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POWER SPECTRUM OF WAVES SCATTERED BY AN IONOSPHERIC LAYER

D. J. Fang

Radio Physics Laboratory
Stanford Research Institute
Menlo Park, California, USA

The purpose of this paper is to study the scattering behavior of radio waves passing through irregularities in the ionospheric medium. The study is based on the diffraction theory given by Ratcliffe[1] considering the incident as well as the scattered waves in an elementary scattering layer as plane waves with a spectrum of angular distribution. It is found that the formal relations between the incident and scattered field spectrum and between the incident and scattered power spectrum are as follows:

$$\mathcal{E}_s(z,t,k_x,k_y) = \int dk_x'\, dk_y' \left[\mathcal{E}_t(z,t,k_x',k_y')\tau(z,k_x',k_y',k_x-k_x',k_y-k_y')\right] \qquad (1)$$

$$P_x(k_x,k_y) = \frac{2\pi}{2z_o}^2 \int dk_{1x}\, dk_{1y}\, dk_{2x}\, dk_{2y} \left[\mathcal{E}_t(z,t,k_{1x},k_{1y})\tau(z,k_{1x},k_{1y},k_x-k_{1x},k_y-k_{1y}) \mathcal{E}_t^*(z,t,k_{2x},k_{2y})\tau^*(z,k_{2x},k_{2y},k_x-k_{2x},k_y-k_{2y})\right] \qquad (2)$$

where

t = time

z = the axis perpendicular to the elementary scattering layer in question

z_o = characteristic impedance of the medium

k_x, k_y = component wave numbers in x-y plane

$\mathcal{E}_t, \mathcal{E}_s$ = total incident and scattered wave spectrum

P_s = scattered power spectrum

$\tau(z,k_x',k_y',k_x,k_y)$ = the Fourier transform (in k_x', k_y') of the scattering function $T(x,y,z,k_x,k_y)$, which represents the field intensity at a point (x,y,z) produced by scattering of an incident plane wave of unit amplitude with directions given by k_x and k_y.

Equations (1) and (2) involve no detailed specifications of the medium by assuming simply the existence of scattering function. In the ionospheric irregularity problem, the appropriate scattering function is[2]

$$T(x,y,z,k_x,k_y) = \frac{k^2}{2\pi} \int_0^L \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx_o\, dy_o\, dz_o \left\{\frac{n_1(x_o,y_o,z_o)}{(z-z_o)\sec\theta} \exp\left[-ikH(x,y,z,x_o y_o z_o)\right]\right\} \qquad (3)$$

where n_1 is the deviation of the refractive index from its mean, H is a complicated distance function, and θ is the polar angle between the wave direction and the z axis.

With the expression of Equation (3), the formal relations of Equations (1) and (2), after taking statistical averages, become:

$$\langle \mathcal{E}_s \rangle = 0$$

$$\langle P_s(k_x,k_y) \rangle = \frac{1}{2z_o} k^2 \mu^2 r_x r_y r_z \pi^{3/2} L$$

$$\iint \mathcal{E}_t(z,t,k_{1x},k_{1y}) \mathcal{E}_t^*(z,t,k_{1x},k_{1y})$$

$$\sec\theta \exp[-\tfrac{1}{4} r_x^2 (k_x-k_{1x})^2 - \tfrac{1}{4} r_y^2 (k_y-k_{1y})^2$$

$$-\tfrac{1}{4} r_z^2 q_1^2 (k_x,k_y,k_{1x},k_{1y})] \, dk_{1x} \, dk_{1y} \tag{4}$$

where

μ = magnitude of the refractive index fluctuation

r_x, r_y, r_z = characteristic scales of the irregularities along x, y, and z coordinates, respectively

L = the thickness of the elementary layer

$$q_1 = \frac{1}{2k \cos\theta_1} \{ (k_x-k_{1x})^2 + (k_y-k_{1y})^2$$

$$+ \tan^2\theta_1 [(k_x-k_{1x})\cos\phi_1 + (k_y-k_{1y})\sin\phi_1]^2 \}$$

$$+ \tan\theta_1 [(k_x-k_{1x})\cos\phi_1 + (k_y-k_{1y})\sin\phi_1]$$

ϕ = the azimuthal angle of the wave.

In a special case where the incident wave is a uniform plane wave, it can be shown that Equation (4) reduces to a form that checks with known results[2]. Equation (4) may prove to be useful in single scattering problems where the incident wave subtends an appreciable incident angle (example, radio star Cassiopeia A), and in multiple scattering problems where the further scattering effect of those waves being already scattered becomes significant. As a matter of fact, the formal relations given in Equations (1) and (2) do not involve any specific detail of the medium. Therefore, the wave propagation and scattering behavior described by the relations may as well be applicable to a general class of problems other than the ionospheric irregularity problem just mentioned. Two particular cases are the problems concerning communications in the interplanetary space and the problems concerning the radio observation of nuclear bursts.

References:

[1] Ratcliffe, J. A., "Some Aspects of Diffraction Theory and Their Applications to the Ionosphere," Reports Prog. Phys., Vol. XIX, pp. 188-267 (1956).

[2] Uscinski, B. J., "The Multiple Scattering of Waves in Irregular Media," Proc. Roy. Soc. A, Vol. 262, pp. 609-640 (1968).