A Simplified Computational Procedure for Transient Response of Thin Cylindrical Antennas

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I. Introduction

Transirnt radition problem may be solved in the time-domain or frequency -domain. In this paper a stratified medium model (transmission line model) of the current distribution is presented for thin cylindrical antennas. By an n inverse Fourier transform the universal current distribution expression in the time-domain is obtained. For an arbitrary applied voltage g (t) , the radiated field is easily found from the impulse response by using convolution. Different from Ref. [1, equations (41)] and [2, equations (15)] , comparing our results with corresponding waveforms obtained from frequency-domain analysis in Ref. [3] and [4] , good agreements are obtained in various radiation directions.

II. Stratified medium model (transmission line model)

To find out the current distribution on thin cylindrcal antennas under a Dirac pulse excitation , imagine the theory model to be a stratified medium problem. The stratified medium model is divided into regions as in Fig.1. We introduce the antenna input boundary and the antenna output boundary (see Fig.1) .

Excitation pulse is

$$F(x,t) = \delta\,(t - x/c_0) \tag{1}$$

which has the frequency-domain representation

$$\hat{f}(x,k_0) = \exp(-jk_0x) \tag{2}$$

where c_0 and k_0 are the phase velocity and wave number in the feed line region, respectively. The current distribution in the antenna region are found by [5]

$$\hat{I}(x,k_1) = B_1\exp(-jk_1x) - A_1\exp(jk_1x) \tag{3}$$

$$A_1 = (A_0+R_{10})/T_{10} = T_{01}R_{12}\exp(-j2k_1L)/(1-R_{10}R_{12}\exp(-j2k_1L)) \tag{4}$$

$$B_1 = (R_{10}A_0+1)/T_{10} = T_{01}/(1-R_{10}R_{12}\exp(-j2k_1L)) \tag{5}$$

$$A_0 = R_{0+} = (R_{12}\exp(-j2k_1L)-R_{10})/(1-R_{10}R_{12}\exp(-j2k_1L)) \tag{6}$$

The current distribution may be written in the following way

$$\hat{I}(x,k_1) = T_{01}\sum_{n=0}^{\infty}R_{10}^nR_{12}^n\exp(-j2nk_1L)\exp(-jk_1X) - T_{01}/R_{10}\sum_{n=1}^{\infty}R_{10}^nR_{12}^n \exp(-j2nk_1L)\exp(jk_1L) \tag{7}$$

where R_{10} and R_{12} are the voltage reflection coefficients, $T_{01}=1-R_{01}=1+R_{10}$ is the voltage transmission coefficient.

$$R_{mn} = (Z_n - Z_m)/(Z_n + Z_m) \tag{8}$$

Where Z_n and Z_m are the characteristic impedance of the transmission line.

By an inverse Fourier transform (7) the current distribution expression in the time-domain are given by

$$I(x,t)=T_{01}\sum_{n=0}^{\infty}R_{10}^{n}R_{12}^{n}\,\delta(t-x/c_1-2nL/c_1)-(T_{01}/R_{10})$$

$$\sum_{n=1}^{\infty}R_{10}^{n}R_{12}^{n}\,\delta(t+x/c_1-2nL/c_1) \qquad (9)$$

Equation (9) may be written as

$$I(x,t)=I_0\{\sum_{n=odd}\Gamma_{on}\,\delta[t-(n-1)L/c_1-x/c_1]-\sum_{n=even}\Gamma_{en}\,\delta[t-nL/c_1+x/c_1]\} \qquad (10)$$

where C_1 is the phase velocity in the antenna region, $I_0=T_{01}$.

$$\Gamma_{on}=(R_{10}R_{12})^{(n-1)/2},\qquad n=1,3,5,\cdots \qquad (11)$$

$$\Gamma_{en}=(R_{10}R_{12})^{n/2}/R_{10}\qquad n=2,4,6,\cdots \qquad (12)$$

Eqation (10) is the current distribution expretion of the thin cylindrical antenna excited by Dirac pulse. In the process of to develop the formula, the thin cylindrical antenna (see Fig. II) be treated as the transmission line (see Fig. I).

III. Results and Conclusions

The impulse response is given by

$$h(t^*,\theta)=\frac{I_0 Z_c}{4\pi cr}\sin\theta\{\sum_{n=odd}\Gamma_{on}\frac{\partial}{\partial t}\int_{-L}^{L}\delta[t^*-(n-1)\frac{L}{c}-\frac{|x|}{c}(1-|\cos\theta|)]dx$$

$$-\sum_{n=even}\Gamma_{en}\frac{\partial}{\partial t}\int_{-L}^{L}\delta(t^*-n\frac{L}{c}+\frac{|x|}{c}(1+|\cos\theta|))dx]\}$$

$$=\frac{I_0 Z_c}{4\pi r}\sum_{n=odd}\Gamma_{on}\{\frac{\sin\theta}{1-\cos\theta}[\delta(t^*-(n-1)\frac{L}{c})-\delta(t^*-(n-1)\frac{L}{c}$$

$$-\frac{L}{c}(1-\cos\theta))]+\frac{\sin\theta}{1+\cos\theta}[\delta(t^*-(n-1)\frac{L}{c})-\delta(t^*-(n-1)\frac{L}{c}$$

$$-\frac{L}{c}(1+\cos\theta))]\}-\frac{I_0 Z_c}{4\pi r}\sum_{n=even}\Gamma_{en}\{\frac{\sin\theta}{1+\cos\theta}[\delta(t^*-(n)\frac{L}{c})$$

$$-\delta(t^*-n\frac{L}{c}+\frac{L}{c}(1+\cos\theta))]+\frac{\sin\theta}{1-\cos\theta}[\delta(t^*-n\frac{L}{c})$$

$$-\delta(t^*-n\frac{L}{c}+\frac{L}{c}(1-\cos\theta))]\} \qquad (13)$$

Here L is the dipole half-length, C is velocity of light, Z_c is the wave impedance , and $t^*=t-r/c$ is the retarded time . The output of a linear time-invariant system is given by the convolution of the input with impulse response of the system , i.e.

$$e(t)=g(t)\circledast h(t) \qquad (14)$$

Here the symbol $\circledast$ means the convolution integral.

The excitation pulse used in calculating the radiation field has the voltage

$$\begin{cases} g(t)=0 & \text{for } t<0 \\ g(t)=V_0\,[1-(1+t/\tau)\exp(-t/\tau)] & \text{for } 0\leqslant t\leqslant T \\ g(t)=V_0\{[1-(1+t/\tau)\exp(-t/\tau)]-[1-(1+(t-T)/\tau)\exp[-(t-T)/\tau]]\} & \text{for } t\geqslant T \end{cases} \quad (15)$$

where T is the pulsewidth , τ is the pulse-rise time , and V_0 is a constant.

The convolution integral of (14) should be calculated in each interval of time for different ranges of the value of θ ($\theta<\cos^{-1}(1-CT/L)$ and $\theta>\cos^{-1}(1-CT/L)$).To permit comparision with Ref.[3] and [4], the same conditions are used.

Fig.3 and Fig.4 show the results of time-domain analysis for the radiation waveform of the dipole in different θ. Comparing these waveforms with corresponding waveforms obtained from freqency-domain analysis in [3] and [4], good agreements are obtained in various radiation directions.

In comparison with frequency-domain solutions, the time-domain solutions have the advantage of simple analysis expression and resolved physical picture. On the other hand, numerical calculation of response in the time-domain is easy.

References

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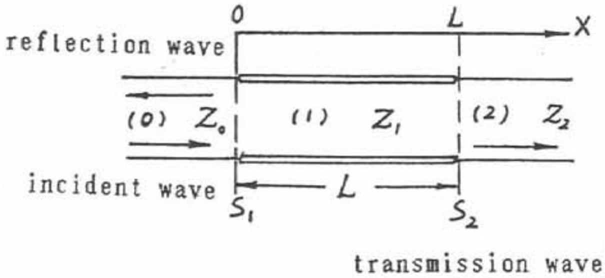

Fig.1 Stratified medium model (transmission line model)
S_1 antenna input boundary
S_2 antenna output boundary
(0) feed line degion
(1) antenna region
(2) free-space region

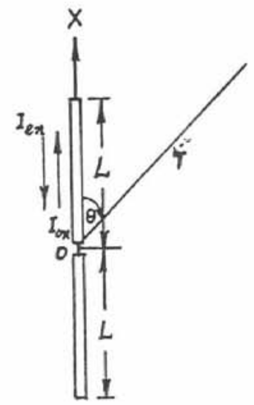

Fig. 2 Thin cylindrical antenna and coordinate system

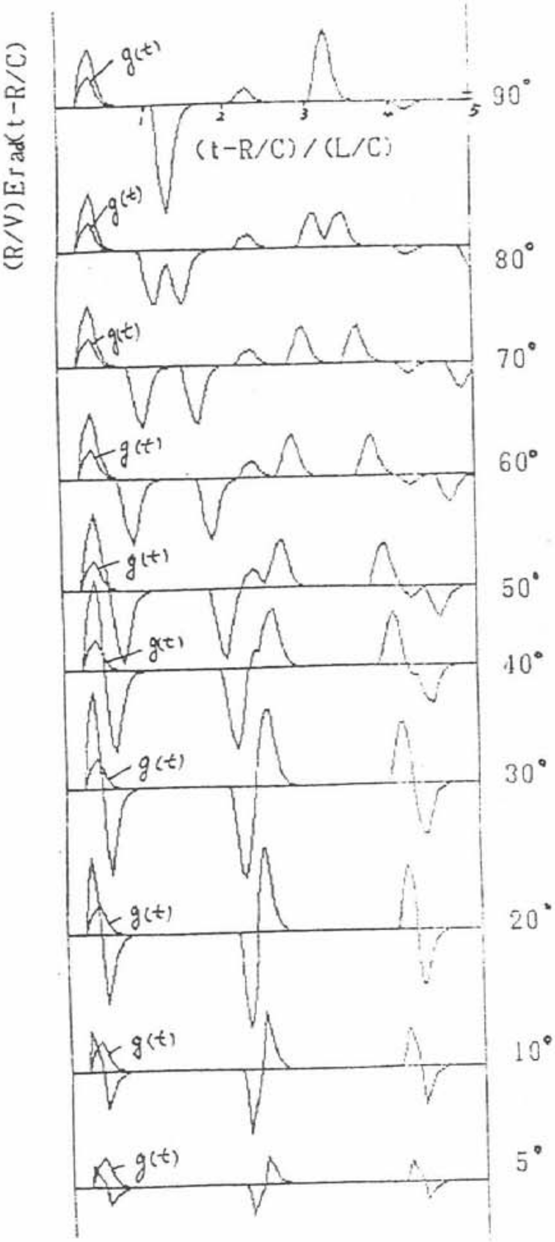

Fig. 4 The time history of the calculated radiation field in various radiation directions, T=0.2L/C, τ =0.05L/C

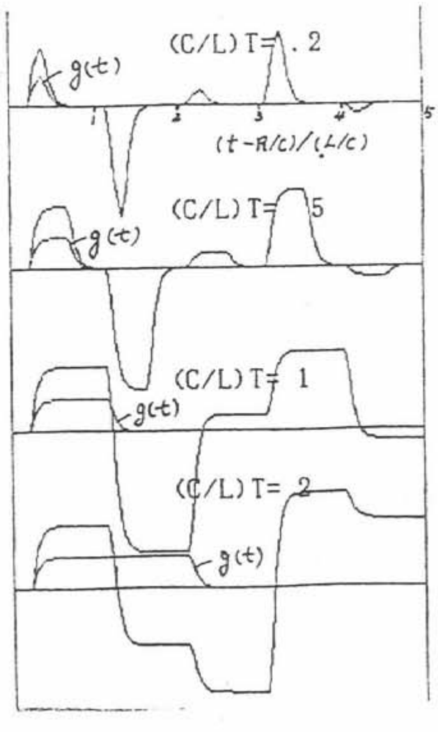

Fig. 3 The time history of the calculated radiation field in θ=90° direction. τ =0.05L/C.