A NEW NUMERICAL METHOD TO SOLVE WIDE VARIETY OF DIFFRACTION PROBLEMS

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It is well-known that many diffraction problems such as step discontinuities in waveguides, some types of diffractional lattices, radiation from open-ended waveguides etc. are reduced to the modified Wiener-Hopf equation:

$$A(\alpha)\Phi_+(\alpha)+B(\alpha)\Phi_+(-\alpha)+\Psi_-(\alpha)=q(\alpha),\ \mathrm{Im}\,\alpha<\delta,\ \delta>0 \qquad (\mathrm{I})$$

or system of equations of similar type.

Here $\Phi_+(\alpha)$, $\Psi_-(\alpha)$ - are unknown functions. They are Fourier transforms of the scattered field components. Functions with $\pm$ are analytic in upper half-plane $\mathrm{Im}\,\alpha>-\delta$ and lower half-plane $\mathrm{Im}\,\alpha<\delta$ respectively. Eq. (I) is unsolvable in closed form. Usually numerical method for (I) based on the behaviour of coefficients $A(\alpha), B(\alpha)$ on the complex plane reduce (I) to an infinite system of linear algebraic equations (s.l.a.e.) / I,2 /. Then the coefficients of s.l.a.e. consist of the infinite products (closed configurations) or contain singular Cauchy type integrals (open configurations). That is why these approaches appear to be not very effective especially when multiwave regimes or resonance regimes are considered.

In this report the alternative approach is suggested. It is based on the fact, that eq. (I) is equvalent to the singular integral equation which can be solved with the modified discrete vortex method. The efficiency of this approach does not depend on the forms of $A(\alpha)$, $B(\alpha)$. This approach can be also used to solve systems of similar equations. It allows one to make the rigorous estimation of convergence rate of numerical solution to exact one using the results of / 3,4/.

After simple transformations eq. (I) can be reduced to singular integral equation (or system of similar equations) / 2 /

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{F(\tau)}{\tau-t}\,d\tau+\frac{1}{\pi}\int_{-\infty}^{\infty}M(t,\tau)F(\tau)\,d\tau=g(t),\ -\infty<t<\infty \qquad (2)$$

where $M(t,\tau)$ is a regular part.

In accordance with / 3 / eq. (2) can be transformed to the following

$$C+\frac{1}{2\pi}\int_{-\pi}^{\pi}f(\varphi)\,\mathrm{ctg}\,\frac{\varphi-\varphi_0}{2}\,d\varphi+\frac{1}{2\pi}\int_{-\pi}^{\pi}Q(\varphi_0,\varphi)f(\varphi)\,d\varphi=G(\varphi) \qquad (3)$$

where $f(\varphi)=F(\mathrm{tg}\frac{\varphi}{2})$, $Q(\varphi_0,\varphi)=M(\mathrm{tg}\frac{\varphi_0}{2},\mathrm{tg}\frac{\varphi}{2})(1+\mathrm{tg}^2\frac{\varphi}{2})$,

$$C = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\varphi)\,tg\,\frac{\varphi}{2}\,d\varphi\,,\quad G(\varphi_0) = g\left(tg\,\frac{\varphi_0}{2}\right),\ -\pi \le \varphi_0 \le \pi$$

The optimal quadrature is used to approximate the singular integral in (3):

$$I(\varphi_{0j}) = \frac{1}{2n+1}\sum_{i=-n}^{n} f(\varphi_i)\,ctg\left(\frac{\varphi_i - \varphi_{0j}}{2}\right) \qquad (4)$$

where $\varphi_i = \frac{2i-1}{2n+1}\pi$, $\varphi_{0j} = \frac{2\pi j}{2n+1}$, $\varphi_{-i} = -\varphi_i$, $\varphi_{0-j} = -\varphi_{0j}$,

$i = 1,\ldots,n$, $j = 0,1,\ldots,n$, $I(\varphi_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\varphi)\,ctg\,\frac{\varphi - \varphi_0}{2}\,d\varphi$.

As a result eq. (3) can be approximated by s.l.a.e.

$$f_{0n} + \frac{1}{2n+1}\sum_{i=-n}^{n} f_n(\varphi_i)\left[ctg\left(\frac{\varphi_i - \varphi_{0j}}{2}\right) + Q(\varphi_{0j}, \varphi_i)\right] = G(\varphi_{0j}) \qquad (5)$$

where f_{0n} is the regularization parameter.

The domination of diagonal coefficients in (5) provides the stability of computer calculations and raises the velocity of computations. The convergence rate can be estimated from the expression:

$$\max\left|f(\varphi_i) - f_n(\varphi_i)\right| \le 2\pi\,(3 + \ln n)\,M_0 \cdot \left(\frac{12}{n}\right)^{\nu + r} \qquad (6)$$

if r-derivation of $M(t,\tau)$, $g(t)$ satisfy the Hölder condition of degree ν . Here M_0 is Hölder's constant, is the exact solution. For diffraction problems $M(t,\tau)$, $g(t)$ are infinitely differentiable with introduction of slight damping in whole space. Therefore our approach appeares to be effective especially when other method are inconvenient.

To illustrate the numerical method let us consider diffraction of a plane wave on the structure as shown in Fig.I. Such a geometry is a key one for a lot of real configurations arising in some microwave devices. For this geometry the functions $M(t,\tau)$ and $g(t)$ have the form / 5,6 /:

$$M(t,\tau) = -\tau\left[2D(t)\left(\lambda(t) + \lambda(\tau)\right)\lambda(\tau)\right]^{-1},\quad D(t) = 1 + \frac{\varepsilon\,\lambda(t)}{\lambda_1(t)},$$

$$g(t) = -\sqrt{\frac{2}{\pi}}\,\kappa \sin\theta\,\lambda(t)\,T(\varepsilon,\theta)\left[D(t)\,\lambda_1(t)\left(\lambda_1(t) + i\sqrt{\varepsilon - \sin^2\theta}\right)\right]^{-1},$$

$$T(\varepsilon,\theta) = 2\varepsilon\cos\theta\left(\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta}\right)^{-1},\quad \lambda(t) = \sqrt{t^2 - \kappa^2},$$

$$\lambda_1 = \sqrt{t^2 - \kappa^2\varepsilon}\,,\quad Re\left[\lambda(t), \lambda_1(t)\right] \ge 0,\ Im\left[\lambda(t), \lambda_1(t)\right] \ge 0,$$

$F(t)$ is the asymmetric part of the Fourier transformation of z -component of the electric field. In numerical calculations we assume the amplitude of the magnetic field of the incident wave to be equal to unity, $\varepsilon = 2$, $\theta = \pi/6$, $\kappa = 1$.

The results of the numerical analysis of (5) are shown in Fig.2. There are an excellent agreement between

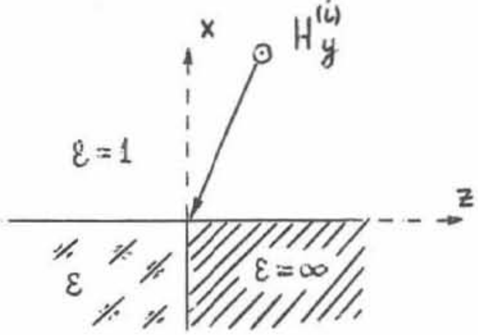

Fig.I.

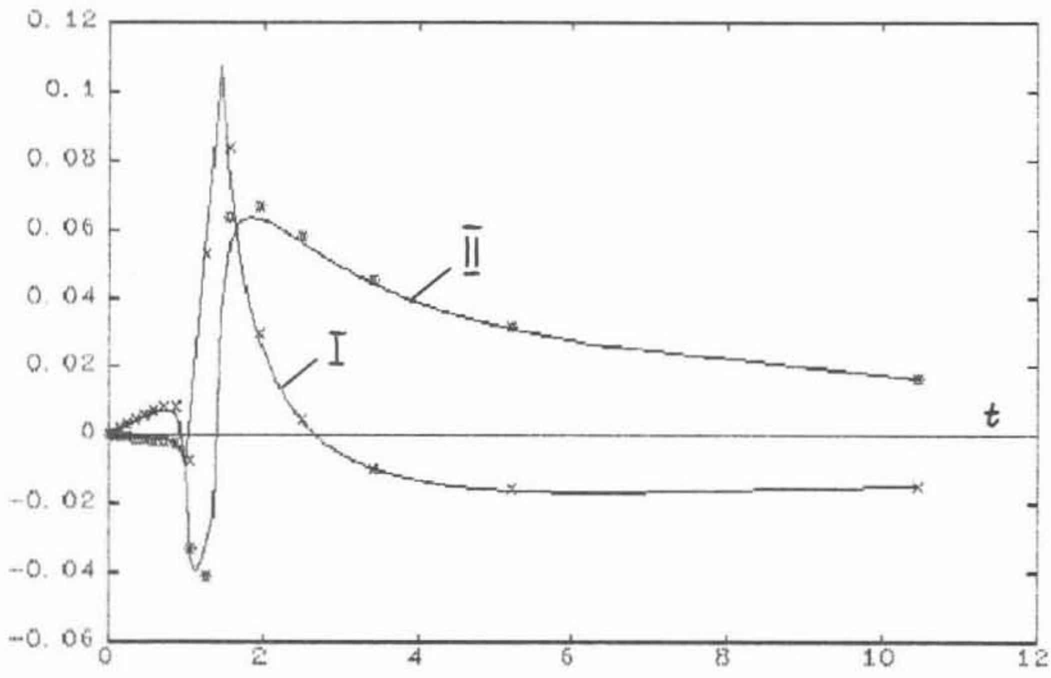

Fig.2.

I - $\mathrm{Re}\,F_n(t_i)$ for $n=48$, x - $\mathrm{Re}\,F_n(t_i)$ for $n=16$,
II - $\mathrm{Im}\,F_n(t_i)$ for $n=48$, * - $\mathrm{Im}\,F_n(t_i)$ for $n=16$.

the results for $n = 16$ and $n = 48$ especially if $t_i = tg(\varphi_i/2)$ are not near the points where $M(t, \tau)$, $g(t)$ have the integrable singularities or are equal to zero: $t = K$ and $t = K\sqrt{\varepsilon}$. Near these points the agreement is not so good, nevertheless it is quite satisfactory. For large t_i $Re\, F_n(t_i)$ and $Im\, F_n(t_i) \sim t_i^{-\nu_0}$, where $\nu_0 = (1/\pi) \arccos \frac{\varepsilon}{1+\varepsilon}$. It is in accordance with the static theory / 5,6 /. The time of numerical calculation for one case with $n = 48$ is equal to about one minute on the computer IBM PC 386/387.

The above example shows a high stability of numerical calculations which us to take large n to achieve required accuracy of the results and at the same time to reduce the errors caused by the computer rounding-off. A possibility to present the coefficients of the s.l.a.e. in terms of elementary functions also makes our approach advantageous. The numerical algorithm is rathe simple, universal and mathematically proved. It is especiallu convenient when the scattered field in the problems under consideration contain a continuous spectrum, because the fied in far region are proportional to $f_n(\varphi_i)$, which could be found directly from system (5).

Note, that this method can be used to solve nonstationary and nonlinear problems, as well as the diffraction problems with complicated geometries.

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