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SCATTERING OF PLANE E-POLARISED WAVE BY A HALF-PLANE AND STRUCTURES MODELLED VIA ITS USING: NEW APPROACH TO THE PROBLEM

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As is known from above investigations [1] there are different direct methods solution to the problem of wave scattering by an infinitesimally thin half-plane. For example, the rigorous solution to the problem can be obtained by means of the Wiener-Hopf method or the singular integral equations method [1].

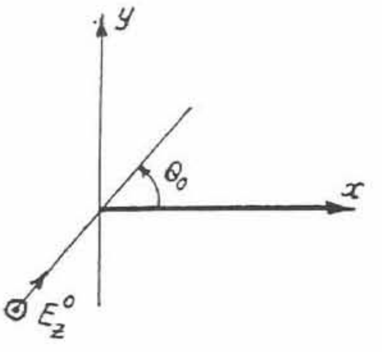

Fig.1

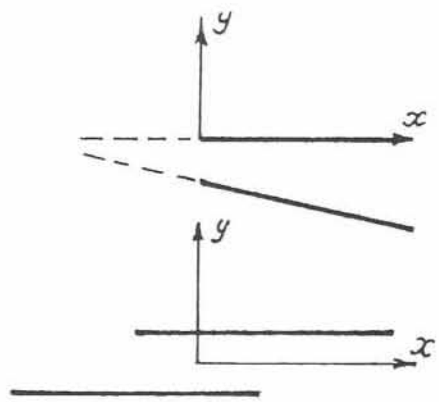

Fig.2

A new approach using the method of orthogonal polynomials (OP) [2] is offered herein. Let an infinitesimally thin and perfectly conducting screen locating in xoy plane $(x = 0, y > 0)$ (see Fig.1) be excited by the plane E-polarized wave such as

$$E_z^0 = e^{ik[\alpha_0 x + \sqrt{1-\alpha_0^2}]}, k = \frac{2\pi}{\lambda}, \alpha_0 = \cos\theta_0 \tag{1}$$

Following [1] this boundary value problem solution is formulated in the form:

$$E_z(x,y) = E_z^0 + E_z^s = E_z^0 - \frac{i}{2}\int_0^\infty \rho(x')H_0^{(1)}(k\sqrt{(x-x')^2+y^2})dx' \tag{2}$$

where $\rho(x')$ is the surface current density, $H_0^{(1)}(x)$ is the Hankel function of the first kind. The function of surface current density is defined from integral equation (IE)

$$\int_0^\infty \rho(x')H_0^{(1)}(k|x-x'|)dx' = -2ie^{ik\alpha_0 x} \tag{3}$$

This IE of the Wiener-Hopf type issues from the Dirichlet boundary condition.

We shall construct the solution of IE (3) by means of the method of OP which is rather straightforward and general one. This method is a particular case of more general scheme of Bubnov method applied to IE but differs from it at two points. It needs firstly to investigate preliminarily the structure of solution near the edge point of the domain of integration and secondly to construct spectral expressions for singular parts of the kernels with OP as eigenfunctions.

Let IE (3) be rewritten in the form:

$$\int_0^\infty \rho(x')K_0(k'|x-x'|)dx' = \pi e^{-k'\alpha_0 x}. \tag{4}$$

Here $k' = -ik$, $K_0(x) = \frac{i\pi}{2}H_0^{(1)}(ix)$ is MacDonald function. Let show that basing on OP method the solution of IE (4) may be obtained analytically. From the edge condition [1] follows that the unknown function $\rho(x')$ is to behave as

$$\rho(x') \underset{x'\to 0}{\sim} O(x^{-\frac{1}{2}}). \tag{5}$$

By introducing the dimensionless parameter $\eta = x'k'$ the function $\rho(\frac{\eta'}{k})$ may be expanded as

$$\rho(\frac{\eta'}{k}) = \frac{e^{-\eta}}{\sqrt{\eta}}\sum_{n=0}^\infty \rho_n L_n^{-\frac{1}{2}}(2\eta). \tag{6}$$

Here $L_n^{-\frac{1}{2}}(2\eta)_{n=0}^\infty$ are the Lagerr polynomials, $\rho_{n=0}^\infty$ are the unknown coefficients.

Expression (6) follows from the fact that the Lagerr polynomials $L_n^{-\frac{1}{2}}(2\eta)$ are the eigenfunctions of integral operator (IO) [2]:

$$\int_0^\infty \frac{K_0(|x-y|)}{e^y\sqrt{y}}L_n^{-\frac{1}{2}}(2y)dy = \frac{\pi}{\sqrt{2}}\gamma_n L_n^{-\frac{1}{2}}(2x)e^{-x} \tag{7}$$

where

$$\gamma_n = \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n+1)} \underset{n\to\infty}{\sim} O(n^{-\frac{1}{2}}) \tag{8}$$

are the eigenvalues of IO.

The orthogonality conditions for the Lagerr polynomials are given by

$$\int_0^\infty \frac{e^{-\eta}}{\sqrt{\eta}}L_n^{-\frac{1}{2}}(\eta)L_n^{-\frac{1}{2}}(\eta)d\eta = \gamma_n\delta_{nk}. \tag{9}$$

From the spectral expression (7) the bilinear expansion for the kernels of IE (4) follows

$$K_0(|x-y|) = \frac{\pi}{\sqrt{2}}e^{-(x+y)}\sum_{n=0}^\infty \gamma_n L_n^{-\frac{1}{2}}(2x)L_n^{-\frac{1}{2}}(2y),$$

$$iH_0^{(1)}(k|x-y|) = \sqrt{2}e^{ik(x+y)}\sum_{n=0}^\infty \gamma_n L_n^{-\frac{1}{2}}(-2ikx)L_n^{-\frac{1}{2}}(-2iky). \tag{10}$$

Let substitute (6) into IE (4). Then taking into account the spectral expression (7) and the orthogonality relation (9) for $(1+\alpha_0) > 0$ we obtain the unknown coefficients

$$\rho_n = 2k' \frac{1}{\gamma_n} \frac{(\alpha_0 - 1)^n}{(\alpha_0 + 1)^{(n+\frac{1}{2})}}, \alpha_0 = \cos\theta_0, \tag{11}$$

Thus, we have analytically obtained the expression for the unknown function:

$$\rho(\frac{\eta'}{k}) = \frac{2k' e^{-\eta}}{\sqrt{\eta(1+\alpha_0)}} \sum_{n=0}^{\infty} \frac{1}{\gamma_n} \left(\frac{\alpha_0 - 1}{\alpha_0 + 1}\right)^n L_n^{-\frac{1}{2}}(2\eta). \tag{12}$$

Unfortunately, this formula is not valid for arbitrary angles of incidence as the series converges only if $0 \leq \theta_0 < \frac{\pi}{2}$ .Nevertheless, this difficulty may be overcome by means of summation using the known formula:

$$\begin{aligned} \rho(\frac{\eta'}{k}) &= \frac{e^{-\eta}}{\sqrt{\pi\eta}} k'(1+\alpha_0) {}_1F_1(1; \frac{1}{2}; (1-\alpha_0)\eta) = \\ &= \frac{e^{-\eta}}{\sqrt{\pi\eta}} k'(1+\alpha_0) \left\{1 - \sqrt{\pi\eta(1-\alpha_0)} e^{(1-\alpha_0)\eta} [1 - erf\sqrt{\eta(1-\alpha_0)}]\right\}. \end{aligned} \tag{13}$$

Here ${}_1F_1(a;b;t)$ is the degenerated hypergeometric function, $erf(x)$ is the probability integral. Note that the representation (13) for surface current density coincides with the other authors results [3] obtained by means of other methods. Using the representation for $\rho(x)$ (13) one may come to the following asymptotes:

$$\begin{aligned} &x \to 0; \rho(x) \approx \sqrt{\frac{2}{\pi kx}} e^{i(kx - \frac{\pi}{4})} k \cos(\frac{\theta_0}{2}) [1 - 4ikx \sin^2(\frac{\theta_0}{2})] + O\left[(kx)^{\frac{3}{2}}\right], \\ &x \to \infty; \rho(x) \approx -ik \sin\theta_0 e^{ikx\cos\theta_0} \left[1 + O(\frac{1}{kx})\right] \end{aligned} \tag{14}$$

Now let define the scattered field $E_z^s(x,y)$ in far zone. For this purpose $E_z^s(x,y)$ is to be put in the form:

$$E_z^s(x,y) = \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{\rho(\alpha)}{\sqrt{1-\alpha^2}} e^{ik(\alpha x + \sqrt{1-\alpha^2}|y|)} d\alpha. \tag{15}$$

where

$$\rho(\alpha) = \int_0^{\infty} \rho(x') e^{-ikx'\alpha} dx' = \frac{1}{k'} \int_0^{\infty} \rho(\frac{\eta}{k'}) e^{\eta\alpha} d\alpha. \tag{16}$$

is the Fourier transform of the function $\rho(x')$.

By changing the coordinate system to the polar one $(x = r\cos\varphi, y = r\sin\varphi)$ and estimating value of the integral in (15) by means of the method of stationary phase for $kr \to \infty$ and $0 < \varphi < \pi$ it is obtained the expresion:

$$E_z^s(r,\varphi) = \sqrt{\frac{1}{2\pi kr}} e^{i(rk + \frac{\pi}{4})} \rho(\alpha), \alpha = \cos\varphi. \tag{17}$$

Substitution of the function $\rho(\frac{\eta}{k'})$ representation (6) into (7) and evaluation of appearing integrals (if $\alpha < 1$) yield such expression for $\rho(\alpha)$:

$$\rho(\alpha) = \sqrt{\frac{1+\alpha_0}{1-\alpha}} \sum_{n=0}^{\infty} \left(\frac{1-\alpha_0}{1+\alpha}\right)^n = \frac{\sqrt{1+\alpha_0}\sqrt{1-\alpha}}{\alpha_0 - \alpha}. \tag{18}$$

Taking into account that $\alpha = \cos\varphi$ and $\alpha_0 = \cos\theta_0$ we get the key result of interest:

$$E_z^s(r,\varphi) = \sqrt{\frac{1}{2\pi kr}} e^{i(rk+\frac{\pi}{4})} \frac{\cos\frac{\theta_0}{2}\sin\frac{\varphi}{2}}{\cos\theta_0 - \cos\varphi}. \tag{19}$$

Thus it would seem reasonable to conclude that the proposed approach may serve a general method solution to the diffraction problems for different structures if only they may be thought of as being prodused by half-planes as shown in Fig.2.

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