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Resonant Frequency of a Circular Microstrip Antenna in a Parallel-plate Waveguide

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1. Introduction

We have been developing a 2.4 GHz point-to-multipoint wireless communication system suitable for use inside mechatronics ICT (Information and Communication Technology) equipment to replace wire harnesses inside equipment with wireless communication as a useful technique of weight reduction and simplification of system assembling [1]. One of the critical problems of such applications is that antennas are installed close to metallic components or in a narrow gap between metallic housings and thus suffer degradation of antenna performance due to effects of the ambient environment. Therefore, in order to maintain desired antenna performance inside equipment, we need to understand the dependence of antenna characteristics on the ambient environment inside equipment and design antennas with a consideration of it.

In this study, to simplify a problem, a circular microstrip antenna (MSA) in a narrow parallel-plate waveguide is considered as a model of a typical installation situation and a representative antenna. So far, we have formulated an analytical expression for the input impedance and have confirmed its validity through comparisons with measurement [2]. In addition to the input impedance, the resonant frequency must be predicted accurately because a MSA has a narrow bandwidth and is usually operated in the vicinity of the resonant frequency. Though, in [2], we have also shown the resonant frequencies, they have been calculated indirectly from the calculation results of the input impedance and their dependence on the parallel-plate waveguide has been not clear. From the antenna designer's point of view, a design formula to calculate the resonant frequency directly is needed. In this paper, we therefore formulate an analytical expression for the resonant frequency and validate it through the comparison with measurement.

2. Theory

Figure 1 shows a circular MSA in an infinite parallel-plate waveguide. The parallel-plate waveguide has a height d and is filled with the air whose permittivity and permeability are denoted by ε_0 and μ_0, respectively. The circular MSA has a radius a and is fed by a line current I_0 on the feed pin at the feed position $(\rho,\phi)=(\rho_0,0)$. The dielectric substrate has a thickness h, a permittivity ε, and the same permeability as the air.

2.1 Electric Field inside the Circular MSA [2]

When we assume that $kh \ll 1$ where $k=\omega\sqrt{\varepsilon\mu_0}$ and ω is the angular frequency, the field inside the circular MSA is uniform in the z direction. The electric field due to the line current I_0 is given by

$$E_z^i(\rho,\phi) = -\omega\mu_0 I_0 \sum_n^{\infty} \frac{2-\delta_{0n}}{4} \begin{cases} J_n(k\rho_0)\left(H_n^{(2)}(k\rho)+A_n J_n(k\rho)\right)\cos n\phi, \rho>\rho_0 \\ J_n(k\rho)\left(H_n^{(2)}(k\rho_0)+A_n J_n(k\rho_0)\right)\cos n\phi, \rho<\rho_0 \end{cases} \quad (1)$$

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$$A_n = -\frac{j\omega\mu_0 Y_{sn} H_n^{(2)}(ka) + kH_n^{(2)\prime}(ka)}{j\omega\mu_0 Y_{sn} J_n(ka) + kJ_n'(ka)} \tag{2}$$

where J_n and $H_n^{(2)}$ are the Bessel function of the first kind and the Hankel function, respectively. δ_{ij} is the Kronecker delta, namely $\delta_{0n} = 1$ for $n = 0$ and $\delta_{0n} = 0$ for $n \geq 1$, and the superscript "i" indicates fields inside the antenna. Y_{sn} are the surface admittances at the side aperture. J_n' and $H_n^{(2)\prime}$ are the derivatives of J_n and $H_n^{(2)}$ with respect to the argument $k\rho$.

The surface admittance of the circular MSA in a parallel-plate waveguide is given by

$$Y_{sn} = \frac{k_0 a\pi Y_0}{2d} \sum_{l=0}^{\infty} \frac{h_l(2-\delta_{0l})}{\beta^2} \left[\beta^2 J_n'(\beta a) H_n^{(2)\prime}(\beta a) + \frac{k_z^2}{k_0^2}\frac{n^2}{a^2} J_n(\beta a) H_n^{(2)}(\beta a) \right] \tag{3}$$

where

$\beta = \sqrt{k_0^2 - k_z^2}$ with $\mathrm{Im}(\beta) \leq 0$, (4) $\quad k_z = \frac{l\pi}{d}$, (5) $\quad h_l = \begin{cases} h, & l = 0 \\ \frac{\sin k_z h}{k_z}, & l \geq 1 \end{cases}$, (6)

and $Y_0 = \sqrt{\varepsilon_0/\mu_0}$.

2.2 Resonant Frequency

In order to obtain the resonant frequency of a circular MSA in a parallel-plate waveguide in an analytical form, we neglect the dielectric and conductor losses, because the contributions of these losses to the resonant frequency are small. Thus, the input impedance can be defined by [3]

$$Z_{in} = \frac{-1}{I_0 I_0^*} \iint_{S_f} \boldsymbol{E}^i \cdot \boldsymbol{J}^* dS \tag{7}$$

where S_f is the surface on the feed pin and $\boldsymbol{J}$ is the surface current on S_f. The superscript "*" indicates the complex conjugate. Because $\boldsymbol{E}^i$ and $\boldsymbol{J}$ are assumed to be uniform in the z direction, Z_{in} is given by

$$Z_{in} = -\frac{hE_z^i}{I_0} = \omega\mu_0 h \sum_{n=0}^{\infty} \frac{2-\delta_{0n}}{4} J_n(k\rho_0)\left(H_n^{(2)}(k\rho_0) + A_n J_n(k\rho_0)\right). \tag{8}$$

Since the contribution of $n \neq 1$ modes to the input impedance is negligibly small in the vicinity of the resonant frequency of the $n = 1$ mode, only the $n = 1$ term in Eq. (8) is evaluated here. We let $\rho_0 = a$ in Eq. (8) because the resonant frequency is almost independent of the feed position. In these conditions, Eq. (8) can be written as

$$Z_{in} = \frac{hJ_1}{\pi a} \frac{g_{s1}J_1 - j(b_{s1}J_1 - Y_c J_1')}{(g_{s1}^2 + b_{s1}^2)J_1^2 - 2Y_c b_{s1} J_1 J_1' + Y_c^2 J_1'^2} \tag{9}$$

where $Y_c = \sqrt{\varepsilon/\mu_0}$ and the arguments ka of J_1 and J_1' are omitted for simplifying the notation. g_{s1} and b_{s1} are the surface conductance and susceptance of the order $n = 1$, respectively. The resonant frequency, therefore, is determined by the real solution of the following equation:

$$b_{s1} J_1(ka) - Y_c J_1'(ka) = 0. \tag{10}$$

Equation (10) is numerically solved by using some iterative method to obtain the resonant frequency because bs1 is also a function of frequency.

To obtain an approximate solution of Eq. (10), now we assume that bs1 is constant as a zeroth-order approximation, that is

$$b_{s1} \approx b_0 \tag{11}$$

where b_0 is a constant approximate surface susceptance. We next introduce the equivalent extension Δa to represent the variation of the resonant frequency. Thus, the resonant frequency can be calculated as

$$f_r = \frac{x_{11}}{2\pi(a+\Delta a)\sqrt{\varepsilon\mu_0}} \tag{12}$$

where x_{11} is the first zero of $J_1'(x)$. We assume that $\Delta a << a$, and use the following approximation:

$$ka = \frac{x_{11}a}{a+\Delta a} \approx x_{11}\left(1-\frac{\Delta a}{a}\right). \tag{13}$$

J_1 and J_1' in Eq. (10) are then expanded in a Taylor series around x_{11} to first order as follows:

$$J_1(ka) \approx J_1(x_{11}) \tag{14}$$

$$J_1'(ka) \approx -x_{11}J_1''(x_{11})\frac{\Delta a}{a} = \frac{x_{11}^2-1}{x_{11}}J_1(x_{11})\frac{\Delta a}{a} \tag{15}$$

where J_1'' is the second order derivative of J_1. By substituting Eqs. (11), (14), and (15) into Eq. (10), the equivalent extension is obtained as

$$\Delta a = \frac{b_0}{Y_c}\frac{x_{11}}{x_{11}^2-1}a. \tag{16}$$

By using Eqs. (12) and (16), the resonant frequency in a parallel-plate waveguide can be easily predicted by the approximate surface susceptance b_0, which may be determined by b_{s1} at the operating frequency or at the zeroth-order resonant frequency $f_r^{(0)}$ [4] in a practical evaluation.

3. Comparisons between Calculations and Measurements

Figure 2 shows the calculated and the measured resonant frequency of the antenna under test for $d < 0.5\lambda_0$ where λ_0 is the wavelength in the air at the resonant frequency measured in the free space. In this calculation, b_0 in Eq. (16) is determined by the surface susceptance at the zeroth-order resonant frequency $f_r^{(0)}$. The antennas under test is printed on a dielectric substrate whose dielectric constant and $\tan\delta$ are 2.6 and 0.0011, respectively. The antenna has a radius of 22.0 mm and a thickness of 0.964 mm ($0.008\lambda_0$). The feed position ρ_0 is 4.9 mm. A good agreement between the calculated and the measured resonant frequency is observed. The relative errors of the calculated resonant frequency to the measured one are less than 1.1%. The calculated resonant frequencies, however, are always higher than the measured ones. This is due to the assumptions that the electric field inside the cavity is uniform in the z direction and the conducting patch is neglected in the formulation of the surface admittance [2].

4. Conclusions

For better understanding antenna properties in a narrow space such as inside mechatronics ICT equipment, a circular MSA in a narrow parallel-plate waveguide was theoretically studied. The resonant frequency was formulated in an analytical form, which couples the resonant frequency with the surface susceptance. In order to validate the presented expression, the calculated resonant frequency was compared with the experimental data. A good agreement between the calculation and the measurement was observed. The relative errors between them were less than 1.1% for the substrate thickness $0.008\lambda_0$.

By using the presented expression, we can easily evaluate and predict variations of the resonant frequency of a circular MSA in a parallel-plate waveguide.

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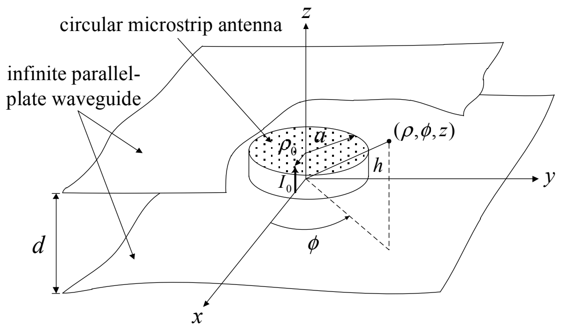

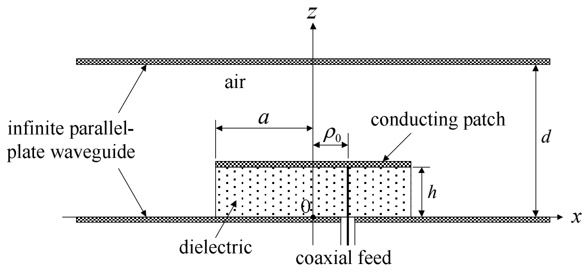

Figure 1: Circular MSA in an infinite parallel-plate waveguide. (a) overview and (b) sideview.

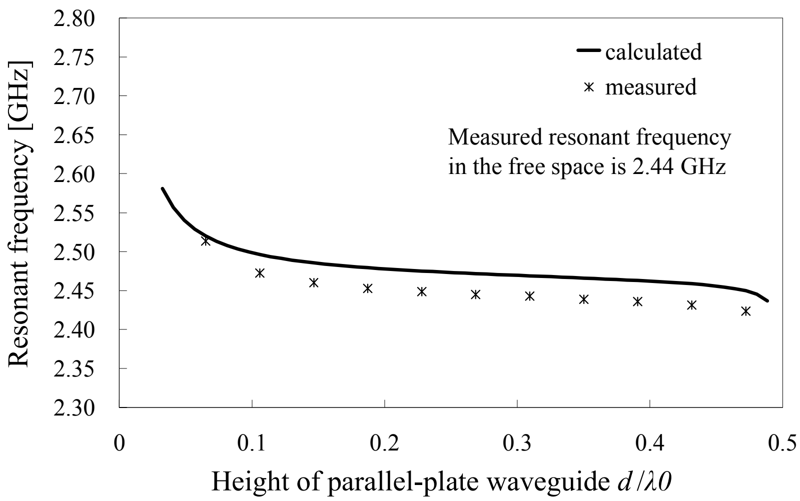

Figure 2: Calculated and measured resonant frequency.

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