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ON THE PROPAGATION OF BEAM WAVE IN AN INHOMOGENEOUS MEDIUM

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§1. Introduction

Theoretical investigations on the propagation of the infinite-plane waves in an inhomogeneous medium have been purblished by many writers[1]. The plane wave is an idealized one, however, in actual cases the waves emitted from antennas are beam waves[2]. Although it is of practical importance to study the propagation of beam waves, investigations on these problems have not been yet reported.

It is now of interest to study how the shape of a beam wave is deformed by a reflection from the ionosphere or atmosphere. In order to obtain a general behaviour of the beam in the inhomogeneous media, fundamental problem where a Gaussian beam waves incident on an imhomogeneous medium in which the permittivity varies linearly in one direction is analyzed and numerically calculated in detail.

§2. Reflected beam waves

For simplicity we shall consider only fields that have no x dependence. As a model of radio wave propagation in the ionosphere or atmosphere, let us consider an inhomogeneous medium in which the permittivity ε is assumed to be given by

$$\varepsilon=\varepsilon_0(1-bz) \quad . \qquad (1)$$

Then the electromagnetic beam waves satisfying Maxwell's equations and boundary conditions can be expressed by taking the inverse Fourier transform. The reflected beam waves in free space is given by

$$E_x^r(y,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j\left[\frac{4k_0}{3b}\left(1-\frac{p^2}{k_0^2}\right)^{3/2}-\frac{\pi}{2}\right]+jpy+j(z-z_0)\sqrt{k_0^2-p^2}}\, E_i(\alpha)\,d\alpha \quad , \qquad (2)$$

where $p=-\alpha\cos\phi-\sqrt{k_0^2-\alpha^2}\sin\phi$ and $E_i(\alpha)$ may be regarded as the spectrum of the aperture field $E_i(\xi)$ and may be represented by

$$E_i(\alpha)=\int_{-\infty}^{\infty}E_i(\xi)e^{-j\alpha\xi}d\xi \quad . \qquad (3)$$

Let us consider the Gaussian beam represented by

$$E_i(\xi)=e^{-a^2\xi^2} \quad . \qquad (4)$$

After some approximations[3], integral in (2) can be determined in a closed form and the amplitude of its reflected beam is given by

$$E_R(y,-L)=\frac{e^{-\frac{\{a\cos\phi(y-R_1)\}^2}{1+X_R^2}}}{\sqrt[4]{1+X_R^2}} \quad , \qquad (5)$$

where $R_1=2\frac{\sin 2\phi}{b}+(z_0+L)\tan\phi$.

We find that the position of the beam wave on the y axis is shifted by R_1 which does not depend on the shape of the beam. In equation (5), parameter X_R is given by

$$X_R=\frac{1}{2}(2a\cos\phi)^2R_2 \, , \quad R_2=\frac{1}{k_0\cos\phi}\left[\frac{4}{b}\cos 2\phi+\frac{(L+z_0)}{\cos^2\phi}\right],$$

which controls the amount of distortion of the beam that becomes larger with increasing value of X_R. On the other hand, when X_R is zero, i.e.,

$$\cos 2\phi\cos^2\phi=-\frac{b}{4}(z_0+L) \quad , \qquad (6)$$

then the amplitude of reflected beam wave becomes

$$E_R(y,-L)=e^{-\{a\cos\phi(y-R_1)\}^2} \quad . \qquad (7)$$

This equation means that the reflected beam wave has the same wave form as incident beam. Under the above condition the reflected beam waves are transmitted without distortion.

The amplitude of the beam wave has been plotted versus the y axis for typical examples in Fig.2,3.

§3. Conclusions

For radio-wave communication with ionospheric or atmospheric propagation, we have studied the characteristics of the beam wave propagating through inhomogeneous media. It is found for the first time that the reflected beam can be transmitted without distortion. In order to confirm the validity of the theory, a model experiment was performed in the millimeter wave range.

References

(1) L.M.Brekhovskikh, Waves in Layered Media, Academic Press, London, 1960

(2) S.Silver, Microwave Antenna Theory and Design, Dover, New York, 1965

(3) S.Kozaki and Y.Mushiake, Propagation of the electromagnetic pulse with Gaussian envelope in inhomogeneous ionized media, IEEE Trans. (Communication) Vol.AP-17, pp.686-688, Sep., 1969

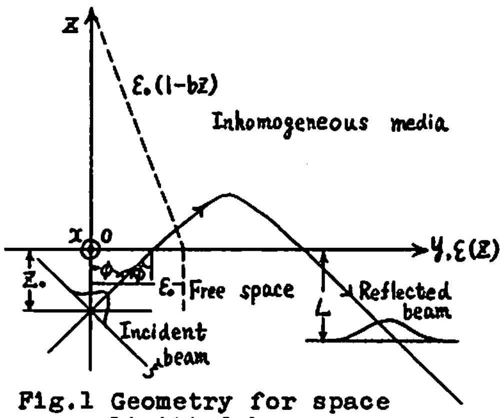

Fig.1 Geometry for space limitted beam

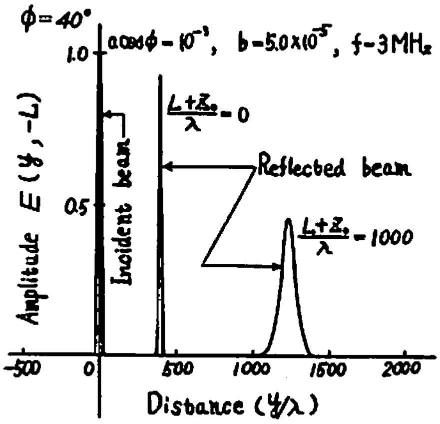

Fig.2 Beam waveforms in various points of observation

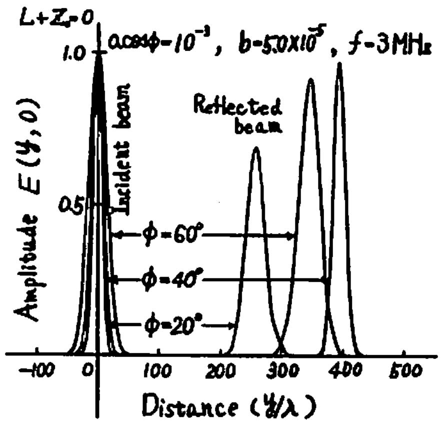

Fig.3 Beam waveforms for various angles ϕ