MODIFIED CHEBYSHEV ARRAYS WITH INCREASED DIRECTIVITY FOR LARGE NUMBER OF ELEMENTS

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1. Introduction

Chebyshev arrays have sidelobes of equal magnitude in their radiation patterns and provide the smallest beamwidth for a given sidelobe level and vice versa. These arrays, however, suffer from directivity saturation and high current peaks at their edges when the number of elements is large [1]. These deficiencies limit the practical applications of large Chebyshev arrays. In this paper a new type of arrays is introduced which, like conventional Chebyshev arrays, provide equal sidelobes but with substantially higher directivity limits. Furthermore, in contrast to large conventional Chebyshev arrays, currents of edge elements in the proposed arrays need not be excessively larger than those of the other elements. These arrays are referred to as *modified Chebyshev arrays.*

The idea of modified Chebyshev arrays is based on the fact that if $f_o(\theta)$ represents an equal sidelobe pattern, $f(\theta)=f_o^m(\theta)$; m=2, 3, . . ., will also represent an equal sidelobe pattern. Let $f_o(\theta)$ be the array factor for a basis array of conventional Chebyshev type, then $f(\theta)$ with $m \geq 2$ represents the array factor for modified Chebyshev arrays. Now, considering two equal sidelobe arrays, one conventional Chebyshev and the other modified Chebyshev type, both with the same number of elements and same sidelobe level, it can be easily shown that the number of sidelobes in the modified Chebyshev array is less than half of those in the conventional one. For large arrays, however, smaller number of sidelobes implies more power in the main beam and thus higher directivity. Another benefit of modified Chebyshev arrays is more equalized current distributions with substantial reduction in currents of the edge elements. This effect is due to i) smaller or no current peaks at the edges of the basis array because of its smaller number of elements, and ii) coefficient mixing brought about when the array polynomial for the basis array is raised to power m to yield a higher order polynomial for the modified Chebyshev array.

2. Array Factor and Element Currents

Let us consider a basis Chebyshev array of N_o equally spaced elements. The array factor, denoted as $f_o(\theta)$, is expressed as [2]

$$f_o(\theta)=\begin{cases} \cos[(N_o-1)\cos^{-1}(\gamma_o\cos\frac{\psi}{2})]/R_o, & |f_o(\theta)| \leq 1/R_o \quad \text{(1a)} \\ \cosh[(N_o-1)\cosh^{-1}(\gamma_o\cos\frac{\psi}{2})]/R_o, & |f_o(\theta)| \geq 1/R_o \quad \text{(1b)} \end{cases}$$

where $\psi = \beta d\ \cos\theta + \alpha$, $\beta = 2\pi/\lambda$; λ being the wavelength, d is the element spacing, α is inter-element phase shift, θ is the angle measured from the line of array, R_o is the side-lobe level, and $\gamma_o=\cosh[\ln(R_o+(R_o^2-1)^{1/2})/(N_o-1)]$. The array factors for a modified Chebyshev array resulting from the above basis array is obtained from

$$f(\theta)=f_o^m(\theta) \quad (2)$$

where m is an integer greater than 1. The sidelobe level of the modified Chebyshev array is $R=R_0^m$ and the number of elements, as will be shown later, is $N=m(N_o-1)+1$. The element spacing and inter-element phase shift for both basis and modified arrays are the same.

To determine the element currents for the modified Chebyshev array, the polynomial representation of $f_o(\theta)$ is used,

$$f_o(\theta)=\sum_{p=0}^{N_o-1} I_p Z^p \qquad (3)$$

where $Z=exp(j\psi)$ and I_p is the current of pth element. Using (3) in (2), yields

$$\left(\sum_{p=0}^{N_o-1} I_p Z^p\right)^m=\sum_{q=0}^{N-1} I_q Z^q \qquad (4)$$

It is clear from (4) that $N=m(N_o-1)+1$. For known values of N_o and R_o, the currents I_p, $p=1, 2, \ldots, N_o$ are determined using the approach described in [2]. Then, by equating the like power terms on both sides of (4), currents I_q, $q=1, 2, \ldots, N$, for a modified Chebyshev array with N elements and sidelobe level of R are obtained.

3. Directivity

The directivity of modified Chebyshev array is obtained from

$$D=4\pi \Big/ \int_{\alpha-\beta d}^{\alpha+\beta d} f_o^{2m}(\psi)d\psi \qquad (5)$$

where $f_o(\psi)$ is given in (1). It is known that the directivity of a conventional Chebyshev array approaches the limit of $2R^2$; R being the sidelobe level, when the number of elements becomes very large [1]. The directivity limit for the modified Chebyshev array can be obtained from (5) by letting N_o approach infinity. The ratio of directivity limit for a modified Chebyshev array (D_∞) to that for a conventional one ($\bar{D}_\infty$), assuming that both arrays have the same sidelobe level, is

$$D_\infty/\bar{D}_\infty=2^{2m-1} \Big/ \binom{2m}{m} \qquad (6)$$

Examination of (6) reveals that the ratio of directivity limits is an increasing function of m. Thus, the larger the value of m the higher the directivity limit for the modified Chebyshev array. This suggests that the number of elements for the basis array, N_o, should be chosen such that m is maximum while $N=m(N_o-1)+1$ is satisfied for integer values of N, N_o and m.

4. Numerical Results

To assess the directivity improvement resulting from the proposed modification, directivities of modified and conventional Chebyshev arrays, with the same number of elements and the same sidelobe levels, are compared. Figure 1 compares directivities for $m=3$ and $R=10$ (sidelobe level $=-20$ dB). The arrays are broadside with $\alpha=0$ and the element spacing is optimum. It is emphasized that optimum element spacing is a function of sidelobe level, R, number of elements, N, and inter-element phase shift, α. More details on optimum element spacing are provided in [2]. It is observed that for $N<80$, the directivity of the modified Chebyshev array, D, is less than that for the conventional one, $\bar{D}$. However, for $N>80$ the situation is reversed and the modified array provides a higher directivity. For $N=487$, $\bar{D}=168.46$ and $D=230.26$, that is an increase of about 36.7% in directivity. The upper limits for directivities for these two arrays are $D_\infty=320$ and $\bar{D}_\infty=200$. Although at a sufficiently large value of N the directivity of the modified Chebyshev array saturates, but, compared to the case of conventional array, this happens at a substantially larger number of elements. Half-power beamwidths of the two arrays were also examined. The beamwidth of the modified Chebyshev array is always greater than that of the conventional one, but when the number of elements is large the beamwidths are essentially equal.

Directivities for several other values of m and sidelobe levels were also examined. Generally, for a smaller value of sidelobe level, $D>\bar{D}$ begins at a smaller number of elements. Also, for smaller values of m saturation of directivity occurs at a smaller value of N. Figure 2 compares directivities of modified and conventional Chebyshev arrays for $m=2$ and $R=\sqrt{10}$ (sidelobe level= -10 dB). The sidelobe level and m for these arrays are smaller than those in Figure 1. Comparison of Figures 1 and 2 provides a qualitative measure of how m and R influence variations of directivity of modified Chebyshev array. The results in Figure 2 also suggest that for moderate sidelobe levels, modified Chebyshev arrays are advantageous even when the number of elements is not too large. It is observed that in this example $D>\bar{D}$ for $N>8$.

To show the effect of proposed modification on current distributions, let us consider, as an example, an array with 21 elements and sidelobe level equal to -20 dB. Although this example is not representative of a large array, but provides some insight on how the modification affects element currents. Choosing $m=2$, the basis array will have 11 elements and a sidelobe level of -10 dB. The current magnitudes for the basis array, normalized to edge element current, are: 1, 0.3235, 0.3602, 0.3880, 0.4054, 0.4114, 0.4054, 0.3880, 0.3602, 0.3602, 0.3235, 1. (Note that current distribution is symmetric with respect to the center element). Using (4) with $m=2$, the currents for the 21 element modified Chebyshev array are obtained. The results are summarized in Table 1. This table also contains the currents for a 21 element conventional Chebyshev array. Due to symmetry, only the currents for the center element and half of the remaining elements are given. Comparison of the two current distributions indicates that the edge currents in the modified Chebyshev array have been reduced substantially.

Table 1. Current distributions for 21 element modified and conventional Chebyshev arrays with sidelobe level of -20 dB. Currents are normalized to the edge element.

Element No.	11	10	9	8	7	6	5	4	3	2	1
Modified	3.2678	1.8077	1.7470	1.6492	1.5196	1.3646	1.1916	1.0090	0.8250	0.6480	1.0
Conventional	0.9374	0.9295	0.9062	0.8683	0.8173	0.7550	0.6836	0.6058	0.5242	0.4414	1.0

5. Conclusion

A new class of phased arrays has been proposed which, similar to conventional Chebyshev arrays, provide equal sidelobes, but offer substantially higher directivities and essentially the same half-power beamwidth for large number of elements. Also, element current distributions of these arrays need not require large peaks at the edges to maintain equal sidelobes.

6. References

[1] R.J. Mailloux, Phased Array Antenna Handbook, Artech House, Boston, 1994, ch. 3.

[2] A. Safaai-Jazi, "A new formulation for the design of Chebyshev arrays," IEEE Trans. Antennas and Propagat., vol. 42, No. 3, pp. 439-443, 1994.

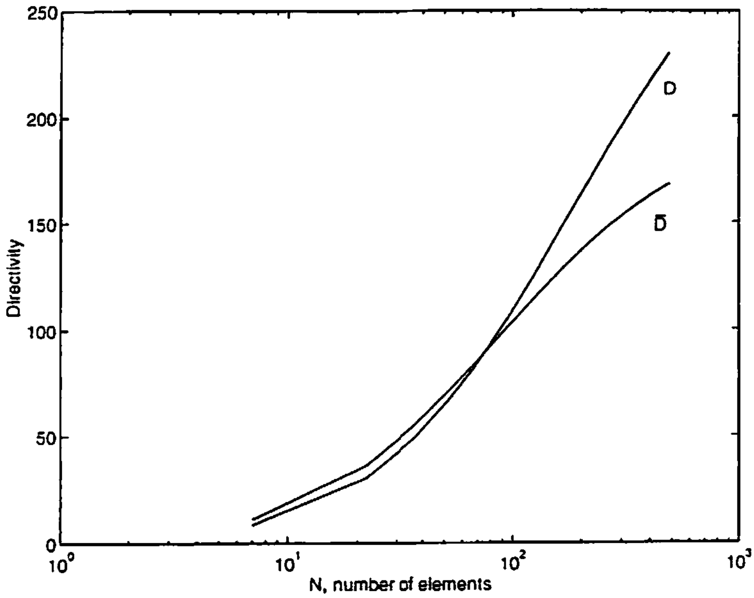

Fig.1 Variations of directivity versus number of elements for conventional ($\bar{D}$) and modified (D) Chebyshev arrays with sidelobe level of -20 dB and m=3.

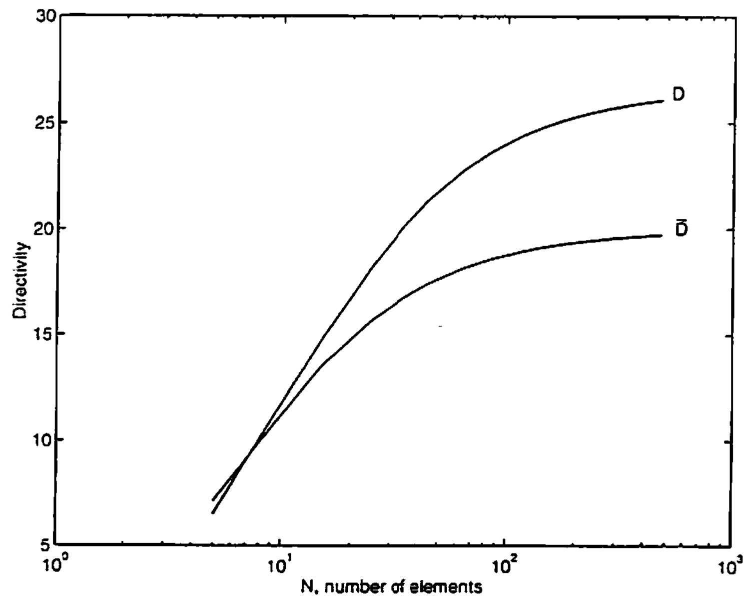

Fig.2 Variations of directivity versus number of elements for conventional ($\bar{D}$) and modified (D) Chebyshev arrays with sidelobe level of -10 dB and m=2.