NUMERICAL INVESTIGATION OF REGULAR OPEN-SLITTED WAVEGUIDES AND CAVITY-BACKED APERTURES WITH ARBITRARY CROSS-SECTION

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Method for solving eigenvalue problems for a longitudinally slitted open waveguide with arbitrary cross-section is presented. Both TM and TE waves of elliptic and Cassini-oval shaped waveguides are investigated numerically. Applications of the treatment are seen in the simulation of leaky-wave antennas and cavity-backed apertures.

FORMULATION OF THE PROBLEM

Hollow metal waveguide with zero-thickness and perfectly conducting walls is considered whose cross-section is formed by arbitrary and in general nonclosed contour L assumed to be a part of some closed not-intersecting smooth curve S. Since waveguide is regular and infinite along z-axis, electromagnetic field can be represented as a normal wave (mode)with dependence on time and z as exp(ihz-ikct) where k is the free-space wave number, h is the complex constant of propagation.

Take transverse wavenumber $g=(k^2-h^2)^{1/2}$ as a spectral parameter located in the Riemann surface C of the function Ln g. In considered waveguide, all the modes are known to divide to TM and TE waves, and further will be investigated separately. Spectral values of g correspond to generalized critical frequencies of the guide, and coincide with complex frequencies of damped resonances of scatterers with cavity-backed apertures.

LEAKY TM WAVES

For these waves initial spectral (i.e.eigenvalue) problem can be reduced to the Dirichlet boundary value problem for 2-D Helmholtz equation for a scalar function u(p) representing E_z component of the field (others are expressed via u(p))

$$(\Delta+g^2)u(p)=0, \qquad p=(x,y)\in R^2\backslash L; \qquad u(p)=0, \qquad p\in L \qquad (1),(2)$$

If the contour L is nonclosed we assume that the function u(x,y) satisfies the Meixner condition in the vicinity of endpoints $p^*_{1,2}$ of the contour L, given by

$$\text{grad } u(p)=|(p-p^*_1)(p-p^*_2)|^{-1/2}h(p), \qquad (3)$$

where h(p) is some function of Holder class in the neibourhood of L. Besides, it is assumed that the function u(p)is subjected to the Reichardt condition at infinity ($\rho=|p| \to \infty$)

$$u(p)= \sum c_n H_n^{(1)}(k\rho)e^{in\phi} \qquad (4)$$

The eigenvalue problem is to define the set $\sigma_g \in C$ for which nontrivial solutions of (1)-(4) do exist.

Function u(p) can be sought as a single-layer potential with unknown density Z(q) satisfying integral equation on L

$$\int_L Z(q) H_0^{(1)}(g|p-q|) dS_q = 0, \qquad p \in L \tag{5}$$

with condition

$$Z(q) = [s(q,p_1^*) s(q,p_2^*)]^{-1/2} \Phi(q), \qquad q \in L \tag{6}$$

where s(p,q) is the arclength between p and q along L, while $\Phi(q)$ is of Holder class on L in corresponding metrics.

As there is a bijective mapping of the interval $[-\pi,\pi]$ on S such that L is the image of interval [-d,d], we can write

$$\int_{-\pi}^{\pi} z(\tau) H_0^{(1)}[gR(\theta,\tau)] d\tau = 0, \qquad \theta \in (-d,d) \tag{7}$$

where

$$z(\tau) = \begin{cases} Z[x(\tau),y(\tau)] l(\tau), & \tau \in [-d,d] \\ 0, & \tau \in [-\pi,\pi] \setminus [-d,d] \end{cases} \tag{8}$$

$$R(\theta,\tau) = \{[x(\theta)-x(\tau)]^2 + [y(\theta)-y(\tau)]^2\}^{1/2}, \quad l(\tau) = \{[x'(\tau)]^2 + [y'(\tau)]^2\}^{1/2}$$

Decomposing the kernel function of (7) as

$$H_0^{(1)}[gR(\theta,\tau)] = H(\theta,\tau) + 2i\pi^{-1} \ln|2\sin[(\theta-\tau)/2]| \tag{9}$$

and expanding both $H(\theta,\tau)$ and the logarithm in terms of angular exponents as

$$H(\theta,\tau) = \sum_{(m)} \sum_{(n)} h_{mn} e^{i(m\theta+n\tau)}, \qquad \ln|2\sin[(\theta-\tau)/2]| = -1/2 \sum_{(n\neq 0)} e^{in(\theta-\tau)}/|n|$$

we come to the dual series equations

$$\sum_{(n\neq 0)} |n|^{-1} z_n e^{in\theta} = 2 \sum_{(n)} e^{in\theta} \sum_{(m)} h_{n,-m} z_m, \qquad \theta \in [-d,d] \tag{10}$$

$$\sum_{(n)} z_n e^{in\theta} = 0, \qquad \theta \in [-\pi,\pi] \setminus [-d,d]$$

Note that if the functions $x(\theta), y(\theta)$ parametrizing S are smooth enough, then the Fast Fourier Transform algorithm can be efficiently applied for the computation of coefficients h_{mn}.

Equations (10) can be regularized by means of the procedure of Ref.1 with the result of the linear algebraic system of the 2nd kind. Thus, the initial spectral problem is equivalently reduced to eigenvalue problem for certain matrix operator-function I+H(g) where H(g) is analytical with respect to g and compact in some Hilbert space for each $g \in C \setminus (0,\infty)$. Then one can conclude that σ_g consists of isolated points of $C \setminus (0,\infty)$ of finite multiplicity. To find them numerically with any desired

accuracy, one can use truncated system of linear equations.

LEAKY TE WAVES

In this case we start with the Neimann boundary problem

$$(\Delta+g^2)v(p)=0, \quad p\in R^2\setminus L; \qquad \partial v(p)/\partial n_p=0, \quad p\in L \qquad (11),(12)$$

where $v(p)=H_z(x,y)$, plus conditions (3) and (4) for $v(p)$. The field function here can be represented as a double-layer potential

$$v(p)=\int_L Z(q)\frac{\partial}{\partial n_q} H^{(1)}(g|p-q|)dS_q, \qquad p\in R^2\setminus L \qquad (13)$$

where the unknown density function satisfies the conditions

$$Z(p_1^*)=Z(p_2^*)=0, \qquad \partial Z(p)/\partial s=[s(p,p_1^*)s(p,p_2^*)]^{-1/2}\Psi(p) \qquad (14)$$

with $\Psi(p)$ from the Holder class on L.

Further treatment exploits the same parametrization of the curve S as for TM modes which leads to an integro-differential equation for the function $Z(\tau)$ completed by the identical zero off L. Proceeding by analogy with the TM case and decomposing the kernel into singular and regular parts expanded in terms of Fourier angular series, we come to the dual series equations differing from (10) by term $|n|$ instead of $1/|n|$ in the left hand part. After using regularization procedure we obtain eigenvalue problem for the set of linear equations which is solvable numerically with the desired degree of approximation.

NUMERICAL RESULTS

Basing on proposed approach, the algorithm for TM and TE-waves computations was developed, effective for waveguides with arbitrary smooth cross-section. There were two main goals of our analysis: to test our results by comparing them with those available from literature, and to investigate some guides not analysed before. As only circular open-slitted waveguides have been studied before (Ref.2), we chosed the guides with cross-sections shaped as Cassini oval and ellipse. These curves depend on some geometric parameters and can vary essentially, e.g., Cassini oval changes from a circle to lemniscate.

Open-slitted waveguide is characterized by following parameters: δ which is the slot position angle, and θ, where $2(\pi-\theta)$ is the slotwidth. The shape of ellipse is defined by its eccentricity ε. For $\varepsilon=1$ ellipse degenerates into a circle. In this case the spectral values of ga must coincide with zeros of integer-n Bessel functions and their derivatives for TM-waves and TE-waves, respectively. We discovered that for both types our algorithm delivers the results (for several lowest modes, and with truncation number N=20) within the accuracy of 10^{-6} as compared with the tabulated ones.

In Figs.1,2 the plots of spectral values as functions of slotwidth are presented for several lower modes in an open-

slitted elliptic waveguide with b/a=0.5 (2a,2b are larger and smaller axes) for two different positions of the slot: $\delta=0,\pi/2$.

Fig.3 demonstrates similar plots for leaky TM-modes of the same elliptic guide. As for Cassini oval cross-section, its shape is defined by the equation $(x^2+y^2+c^2)^2-4c^2x^2=d^4$. Fig.4 shows complex wavenumbers of TE_{oo}-mode (so-called slot mode) in this waveguide as functions of the parameter $q=c/d$ while 2a is again the larger axis.

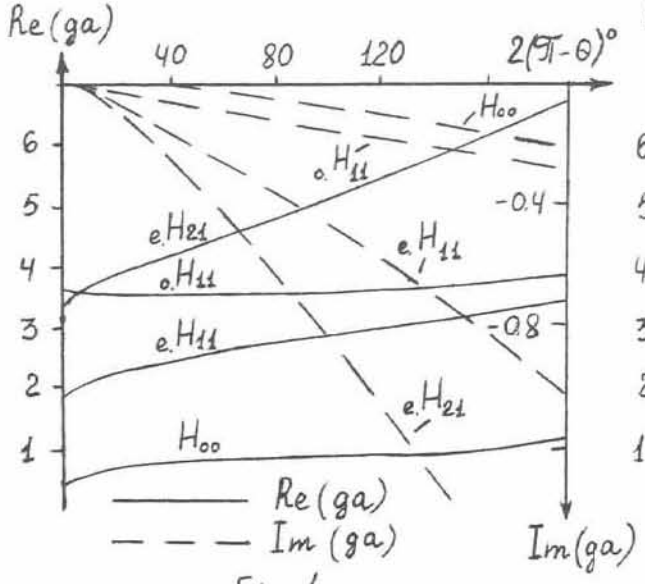

Fig. 1

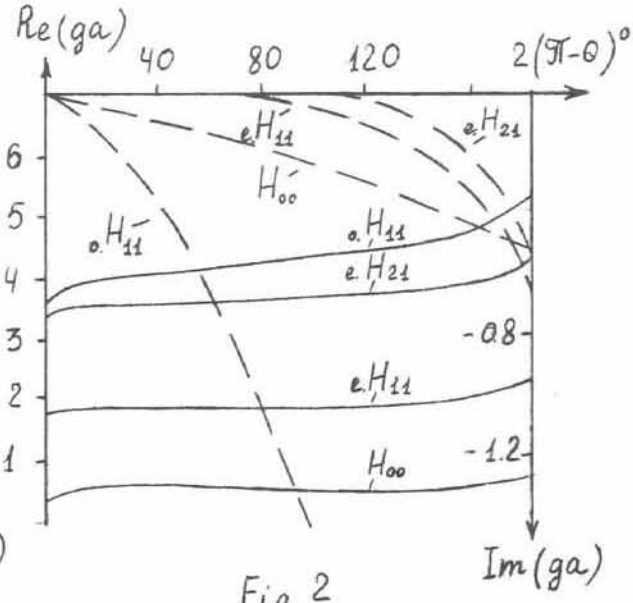

Fig. 2

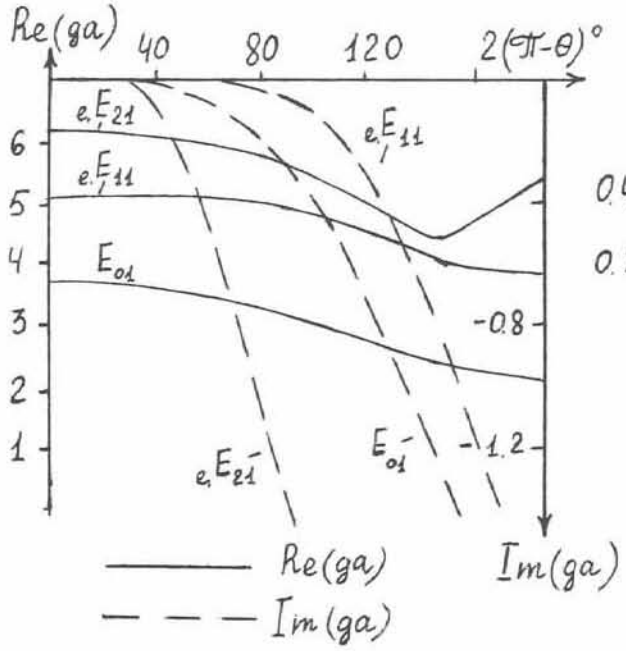

Fig. 3

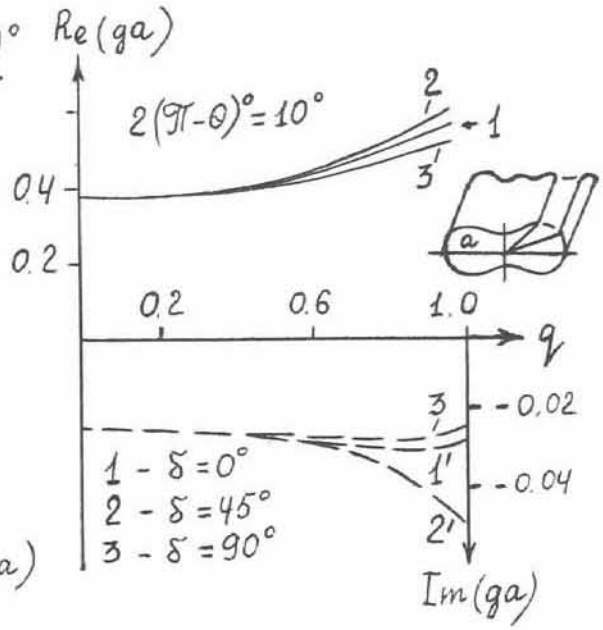

Fig. 4

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