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A SIMPLE CONDITION ON THE VALIDITY OF THE RAYLEIGH-GANS-BORN APPROXIMATION IN THE IONOSPHERE

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Introduction: Previous approaches to the problem of a full wave solution for electromagnetic propagation in the ionosphere have for the most part assumed horizontal stratification. Such an assumption is inappropriate to such phenomena as: 1. Propagation in a dipole geomagnetic field; 2. Propagation along field ionization; 3. Interaction of waves with a modified ionosphere[1]; 4. Radio wave propagation in the presence of an ionization trough[2].

An exact solution to such problems is hindered merely by the spatial dependence and anisotropy of the constitutive parameters. The Neuman series solution, whose first term is equivalent to the Rayleigh-Gans-Born (R-G-B) approximation, gives a sufficiently general and cogent procedure to treat the problem. The R-G-B makes a simple analytic approximation which is desirable for engineering use[3,4,5]. It is qualitatively known that this approximation can be applied only to plasmas of relatively low density. This conjecture was shown[6,7] to be overly stringent in some cases of isotropic media. What is needed now is a rigorous criterion on the validity of R-G-B approximation in anisotropic and inhomogeneous media.

Analysis: In this work, the general problem is formulated as a vector integral equation using the Stratton-chu formula:

$$E_j(\bar{r}) = E_j^{(o)}(\bar{r}) + \sum_{n=1}^{3}\int d\bar{r}'\, K_{jn}(\bar{r},\bar{r}')E_n(\bar{r}') \qquad j = 1,2,3$$

$$K_{jn}(\bar{r},\bar{r}') = \alpha G_{ji}(\bar{r},\bar{r}')\,\sigma_{in}(\bar{r},\bar{r}')$$

$$\alpha = k^2 / i\omega\epsilon_o$$

where $G_{ij}(\bar{r},\bar{r}')$ and $\sigma_{in}(\bar{r},\bar{r}')$ are respectively the Green's and the conductivity's tensors; $E_j^{(o)}(\bar{r})$ is the free term. This resulting system of coupled and weakly singular integral equations is solved formally by an iteration process, subject to a general convergence condition, $B < 1$:

$$B = \left[\max_{1\leq j\leq 3} \sum_{n=1}^{3} \iint \left| K_{jn}(\bar{r},\bar{r}')\right|^2 d\bar{r}\, d\bar{r}'\right]^{\frac{1}{2}}$$

and

$$E_j^{(S)} = E_j^{(S1)} + E_j^{(SM)} \qquad j = 1,2,3$$

where

$$E_j^{(S1)} = \sum_{n=1}^{3}\int d\bar{r}'\, K_{jn}(\bar{r},\bar{r}')\, E_n^{(o)}(\bar{r}') \quad \text{(R-G-B contribution)}$$

$$E_j^{(SM)} = \sum_{n=1}^{3}\int d\bar{r}'\, K_{jn}(\bar{r},\bar{r}')\, E_n(\bar{r}') \quad \text{(Higher order contribution)}$$

A measure of the importance of energy contributions from high order iterations is derived. This measure is simple and appears useful as a criterion in establishing the adequacy of the widely used R-G-B approximation. The measure is expressed in terms of physical parameters using the inequalities appropriate to quadratically summable functions:

$$\frac{\text{Higher Order contribution}}{\text{R-G-B contribution}} \leq \sum_{n=1}^{\infty} (n+1)B^n$$

Application of the results to ionospheric propagation is carried out, for an earth magnetic field assumed to lie along the Z-axis. It is shown that the applicability of R-G-B approximation is dependent upon the plasma parameters, the volume of the inhomogeneity, and the relative orientation of the earth magnetic field and the incident wave number, $\bar{k}$:

$$B^2 = |\alpha|^2 \iint \frac{(|U|^2 + Y^2)\, X^2}{R^2\ |(U^2 - Y^2)|^2}\, d\bar{r}\, d\bar{r}'$$

$\hat{a}_x, \hat{a}_y$ components

and

$$B^2 = |\alpha|^2 \iint \frac{X^2}{|U|^2}\, d\bar{r}\, d\bar{r}'$$

$\hat{a}_z$ component

where U,X,Y have the usual meanings of magneto-ionic theory. Discussion and evaluation of the measure are easily performed for a broad class of equilibrium variations in electron density and collision frequency. It is found that collisions ($\Omega_e \neq 0$), anisotropy ($Y > \sqrt{3}$), and low electron density relative to the effective volume of the inhomogeneity weaken the multiple scattering process and thus improve the speed of convergence of the series solution, implying that the R-G-B approximation will give a better result.

Conclusion: The validity of the R-G-B approximation, for electromagnetic scattering in the ionosphere appears satisfactory for reasonably low B, the sufficiency condition for the convergence of the series solution.

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