# THE DISTUBED ZONE NEAR A CYLINDRIC LOW FREQUENCY ANTENNA AND ITS CAPACITANCE IN COLLISIONLESS PLASMA

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Cylindric antennas are widely used both for measuring electric fields and as sensors for determining plasma parameters in ionosphere. The impedance of a low frequency antenna is determined not only by the parameters of ambient plasma but also by an ionic shield (disturbed zone) near its surface. The most correct treatment of this zone and of the electric characteristics related to it for the cylindric electrode of a large radius with a constant potential at their surface is given in [1] . In the present paper an electric field in the vicinity of a cylindric antenna of a large and a small radii is found with regard to the particle absorption at the surface. This effect is essential for the bodies whose size exceeds the Debye length of plasma [2] or the bodies which have a small surface potential. The results obtained are used for the calculation of a static and a dynamic capacitances of the antennas of such type.

Consider a cylindric antenna of radius $r_0$ and length $\ell \gg r_0, D$ ( $D = [T/4\pi e^2 N_0]^{1/2}$ - is the Debye length of plasma) in collisionless plasma. The particle distribution over velocities of the non-disturbed plasma is set to be maxwellian with concentration $N_0$ and temperature $T$ . We shall assume that at the antenna surface all the electrons, having reached the antenna, are absorbed, and the ions neutralized.

The near zone of an antenna can be described in a quasistatic approximation. An electric field potential $\varphi$ is defined in this case by the Poisson equation which has to be solved together with kinetic equations for the distribution functions of particles which determine their concentrations $N_{i,e}$ .

Let us examine the case when the potential at the antenna surface $\varphi_0$ is negative, and the frequency of its variation $\omega$ satisfies the condition $\omega_{Li} \ll \omega \ll \omega_{Le}$ ( $\omega_{Li,e}$ are plasma frequencies of ions and electrons). In this case the electrons are repulsing particles in a quasistatic electric field, and for their concentration far from the ends of the antenna booms (their influence is neglected) we have [3]

$$n_e = N_e/N_0 = e^{-\psi} - \pi^{-1}\int_{\psi_0}^{\infty} \arcsin\left(\xi^{-1}\left[\frac{x-\psi_0}{x-\psi}\right]^{1/2}\right) e^{-x}\,dx\,,\ \xi = r/r_0\,,\ \psi = -e\varphi/T. \quad (1)$$

The alternating electric field does not affect the motion of ions, therefore $n_i$ is determined by an expression similar to that in the vicinity of a cylinder with a fixed surface potential, which has a weak dependence on the magnitude of this potential [4] . Therefore, assuming that $|e\varphi_0| \ll T$, $n_i$ can be described by the expression for neutral particles with regard to their absorption at the surface

$$n_i = N_i / N_0 = 1 - \pi^{-1} \arcsin \xi^{-1}. \tag{2}$$

Now the problem goes to the solution of the Poisson equation which in terms of dimensionless variables has a form

$$\xi^{-1} \frac{d}{d\xi}\left( \xi \frac{d\psi}{d\xi} \right) = \left( \frac{\tau_0}{D} \right)^2 (n_i - n_e), \tag{3}$$

with boundary conditions $\psi|_{\xi \to 1} = \psi_0$, $\psi|_{\xi \to \infty} = 0$.

In the cases, which are most important for practical application when, $\tau_0 \gg D$ and $\tau_0 \ll D$, the solution (3) can be done via asymptotic methods.

For $\tau_0 \gg D$ (the cylinder with a large radius) (3) is equation with a small parameter proceding the highest derivative. Its solution everywhere, where $|d\psi/d\xi| \ll (\tau_0/D)\psi$ coincides with the solution of a quasineutrality equation $n_i(\xi,\psi) = n_e(\xi,\psi)$ with precision to the small terms. Taking into account (1) and (2) its solution is

$$\psi = -\ln\left\{ 1 + \pi^{-1} \left[ \int_{\psi_0}^{\infty} \arcsin\left( \xi^{-1} \left[ \frac{x - \psi_0}{x - \psi} \right]^{1/2} \right) e^{-x} dx - \arcsin \xi^{-1} \right] \right\}. \tag{4}$$

For $\xi \gg 1$, by expanding (4) over $\psi$ and $\xi^{-1}$, we have

$$\psi = \psi_\infty \xi^{-1}, \quad \psi_\infty(\psi_0) = \pi^{-1} \left[ 1 - \int_{\psi_0}^{\infty} e^{-x} \left[ \frac{x - \psi_0}{x} \right]^{1/2} dx \right]. \tag{5}$$

Near the antenna surface, in the region of a double sheath whose thickness under our conditions is of the order of $D$, where derivative $d\psi/d\xi$ is large, one needs to solve a full equation (3).

By neglecting the antenna surface curvature in this region we have that $n_e$ is determined by the expression (2) for $\xi = 1$, and $n_i = 1/2$. Thus, for the potential in the double sheath we have equation

$$d^2\psi/d\zeta^2 = f(\psi), \quad f(\psi) = n_i - n_e(\psi) \tag{6}$$

with boundary conditions $\psi|_{\zeta \to 0} = \psi_0$, $\psi|_{\zeta \to \infty} = \psi_1$, where $\psi_1$ is solution (4) for $\xi = 1$, and $\zeta = (\tau - \tau_0)/D$. In the nonexplicit form the solution of this equation is

$$\zeta = \int_{\psi}^{\psi_0} \left[ 2 \int_{\psi_1}^{\psi} f(\psi)\, d\psi \right] d\psi .$$

By matching the solutions obtained in the quasineutrality domain $\psi_{quas.}$ and in the double sheath $\psi_{d.sh.}$, that are asymptotic expansion of the solution (3) in the corresponding regions, we shall obtain a solution equally suitable for any values of the variable [5]. With precision to the terms of the order of $(D/\tau_0)^2$ we have $\psi = \psi_{quas.} + \psi_{d.sh.} - \psi_1$. A uniformly suitable approximation in case of $D/\tau_0 = 0.1$ and $\psi_0 = 0,5$ is demonstrated in Fig. 1. A dashed line is used to indicate the solutions of equations (4)(curve 1) and (6) (curve 2). A horizontal dashed line corresponds to the $\psi_1$ value.

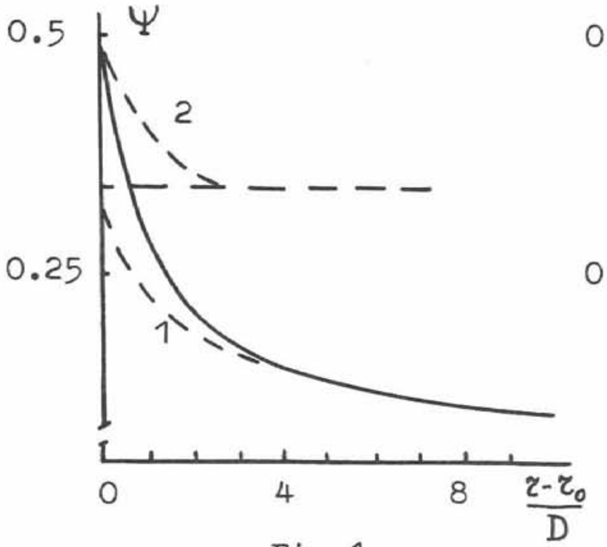


Fig.1

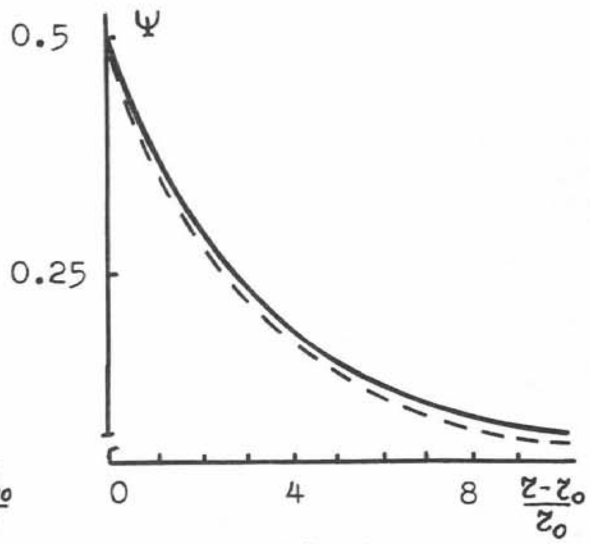


Fig.2

It should be noted, that such method permits to obtain the solution (3) at any large $r_0/D$ ratios, having once carried out simple calculations in the double sheath and the quasineutrality domain at a given $\psi_0$ .

For $r_0 \ll D$ (the cylinder of a small radius) we have a small parameter in the right side of (3), and its solution can be obtained via a method of composed equation. In this case the expressions for $n_{i,e}$ , valid in the region of $r \gg D$ , should be used for all $r$ values, the same as for $r < D$ , where the potential derivatives are large, the form of $n_{i,e}$ is irrelevant. Taking into account that for $r \gg D$ the $n_{i,e}$ and $\psi$ disturbances are small, a composed equation looks as

$$d^2\psi/dx^2 + x^{-1}\,d\psi/dx - \psi = -(r_0/D)\,\psi_\infty/x ,$$

where $x = r/D$ , $\psi_\infty$ has been determined in (5). The solution of this equation is

$$\psi = \psi_0\frac{K_0(x)}{K_0(x_0)} + \psi_\infty x_0\left[I_0(x)\int_x^\infty K_0(t)\,dt + K_0(x)\int_{x_0}^{x} I_0(t)\,dt - K_0(x)\frac{I_0(x_0)}{K_0(x_0)}\int_{x_0}^{\infty} K_0(t)\,dt\right]. \qquad (7)$$

where $x_0 = r_0/D$ ; $I_0(x)$ and $K_0(x)$ are modified Bessel functions [6] . The form of the field potential at $\psi_0 = 0.5; r_0/D = 0.1$ is shown in Fig.2. For $r < D$ the main role in eq. (7) belongs to the first term which is shown in Fig.2 by a dashed line. For $r \gg D$ $\psi = \psi_\infty/\xi$ .

The calculation of the electric field at the antenna surface permits to determine the magnitude of its capacitance. The Gauss theorem provides a relation between surface charge density $\sigma$ and field strength at the surface $\mathcal{E}_0$ : $\sigma = \mathcal{E}_0/4\pi$ . For the static and dynamic capacitance per surface unity we have then

$$C_{st} = (4\pi)^{-1}\,\mathcal{E}_0/\varphi_0 = \varkappa_{st}/4\pi D\ , \quad \varkappa_{st}(\psi_0) = E_0/\psi_0\ ;$$
$$C_d = (4\pi)^{-1}\,d\mathcal{E}_0/d\varphi_0 = \varkappa_d/4\pi D\ , \quad \varkappa_d(\psi_0) = dE_0/d\psi_0\ .$$

The $\varkappa_{st,d}$ dependences for the cylinder with a large radius are shown in the Fig.3. For small potentials at the antenna surface $\psi_0 \to 0$ $E_0 = (\pi/12)^{1/2}\psi_0^{3/2}$, and for $\varkappa_{st,d}$ we obtain $\varkappa_{st} = 2/3\,\varkappa_d = (\pi/12)^{1/2}\psi_0^{1/2}$. For the capacitance of the cylinder with a small radius we have $\varkappa_{st} = \varkappa_d = -\ell/2\ln(r_0/D)$ .

The results obtained can be used for interpreting the impedance measurements of plasma parameters carried out employing the antennas of such type.

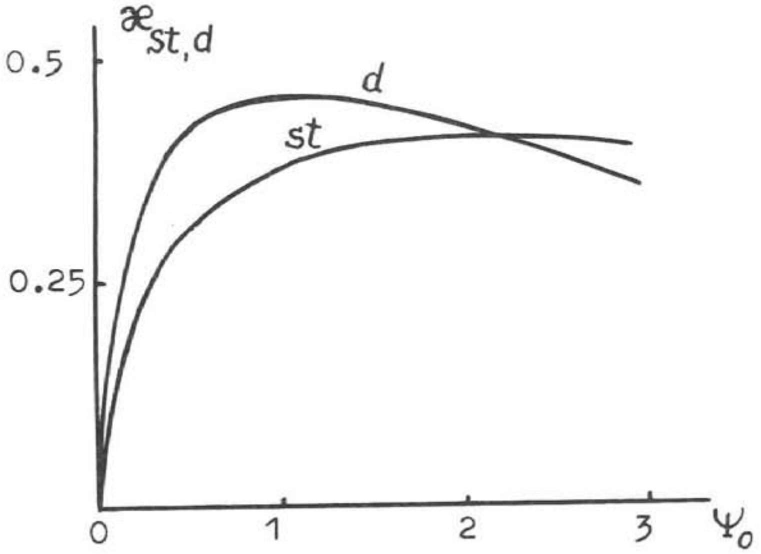

Fig. 3

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