

A worldwide conversion method for  
different integration time rain rates by using M distribution and thunderstorm ratio

Chieko ITO and Yoshio HOSOYA

Dept. of Electrical and Electronic Eng., Kitami Institute of Technology

Kitami-shi, Hokkaido, 090-8507, Japan

E-mail Ito-Chieko/elec@king.cc.kitami-it.ac.jp (Ito)

HOSOYA-Yoshio/elec@king.cc.kitami-it.ac.jp (Hosoya)

## 1. Introduction

The most serious transmission impairment is caused by the rain attenuation on the radio links using frequencies above about 10 GHz. Therefore, it is required to predict the rain attenuation distributions, and accordingly it is necessary to know one-minute rain rate distribution. But, rain rate is usually measured with a long integration time (e.g. 60-min. of AMEDAS : Automated Meteorological Data Acquisition System of Japan Meteorological Agency), and measurements of 1-min. rain rate distribution are rare in the world. Therefore, it is important to develop the method for converting the rain rate distributions with a long integration time to 1-min. rain rate distribution.

A few conversion methods are currently proposed. They are Segal method<sup>[1]</sup>, Damosso method<sup>[2]</sup> and Morita method<sup>[3]</sup>. But these methods are derived as empirical ones from relations between rain rate distributions with different integration times for a specified percentage of time. Therefore, it must be cautious to apply these methods to other locations and integration times that were not used to derive these methods.

In Japan, a method that can predict 1-min. rain rate distribution from 60-min. distribution was already proposed<sup>[4]</sup> by using simplified Moupfouma distribution (termed as M distribution). This method uses autocorrelation characteristics of rain rate, and it can easily be expanded to arbitrary integration time. Therefore a research was conducted to apply this method to various worldwide locations by using the newly constructed databank for 49 locations in 21 countries.

In this paper, the autocorrelation characteristics of rain rate for various locations were analyzed. As a result of this analysis, it was found that the autocorrelation characteristics of rain rate were different for each location, and that a good conversion accuracy was obtained by using suitable autocorrelation characteristics for each location. Moreover, it was found that this autocorrelation characteristics could be estimated by using regional climatic parameters such as thunderstorm ratio and so on, and it was shown that a good conversion accuracy of different integration time rain rate could be obtained.

## 2. Worldwide databank of rain rate with different integration time

For analysis in this study, A databank of rain rate with different integration time was newly constructed from many literatures. It contains data from 49 locations in 21 countries. Those are : 1.Stockholm(SE), 2.Helsinki(FI), 3.Keimola(FI), 4.Chilbolton(GB), 5.Granfield(GB), 6.Kelvedon Hatch(GB), 7.Slough(GB), 8.Wotton(GB), 9.Copenhagen(DK), 10.Paris(FR), 11.Barcelona(ES), 12.Darmstadt(DE), 13.Mseno(CZ), 14.Warsaw(PL), 15.Bari(IT), 16.Bologna(IT), 17.Fucino(IT), 18.Ghilaradona(IT), 19.Gera Lario(IT), 20.Milano(IT), 21.Rome'84-'88(IT), 22.Rome'75-'76(IT), 23.Rome'75-'77(IT), 24.Torino(IT), 25.Udine(IT), 26.Kafallinia(GR), 27.New Delhi-WM(IN),

28.New Delhi-MON(IN), 29.Yokosuka(JP), 30.Nagoya(JP), 31.Ile-Ife(NG), 32.Kingston(CA), 33.Holmdel(US), 34.Miami'73(US), 35.Miami'57-'58(US), 36.New York(US), 37.Palmetto(US), 38.Urbana(US), 39.Kourou(GF), 40.Belem(BR), 41.Brasilia(BR), 42.Manaus(BR), 43.Rio de Janeiro(BR), 44.Santa Rita(BR), 45.Iquitos(PE), 46.Ipoh(MY), 47.Ipoh'92-'95(MY), 48.Kuala Lumpur(MY), 49.Singapore(SG). This databank also includes the average annual total rainfall, the average number of thunderstorm days, and the thunderstorm ratio, etc.

### 3. Simplified Moupfouma distribution (M distribution)

The probability density function of Moupfouma distribution  $f(R)$  is eqn.(1). Parameters  $p, u, b$  are positive values.  $R_{\min}$  is the lower limit value of  $R$ <sup>[5]</sup>. In general, this distribution can approximate very well the rain rate  $R(\text{mm/h})$  distributions with different integration times for wide time-percentage range required in radio link design. Unfortunately, relations between mean/variance and distribution parameters can not be calculated easily, and the distribution of the sum of Moupfouma variate can not be calculated simply. However, it is known that parameter  $b$  is nearly equal to 1, and that the calculation of the above relations are comparatively easy. Thus, the Moupfouma distribution with  $b = 1$  is used for the following analysis and is termed as M distribution<sup>[4]</sup>.

$$f(R) = \frac{p}{R^b} \exp(-uR) \left( \frac{b}{R} + u \right) \quad (R_{\min} \leq R < \infty) \quad (1)$$

### 4. The conversion method using M distribution<sup>[4]</sup>

The probability distribution function of M distribution  $F(R)$  is eqn.(2). Parameters  $p, u$  are positive values.  $R^*$  is the solution of eqn.(3) from the condition that  $F(R^*) = 1$ , and an approximation equation for  $R^*$  was obtained<sup>[4]</sup>.

$$F(R) = \frac{p}{R} \exp(-uR) \quad (R^* \leq R < \infty) \quad (2)$$

$$p = R^* \exp(uR^*) \quad (3)$$

Fig.1 shows the comparison of measured rain rates with M distribution approximated value for all data in the databank

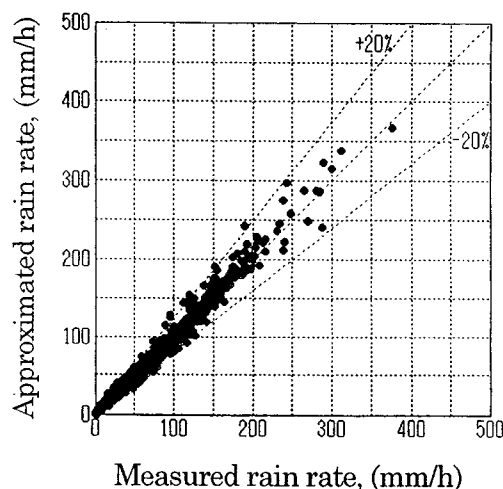


Fig.1 The comparison of measured rain rates with M distribution approximated value for all data in the databank.

From Fig.1, it is found that M distribution has good approximation accuracy for any location and any integration time. Therefore, the conversion method using M distribution is used in the following analysis. This method is as follows;

Mean  $m$  and variance  $\sigma^2$  of M distribution are given as eqns.(4) and (5), respectively.  $E_1(x)$  is the integral exponential function.

$$m = p \{ \exp(-uR^*) + E_1(uR^*) \} \quad (4), \quad \sigma^2 = p(R^* + 2/u) \exp(-uR^*) - m^2 \quad (5)$$

Parameters  $p, u$  can also be calculated from  $m$  and  $\sigma^2$ , inversely<sup>[4]</sup>.

In general,  $n$ -min. rain (mm) is the depth of rain accumulated at the Earth surface within  $n$ -min. integration time, and rain rate (mm/h) is  $n$ -min. rain multiplied by  $60/n$ . Therefore,  $n$ -min. rain is the sum of  $n$  piece of 1-min. rain. So mean  $m_n$  and variance  $\sigma_n^2$  of  $n$ -min. rain can be calculated using eqns. (6) and (7) from  $m_1$  and  $\sigma_1^2$  of 1-min. rain.  $\rho(t)$  is the correlation coefficient

(autocorrelation function) between two 1-min. rains separated by t-min. It was known that  $\rho(t)$  could be approximated by eqn.(8) from 1-min. rain data analysis, and  $\alpha = 0.247$  in Japan<sup>[4]</sup>.

$$m_n = n \cdot m_1 \quad (6), \quad \sigma_n^2 = \sigma_1^2 \left\{ n + 2 \sum_{t=1}^{n-1} (n-t) \rho(t) \right\} \quad (7), \quad \rho(t) = \exp(-\alpha t^{1/2}) \quad (8)$$

From the above, the required rain rate conversion can be done by using appropriate  $\alpha$ .

### 5. Examination of autocorrelation parameter $\alpha$ for 1-min. rain

We converted 1-min. rain rate distribution into n-min. distributions for each location by using the databank and this method. The parameter  $\alpha$  was determined to minimize the conversion error. As the parameter  $\alpha$  can not be calculated for locations without 1-min. rain data, (No.6, 8, 9, 13, 18, 27, 28, 36, 39, 45), we excluded them from the analysis. For No.38 (n = 2 min.)  $\alpha$  was 0. This means that rain does not change with time. This is not realistic in practical situations. So, we excluded it from the analysis.

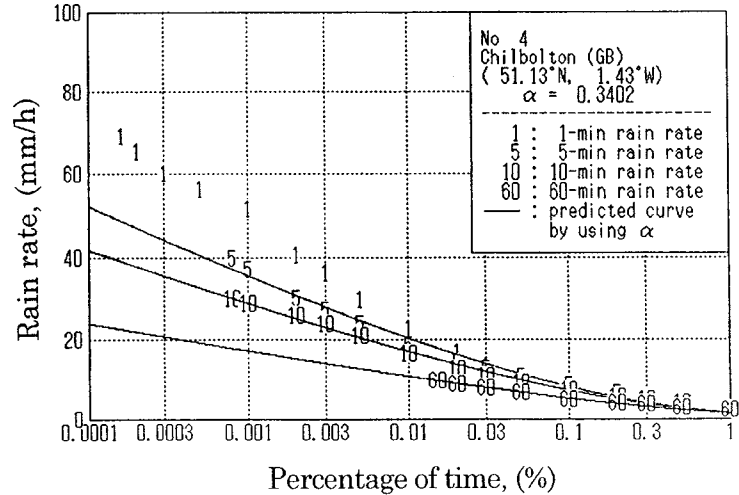


Fig.2 Measured rain rates data and predicted values by using  $\alpha$  in No.4.

Fig.2 shows measured 1-min. and n-min. rain rate data in No.4, and predicted distribution curve by using  $\alpha$ . It is found that a good conversion accuracy is obtained by using suitable  $\alpha$  for each location. However, it was found that the differences in  $\alpha$  values are large from location to location. Therefore, it should be considered that the parameter  $\alpha$  has the regional dependence.

### 6. Examination of the relation between $\alpha$ and the regional climatic parameters

The range of the calculated  $\alpha$  in the above was from 0.092 to 0.893. We carried out multiple regression analysis by using 7 regional climatic parameters that seem to influence  $\alpha$ . As the result, a prediction equation of  $\alpha$  was obtained as eqn. (9).

$$\alpha = -0.0006613 \phi - 0.0003005 \lambda - 0.0074150 R_{0.01} + 0.0043216 R_{0.001} + 0.0001083 M_{\text{year}} + 0.0030336 D_{\text{th}} - 0.5476583 \beta + 0.1593369 \quad (9)$$

$\phi$  ( $^{\circ}$ ) is latitude in that north is "+", and south is "-".  $\lambda$  ( $^{\circ}$ ) is longitude in that east is "+", and west is "-".  $R_{0.01}$ (mm/h) and  $R_{0.001}$ (mm/h)<sup>[6]</sup> are 1-min. rain rates for 0.01% and 0.001% of percentage of time, respectively.  $M_{\text{year}}$ (mm) is the average annual total rain<sup>[7]</sup>.  $D_{\text{th}}$ (day) is the average number of thunderstorm days<sup>[8]</sup>.  $\beta$  is the thunderstorm ratio of Dutton<sup>[9]</sup>. The thunderstorm ratio  $\beta$  is defined as  $\beta = M_t / M_{\text{year}}$ , where  $M_t$ (mm) is the average annual total rain arising from thunderstorm.

Fig.3 shows the comparison of measured  $\alpha$  with predicted  $\alpha$  by eqn.(9). The correlation coefficient between them is 0.77. Table 1 shows partial correlation coefficients between  $\alpha$  and regional climatic parameters. It is shown that correlations between  $\alpha$  and  $R_{0.01}$ ,  $R_{0.001}$ ,  $\beta$  are

especially large.

Table 1. Partial correlation coefficients between  $\alpha$  and regional climatic parameters.

Climatic Parameters	partial correlation coefficients
$\phi$ ( $^{\circ}$ )	-0.09941
$\lambda$ ( $^{\circ}$ )	-0.17894
$R_{0.01}$ (mm/h)	-0.64784
$R_{0.001}$ (mm/h)	0.73175
$M_{year}$ (mm)	0.31977
$D_{th}$ (day)	0.48966
$\beta$	-0.57052

7. Comparison of measured 1-min. rain rates with values converted by the proposed method

Fig.4 shows comparison of measured 1-min. rain rates with values converted from n-min. rain rates by using M distribution and eqn.(9). From Fig.4, it is found that 1-min. rain rate distribution can be predicted within conversion error of about 25%.

### 8. Conclusion

From theoretical analysis and databank, it was found that good conversion accuracy could be obtained by using M distribution and suitable 1-min. rain autocorrelation parameter  $\alpha$  for each location. Moreover, the  $\alpha$  has regional dependence, and can be estimated by regional climatic parameters. By using estimated  $\alpha$  and M distribution, 1-min. rain rate distribution can be predicted within conversion error of about 25%. This conversion method can be easily expanded to arbitrary integration times.

### [References]

- 1 B. Segal : J. Atmos. Ocean. Technol., vol. 3, pp.662-671, Dec. 1986.
- 2 E. Damosso, et al : CSELT Tech. Rep. , vol. 8, no. 4, pp.299-302, Aug. 1981.
- 3 K. Morita : ECL Tech. Jour., NTT, Jap., vol.27, no.10, pp.2249-2266, Oct. 1978.
- 4 Y. Hosoya : IEICE Trans., vol. J71-B, no. 2, pp.256-262, Feb. 1988.
- 5 F. Moupfouma : IEE Proc., Vol.134, Pt. H, No.6, pp.527-537, 1987.
- 6 C. Ito and Y. Hosoya : Electron. Lett., vol.35, no18, pp.1585-1587, 2nd Sept. 1999.
- 7 Japan Meteorological Agency : Technical data series, no.59, Tokyo, 1994.
- 8 World Meteorological Organization : WMO/OMM, No.117, TP.52, Geneva, 1962.
- 9 E.J. Dutton, H.T. Dougherty and R.F. Martin, Jr. : NTIS Rep. ACC-ACO-16-17, Aug. 1974.

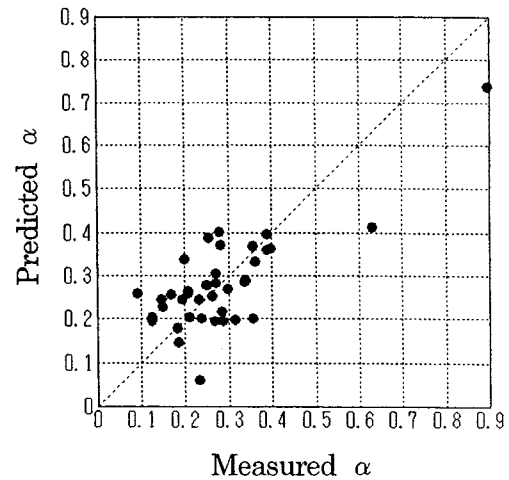


Fig.3 The comparison of measured  $\alpha$  with  $\alpha$  predicted by eqn.(9).

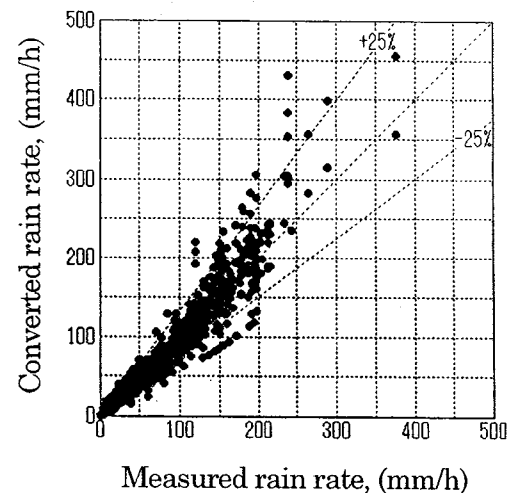


Fig.4 Comparison of measured 1-min. rain rates with values converted by the proposed method.