which can be solved by using a direct solver or an iterative solver.

The direct solver requires $O(N^3)$ operations to solve the matrix equation while an iterative solver needs only $O(N^2)$ operations for the matrix-vector multiplication in each iteration. The memory requirement for these two solvers is in the order $O(N^2)$. Such computation complexity and memory requirement are too expensive to solve a radiation problem of electrically large antenna. However, several fast algorithms, such as fast multipole method (FMM) [2,3], adaptive integral method (AIM) [4,5] and its variant, precorrected-FFT (P-FFT) [6], have been proposed to reduce the memory requirement of storage and to speed up the matrix-vector multiplication in the iterative solver.

In this paper, the AIM will be applied for computing the radiation problem of electrically large antennas. We will give the formulation of the integral equation for a perfectly conducting object, a brief description of AIM and numerical results to demonstrate its capability.

II. Formulation

A. Method of Moments

To analyze the radiation problem of an open structure like reflector antenna, electric field integral equation (EFIE) would be used. Consider the tangential component of the electric field on the surface of perfectly conducting object, we can obtain

$$\hat{\boldsymbol{n}} \times \boldsymbol{E}^{inc} = \hat{\boldsymbol{n}} \times \left\{ -j\omega\mu \boldsymbol{A} + \frac{1}{j\omega\epsilon} \nabla\nabla \cdot \boldsymbol{A} \right\}, \qquad (1)$$

where the ω, ϵ and μ are angular frequency, free space permittivity and free space permeability, respectively. $\boldsymbol{A}$ is the vector magnetic potential which is given by

$$\boldsymbol{A} = \int \boldsymbol{J} \frac{e^{-jk|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|} dS', \qquad (2)$$

where $\boldsymbol{J}$ and k are the induced surface current and free space wave number respectively.

The geometry of the structure is modelled using triangular patches. The Rao-Wilton-Glisson (RWG) vector basis function [7], which defined on a pair of triangular patches, is used to expand the electric current and to discretize the integral equation. We apply Galerkin's scheme to test the equation and thus convert it to a system of linear equations

$$\boldsymbol{ZI} = \boldsymbol{V}, \qquad (3)$$

where $\boldsymbol{Z}$, $\boldsymbol{V}$ and $\boldsymbol{I}$ are impedance matrix, excitation vector and coefficients of induced current, respectively. The element in the impedance matrix $\boldsymbol{Z}$ is given by the following:

$$Z_{mn} = \frac{1}{j\omega\epsilon} \int \boldsymbol{f}_m \cdot \int \left(k^2 + \nabla'\nabla' \cdot\right) \boldsymbol{f}_n \frac{e^{-jk|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|} dS' dS, \qquad (4)$$

where $\boldsymbol{f}_n$ denotes the RWG vector basis function.

B. Adaptive Integral Method

The memory requirement and computational complexity of the MoM are $O(N^2)$ and $O(N^3)$ respectively if a direct solver is used to solve the matrix equation. The computation cost is prohibitively expensive for solving problems with a large number of unknowns. The AIM was proposed to reduce the memory requirement and to accelerate the matrix-vector multiplication in an iterative solver. The basic idea of AIM is to split the matrix-vector multiplication into two parts, *i.e.*

$$\boldsymbol{ZI} = \boldsymbol{Z}^{near}\boldsymbol{I} + \boldsymbol{Z}^{far}\boldsymbol{I}, \qquad (5)$$

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where $\boldsymbol{Z}^{near}\boldsymbol{I}$ and $\boldsymbol{Z}^{far}\boldsymbol{I}$ represent near-zone interaction and far-zone interaction respectively. The far-zone interaction is approximated using fast Fourier Transform (FFT) while the near-zone interaction is computed directly using the MoM.

To employ AIM, the object is enclosed in a rectangular grid and then recursively subdivided into small rectangular grids. In order to use FFT to approximate the far-zone interaction, we need to transform the RWG basis function, $\boldsymbol{f}_n$ to the Cartesian grids. We also note that the matrix elements in Eq. (3) can be expressed as a linear combination of the following form

$$Z_{mn} = \int_{T_m}\int_{T_n} \psi_m(r) g(r,r')\psi_n(r')dr'dr, \qquad (6)$$

where the transformation function, $\psi_n(r) = \{\boldsymbol{f}_n, \boldsymbol{\nabla}\cdot\boldsymbol{f}_n\}$. The transformation function, $\psi_n(r)$ can be approximated as a linear combination of Dirac delta functions,

$$\psi_n(r) \approx \widehat{\psi}_n(r) = \sum_{u=1}^{(M+1)^3} \Lambda_{nu}\delta(r-r'), \qquad (7)$$

where M is the expansion order and Λ_{nu} are the expansion coefficients of $\psi_n(r)$. Λ_{nu} can be determined using multipole expansion [5], which is based on the criteria that the coefficients Λ_{nu} produce the same multipole moments of the original basis function. Once the transformation function has been determined, the matrix elements can be approximated as

$$\widehat{Z}_{mn} = \sum_{v=1}^{(M+1)^3}\sum_{u=1}^{(M+1)^3} \Lambda_{mv} g(r_v - r'_u)\Lambda_{nu} \qquad (8)$$

.

By using the transformation function, now we able to compute the two components in the matrix-vector multiplication in Eq. (5) with

$$\boldsymbol{Z}^{far} = \boldsymbol{\Lambda g \Lambda}^T \qquad (9a)$$
$$\boldsymbol{Z}^{near} = \boldsymbol{Z}^{MoM}_{nz} - \boldsymbol{Z}^{far}, \qquad (9b)$$

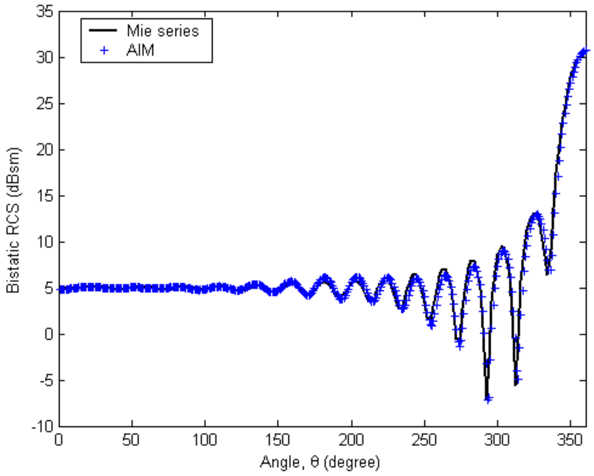
where $\boldsymbol{Z}^{MoM}_{nz}$ is the matrix contains only the direct interaction of neighbor elements and $\boldsymbol{\Lambda}$ represents the basis transformation matrix of the elements. The matrix $\boldsymbol{g}$ is Toeplitz, and this enable the use of FFT to compute the 3-D convolution in the Eq. (8) efficiently. Hence we can represent the matrix-vector multiplication as

$$\begin{aligned}\boldsymbol{ZI} &= \boldsymbol{Z}^{near}\boldsymbol{I} + \boldsymbol{Z}^{far}\boldsymbol{I}\\ &= \boldsymbol{Z}^{near}\boldsymbol{I} + \boldsymbol{\Lambda}\mathcal{F}^{-1}\left\{\mathcal{F}\{\boldsymbol{g}\}\cdot\mathcal{F}\left\{\boldsymbol{\Lambda}^T\boldsymbol{I}\right\}\right\}, \qquad (10)\end{aligned}$$

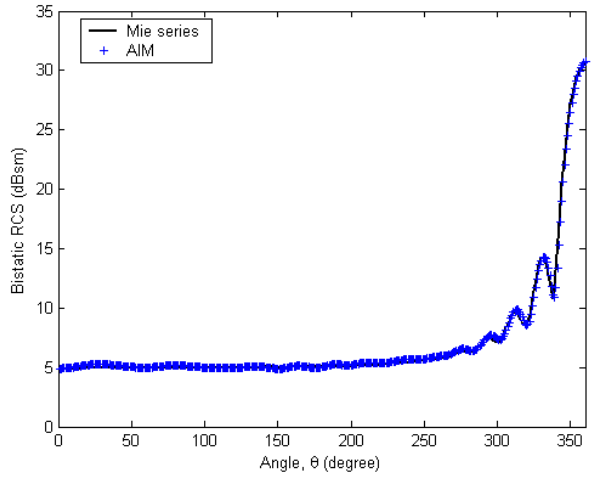
where $\mathcal{F}\{\bullet\}$ and $\mathcal{F}^{-1}\{\bullet\}$ stand for FFT and inverse FFT, respectively.

III. Numerical Results

In order to validate the accuracy of our code, we first present the radar cross section of a perfectly conducting sphere having radius of 3λ. The bistatic RCSs for VV- and HH-polarizations are shown in Fig. 1. The results are compared with Mie series solution and excellent agreements are observed.



(a) VV-polarization

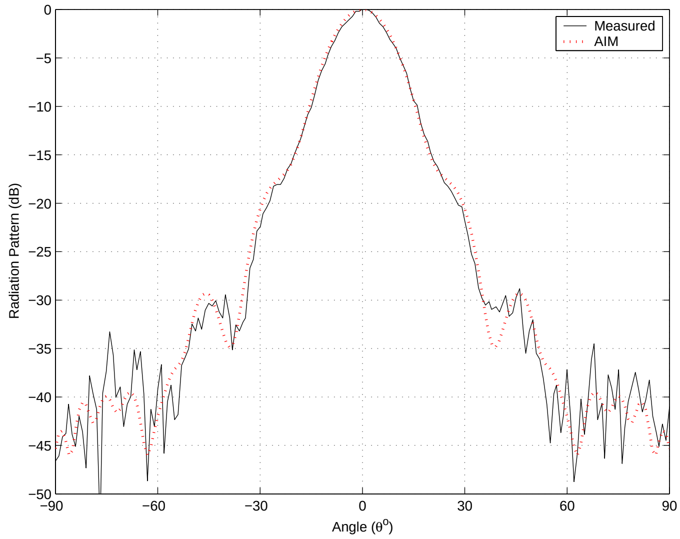


(b) HH-polarization

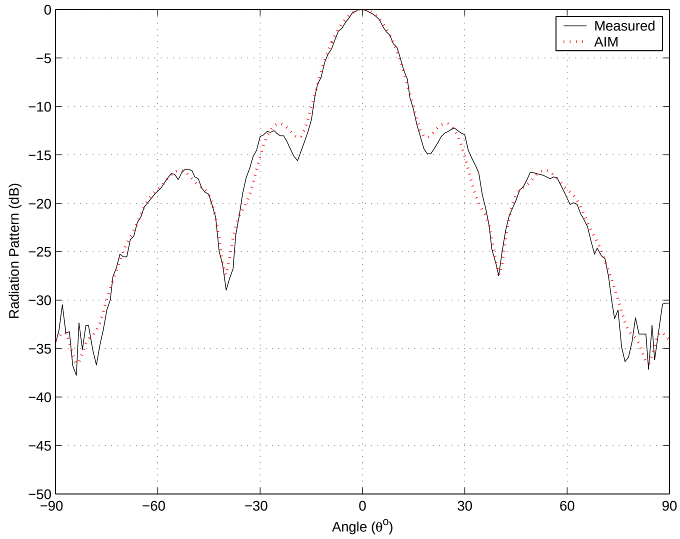
Fig. 1. . Bistatic RCSs of a perfectly conducting sphere of radius 1 m at 0.9 GHz

Next we consider a horn antenna with the aperture dimensions of $3\lambda \times 4\lambda$ and height of 8.2λ. The horn antenna is fed by a dipole antenna placed at the center of the waveguide. The discretization of the horn antenna using triangular patches resulting 15101 RWG basis functions. The radiation patterns for E- and H-plane are shown in Fig. 2. The results are compared with measurement data.

The third example we consider is a parabolic reflector having a diameter of 5λ. The F/D ratio of the reflector is 0.375. The reflector is fed by a dipole backed by a circular dish. The parabola reflector is modelled using triangular patches, and resulting 7157 RWG basis functions. The computed E- and H-plane radiation patterns by the AIM are shown in Fig. 3. The results are again compared with the measurement data [8]. The induced surface current

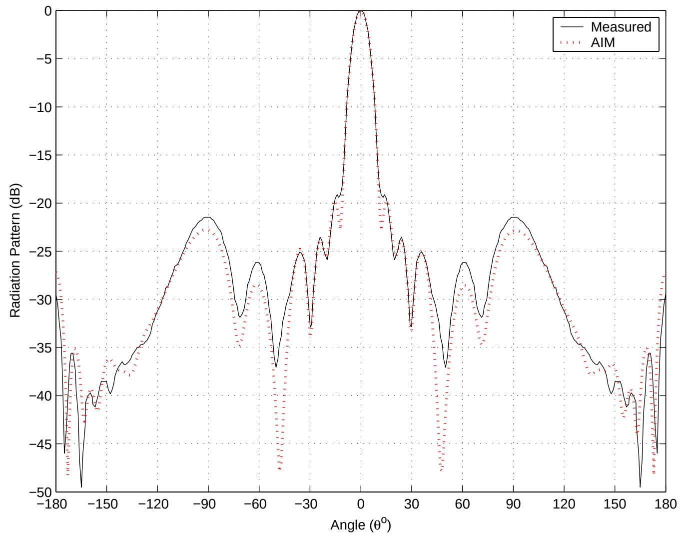


(a) E-plane

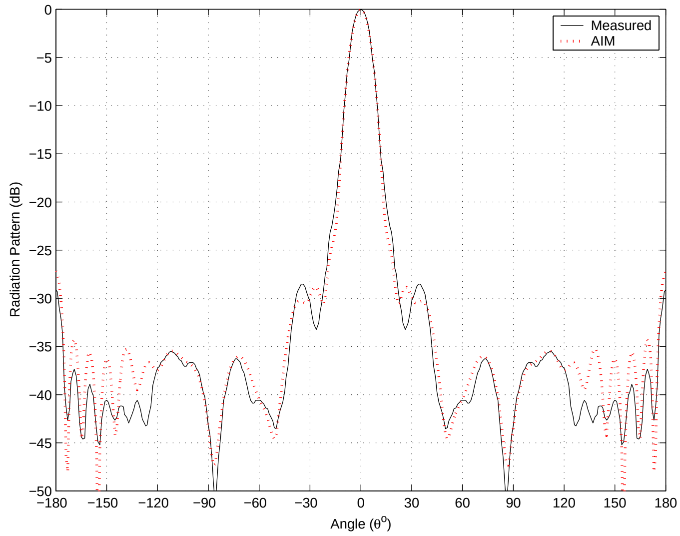


(b) H-plane

Fig. 2. Radiation patterns of a rectangular horn antenna.



(a) E-plane



(b) H-plane

Fig. 3. Radiation patterns of parabolic reflector.

density on the parabolic reflector is shown in Fig. 4

Lastly, we consider a rectangular horn-fed parabolic reflector with different F/D ratios. The parabolic reflector is assumed to have the F/D ratios of 0.3, 0.375 and 0.4, and their respective diameters are 21λ, 17λ and 16λ. The aperture dimensions and height of the rectangular horn for feeding are $1.2\lambda \times 1.6\lambda$ and 3.5λ, respectively. The discretization of these configurations results in 126948, 78975 and 69379 RWG basis functions, respectively. The computed E- and H-plane radiation patterns are shown in Fig. 5 for different F/D ratios.

Table 1 shows the comparison of memory requirements of storage for the MoM and the AIM needed for the examples 2, 3 and 4. From Table 1, we can see that the memory requirement of storage is reduced significantly by using the AIM.

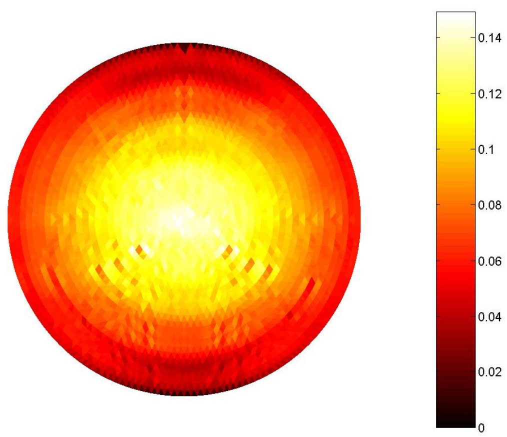
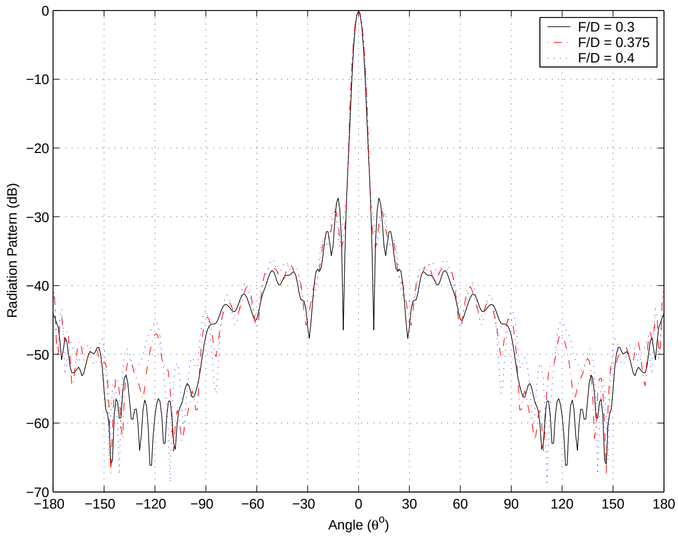
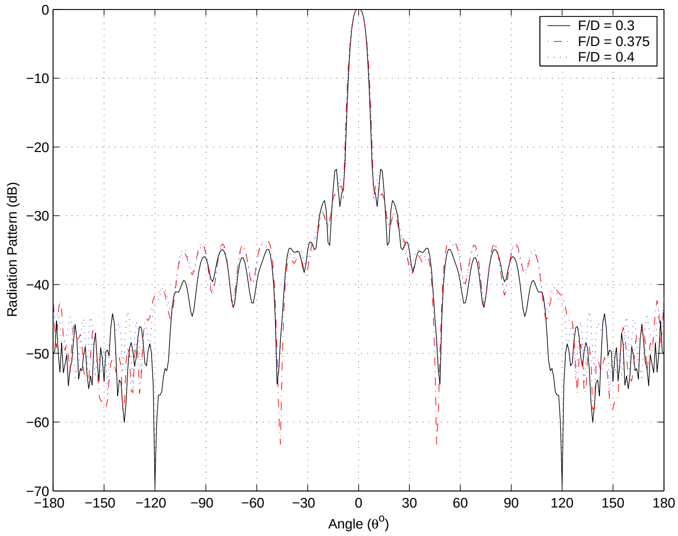


Fig. 4. Induced surface current density on the parabolic reflector.



(a) E-plane



(b) H-plane

Fig. 5. Radiation patterns of rectangular horn-fed parabolic reflector with different F/D ratios.

TABLE I

Comparison of memory requirements of storage for MoM and AIM

Example	Unknowns	MoM	AIM
Fig. 2	15101	3.4 GB	21 MB
Fig. 3	7157	780 MB	9 MB
Fig. 5 $F/D = 0.4$	69379	71.7 GB	98 MB
Fig. 5 $F/D = 0.375$	78975	92.9 GB	112 MB
Fig. 5 $F/D = 0.3$	126948	240.1 GB	190 MB

Note: 1 MB = 1048576 bytes and 1 GB = 1024 MB

IV. Conclusion

In this paper, the AIM has been successfully implemented to solve the radiation problem of electrically large-sized reflector antennas and horn antennas and to analyze their radiation characterizations. The problem is formulated using the EFIE approach. Then, the MoM has been used to discretize the integral equation and to convert it into a matrix equation. The AIM is employed in the iterative solver to reduce the memory requirement and to speed up the matrix-vector multiplication, so that large scaled antenna radiation problems can be handled systematically and accurately by the approach. Numerical results are presented to demonstrate the accuracy of the proposed method and its broad applicability of the present approach.

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