

DOPPLER EFFECT IN ANISOTROPIC MEDIUM.
K.A. Barsukov.

Moscow Lenin State Teacher's Training University,
Moscow USSR.

We would like to discuss here some peculiarities in the radiation of the source, moving uniformly through an anisotropic medium. Such problems arise, for example, while investigating of the ionosphere with the properties of a girotropic and anisotropic crystal by the radiator placed on a fast moving rocket when all the information is communicated by the radio waves. We'll examine below the so-called Doppler effect along the ray direction when the Doppler frequency is considered to be the function of the angle between the source velocity and the ray direction. The Doppler effect along the ray direction possesses of a number of the interesting peculiarities, which we'll illustrate here by the example of the uniaxial anisotropic crystal with ϵ_{11} and ϵ_{\perp} as constants. For example, for a cold electronic plasma in a strong magnetic field $\epsilon_{11} = 1$ and $\epsilon_{\perp} = 1 - \omega_p^2/\omega^2$, where ω_p is the plasma frequency. In this case the Doppler formula for the extraordinary waves may be written in the form

$$\omega = \Omega / 1 -$$

$$- \beta \sqrt{\epsilon_{11}} \frac{s \cos \vartheta + (1-s) \cos \alpha \cos \varphi}{\sqrt{\cos^2 \varphi + s \sin^2 \varphi}} \quad (1)$$

where $s = \epsilon_{\perp} / \epsilon_{11}$ and α, φ denote the angles between the axis of the crystal, the velocity and the ray respectively, $\Omega = \Omega \sqrt{1 - \beta^2}$ and Ω is the frequency of the

radiator in the co-moving frame. There are two interesting consequences followed then from (1). In the first place, if the ray is perpendicular to the velocity direction of the source ($\vartheta = \pi/2$, the transverse Doppler effect) the frequency shift will appear to be proportional to the source velocity and this shift being highest possible for $\varphi = \frac{\pi}{2} \pm$

$$\text{equals } \frac{\pm \arcsin \left[\sqrt{\frac{\epsilon_{11}}{\epsilon_{\perp}}} + \sqrt{\epsilon_{\perp}} \right]}{\frac{\pi}{2} \pm \alpha}$$

$$\left(\frac{\omega - \Omega}{\omega} \right)_{\max} = \pm \beta (\sqrt{\epsilon_{11}} - \sqrt{\epsilon_{\perp}}).$$

(2)

For the strong magnetoplasma we have

$$\varphi = \frac{\pi}{2} \pm \arcsin \left[1 + \sqrt{1 - \omega_p^2 (\Omega + \beta \omega)^2} \right]^{-1}$$

and

$$(\omega - \Omega)_{\max} = \beta \omega_p.$$

There is also the second remarkable peculiarity of the Doppler effect, when, for the example, the ray direction forms an acute angle with the velocity direction, but the wave vector - an obtuse angle. The peculiar inverted Doppler effect arises in this case - that is the frequencies less than Ω are radiated forward, although in the usual Doppler effect they are radiated backward. The inverted Doppler effect can be occurred for an obtuse angle between the ray and velocity (the backward radiation) too. In general case there is the sufficient condition for the inverted Doppler effect of the form

$(\vec{k}, \vec{v}) / (\vec{w}, \vec{v}) < 0$, where $\vec{k}, \vec{v}, \vec{w}$ denote the wave vector, the source velocity and the group velocity vector respectively. The maximum range of the angles, where the inverted Doppler effect can exist is occurred for $\alpha = \alpha \text{ctg } \sqrt{S}$ and $|\text{tg } \vartheta| > 2\sqrt{S} / |1 - S|$. The peculiarities of the Doppler radiation, showed above, exist also in a cold electronic plasma with a magnetic field. So, if we consider the radiator moving along the uniform, homogenous magnetic field in plasma, the inverted Doppler effect for the extraordinary waves will occur in the following frequency range and the angles

$$\omega_1(\vartheta_1) < \omega < \frac{1}{2} \left\{ \sqrt{\omega_0^2 + \omega_H^2 + 2\omega_0\omega_H \cos \vartheta_1} + \sqrt{\omega_0^2 + \omega_H^2 - 2\omega_0\omega_H \cos \vartheta_1} \right\} \quad (3)$$

where ω_H denotes the gyro-magnetic frequency of the plasma, ϑ_1 is the angle between the wave vector and the magnetic field H_0 and $\omega_1(\vartheta_1)$ is determined by the equation

$$\begin{aligned}
 \omega_H^2 \left[1 - \frac{\omega_0^2}{\omega_1(\omega_1^2 - \omega_H^2)^{1/2}} \right] \sin^2 \vartheta_1 &= \\
 = 2(\omega_1^2 - \omega_0^2) \left(1 + \frac{1}{\omega_1} \sqrt{\omega_1^2 - \omega_H^2} \right) &\quad (4)
 \end{aligned}$$

with $\omega_1(\vartheta_1)$ being changed in the range:

$$\begin{aligned}
 \max \left(\frac{\omega_0}{\omega_H} \right) \leq \omega_1(\vartheta_1) \leq &\quad (5) \\
 \leq \sqrt{0,5(\omega_H^2 + \sqrt{\omega_H^4 + 4\omega_0^4})} &
 \end{aligned}$$