A MODIFIED MODEL FOR THE PREDICTION OF THE PERFORMANCE OF A DOUBLE SITE DIVERSITY EARTH SPACE SYSTEM LOCATED IN HEAVY RAIN CLIMATIC REGIONS

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1.Introduction

Site diversity is considered to be an effective technique to overcome severe rain attenuation in satellite communication links. In the present paper, a modification of an existing method to predict the improvement in using dual site diversity techniques is proposed. The modified method assumes a more realistic model for the description of the rain height. It is also based on a convective raincell model and the assumption that the point rainfall statistics follows a gamma form. The difference between the existing predictive results and the deduced ones after use of the modified procedure, is examined for an Earth-space double diversity system locating in M climatic zone.

2. The Analysis

Site diversity is a well known technique which involves the deployment of two (or more) spatially separated, but interconnected, earth terminals to provide alternate propagation paths with the capability of switching to the least impaired path as required. A site diversity configuration between two Earth stations 1 and 2 is shown in Figure 1 where S is the separation of the two Earth-stations, D is their horizontal separation and φ is the common elevation angle. Our objective is the calculation of the following joint exceedance probability:

$$P_{1,2} = P\left(A_{S_1} \geq x_S, A_{S_2} \geq x_S\right) \tag{1}$$

where A_{S_i} (i=1,2) are the slant path attenuations and x_S is the corresponding outage level (in dB). The following assumptions are taken into account:

a)We first employ the Crane's simplified consideration for the spatial rainfall structure [1]. Following this consideration, a uniform rain structure from the ground up to an effective rain height He is assumed. For the determination of the He, the model proposed by Stutzman and Dishman [2] is adopted. According to this model, the He is dependent not only upon the geographic latitude Λ of the location, but also upon the specific value of the point rainfall rate R. Consequently we have :

$$\left.\begin{aligned} H_e &= H && \text{for } R \leq 10\text{mm/hr} \\ H_e &= H + \log\left(\frac{R}{10}\right) && \text{for } R \geq 10\text{mm/hr} \end{aligned}\right\} \tag{2}$$

where

$$\begin{aligned} H &= 4.8\text{Km} && |\Lambda| \leq 30^\circ \\ H &= 7.8 - 0.1|\Lambda| && |\Lambda| \geq 30^\circ \end{aligned} \tag{3}$$

The application of the above consideration leads to some cumbersome and very complicated calculations but this situation can be avoided by assuming the homogeneity of the rainfall medium inside the part extending the constant rain height at the 0° isotherm. Following this assumption, the effective length of the satellite path is given by :

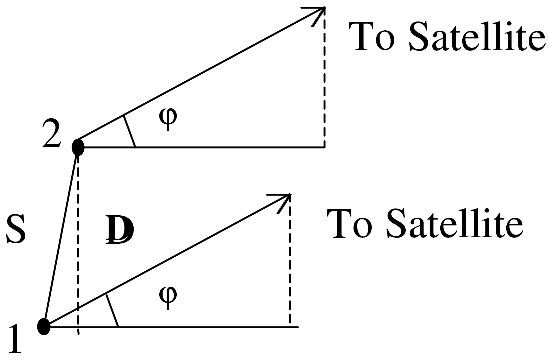

Figure1: Configuration of the problem

$$Ls = \frac{H_e - H_o}{\sin\varphi}, \qquad \varphi \geq 10^o \tag{4}$$

where H_e depends only upon the point rainfall rate of the 0^o isotherm height. This assumption can be shown to be valid, for elevation angles greater than about 10^o leading to the projected straightline parts of the order of few kilometers. Furthermore, H_o is the average height of the Earth-station above sea level.

According to the assumption of uniform vertical rain structure, the joint exceedance probability (expression (1)) can be obtained as:

$$P_{1,2} = P(A_1 \geq x_D, A_2 \geq x_D) \tag{5}$$

with

$$x_D = x_S \cos\varphi \tag{6}$$

and A_i (i=1,2) are the surface projected attenuations as calculated for an hypothetical terrestrial link with path length $L_D = Ls\cos\varphi$.

b)The gamma form for the unconditional, point rainfall rate R and attenuation A distribution is adopted. The motivation for using this assumption follows from the fact that the rainfall rate in various climatic zones of the world such as (J,M,N,P,Q zones) is better fitted to gamma than the lognormal form[3]. As a result, the application of the predictive procedure to locations belonging to the above zones can only be handled by employing an appropriate two dimensional gamma distribution.

c) All the other assumptions concerning the specific attenuation and the spatial rainfall structure [4] are the same as presented in [5].

Following the previous considerations the joint exceedance probability (expression (1)) can be obtained as:

$$P_{1,2} = \frac{(1-\rho)^v}{\Gamma(v)} \cdot \sum_{i=0}^{\infty} \frac{\rho^i}{i!} \cdot \Gamma(v+i) \cdot \left[1 - \frac{\gamma(v+i, \beta' x_D)}{\Gamma(v+i)}\right]^2 \tag{9}$$

where

$$\left. v = \frac{\mu_{A_1}^2}{\sigma_{A_1}^2}, \ \beta = \frac{\mu_{A_1}}{\sigma_{A_1}^2}, \beta' = \frac{\beta}{1-\rho} \right\} \tag{10}$$

in terms of the mean value μ_{A_1} and standard deviation σ_{A_1} of the variables A_{Si}(i=1,2), which are considered identical, and ñ which is the path correlation. In the above expressions $\Gamma(\ )$ and $\gamma(\ )$ are the gamma and the incomplete gamma function [6]. The consideration of the novel assumptions for the rain height reflects upon the calculation of the above parameters. Following a cumbersome but straightforward analysis one is able to express μ_{A_1}, σ_{A_1} by means of the gamma statistical parameters v_R and β_R of the point rainfall distribution, the constants a and b of the specific attenuation and the characteristic distance G, as :

$$\mu_{A_1} = am_b L_D + \frac{a}{\tan\varphi}\left\{\sum_{k=0}^{m} \frac{1}{\Gamma(\nu_R)} \frac{1}{\beta_R^{b+k}} \left[\Gamma(b+k+\nu_R) - \gamma(b+k+\nu_R, 10\beta_R)\right]\right\} \quad (11)$$

where

$$m_b = \frac{\Gamma(\nu_R + b)}{\beta_R^b \Gamma(\nu_R)} \quad (12)$$

In addition m is the rank of the polynomial regression fitting concerning the rainrate depending term of H_e in equation (2).

Moreover

$$\sigma_{A_1}^2 = E[A_1^2] - \mu_{A_1}^2 \quad (13)$$

$$E[A_1^2] = P_{11} + 2P_{1d} + P_{dd1} \quad (14)$$

$$P_{11} = a^2 \sigma_b^2 H_{11} + a^2 m_b^2 L_S^2 \quad (15)$$

$$\sigma_b^2 = m_{2b} - m_b^2 \quad (16)$$

$$m_{2b} = \frac{\Gamma(\nu_R + 2b)}{\beta_R^{2b} \Gamma(\nu_R)} \quad (17)$$

The analytical expression of H_{11} can be found elsewhere [5]. The factors P_{1d} and P_{dd1} are complicated but analytical expressions of the known parameters of the problem related to the rainfall characteristics, frequency and incident polarization. Another crucial point of the whole analysis is the calculation of the path correlation coefficient ñ which is expressed as :

$$\rho = \frac{E(A_1 A_2) - \mu_{A_1}^2}{\sigma_{A_1}^2} \quad (18)$$

Following again a similar analysis as before, one gets :

$$E[A_1 A_2] = R_{12} + R_{1d} + R_{2d} + R_{dd} \quad (19)$$

where

$$R_{12} = a^2 \sigma_b^2 H_2 + a^2 m_b^2 L_S^2 \quad (20)$$

and the analytical derivation of the H_2 can be found elsewhere[5]. The other factors R_{1d} , R_{2d} and R_{dd} are again complicated but analytical expression of the given parameters.

3. Numerical Results and Discussion

Numerical results are presented for a hypothetical Earth-space double diversity system located in M climatic zone where gamma form is the best model for the representation of the rainrate distribution [3]. In Figure 2 the exceedance probability (for both single and double diversity scheme) has been drawn versus the attenuation threshold (in dB) in comparison with the model adopting the constant rain height model [7]. The geometrical and electrical characteristics of the above communication systems such as frequency, elevation angle, latitude of the location, Earth station ground height along with the appropriate values for the point rainfall parameters are presented in Table 1. As can be seen, there is significant difference between the existing predictive results and the proposed in this paper, thus the employment of the modified procedure is necessary. The proposed model is qiuite flexible and it is oriented to be applicable to any location of the world where the previous assumptions are satisfied.

Zone	f	Λ	φ	H_o	S	D	v_R	β_R
M	12GHz	35°	30°	0.2Km	10Km	10Km	0.0066	0.0399

Table 1:Parameters of the communication systems under consideration

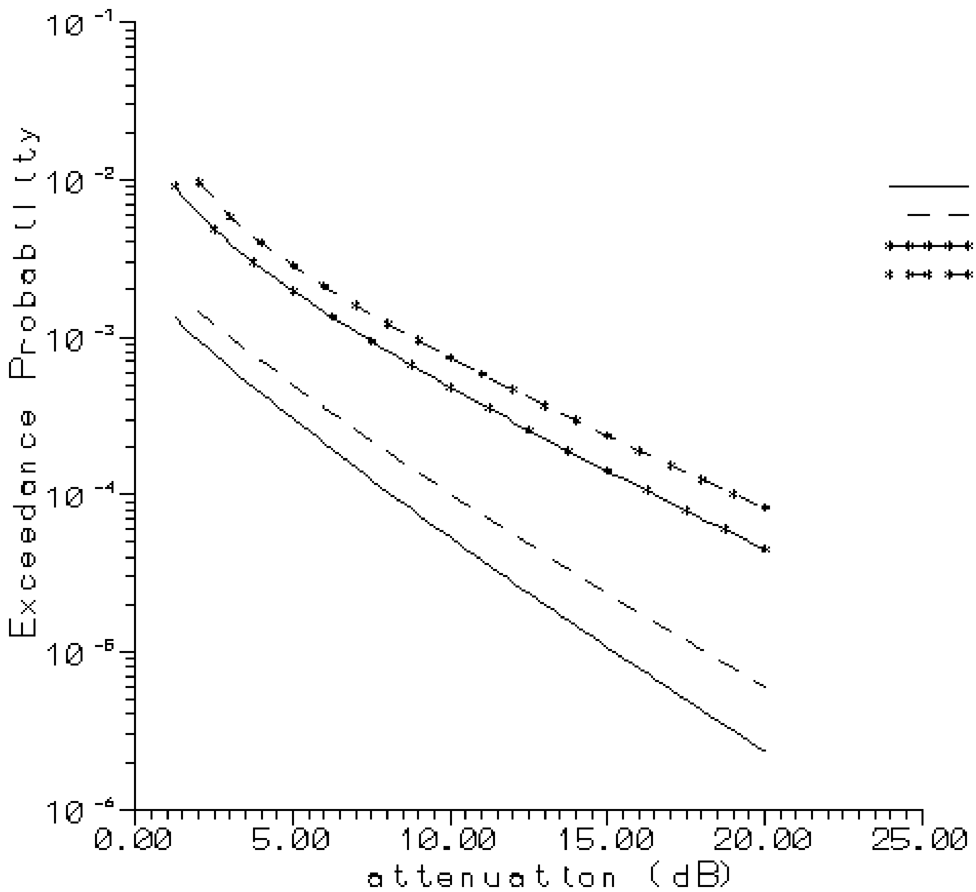

Figure 2: Exceedance attenuation probability for single and dual site diversity schemes
(A) : dual site diversity using constant rain height model, (B): dual site diversity using modified model
(C): single site constant rain height model, (D): single site using modified model

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