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# BOUNDARY-CONTACT PROBLEMS IN ELECTROMAGNETIC DIFFRACTION THEORY

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## 1. Introduction

The idea of usage of the impedance boundary conditions (IBC) in e.m. diffraction theory has maintained in the 40'th years of the century . The IBCs allow to simulate scattering body with the high degree of accuracy and to exclude the investigation of complex wave processes inside the scatterer . These ideas are succesfully applied in radiophysics, and the corresponding IBCs were named Stchukin-Leontovich's boundary conditions . The necessity of usage of more accurate models for the scattering surfaces has arised in the last few years due to development of new technoloqies . The corresponding boundary conditions were named the generalized impedance boundary conditions ( GIBC ) . The applications of the GIBCs we link with the following directions of the diffraction theory : the scattering of optic radiation by metallic bodies [1,2] and the scattering of radio waves by thin multylayered coatings [3,4,5] . The GIBCs in the scalar diffraction problem contain the tangent derivatives of high order (the fourth order in our case), beside the wave field and their normal derivative .

The formulation of the problem becomes more complicated when the coefficients in the GIBCs have jump in the fixed point ( for the 2-dimensional problem ) or on the curve ( for the 3-dimensional problem ) . In this case we are forced to formulate the additional conditions in these points (the contact conditions ) . The constants , included in the contact conditions , we name the contact impedances . The contact impedances are the local characteristics of the point of junction for the different material coatings . The contact impedances are supposed to be known and included in formulation of the problem . The appropriate choice of the contact impedances is the essential step to formulate the diffraction problem .

## 2. The formulation of boundary-contact problems

Let us consider cylindrical e.m. diffraction problem . The wave field U ( where $U=E_z$ for TE polarization , $U=H_z$ for TM polarization ) satisfies the Helmholtz's equation beyond the absorbing body or dielectric metal backed layer Ω and the boundary condition on the surface S of Ω

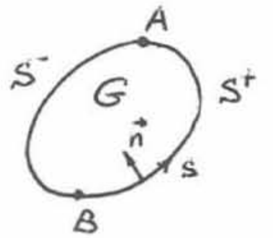
$$\left.\frac{\partial U}{\partial n}\right|_S = \left\{ \left(\alpha(s,\delta) + \frac{\partial}{\partial s}\beta(s,\delta)\frac{\partial}{\partial s} + \frac{\partial^2}{\partial s^2}\gamma(s,\delta)\frac{\partial^2}{\partial s^2}\right) U\right|_S + O(\delta^4) \qquad (1)$$

U is the sum of known incident and unknown scattered field . The solution must satisfy the principle of absorbtion. The coefficients $\alpha,\beta,\gamma$ in the operator $l(s,\partial/\partial s)$ (1) have the form

$$\alpha(\delta,s) = \frac{1}{\kappa_0}\,[\,1 - \delta^2(\chi/3) - \delta^4(\chi^2/45)\,]$$

$$\beta(\delta,s,\partial/\partial s) = \frac{1}{\kappa_0}\left[-(\delta^2/3)\right], \quad \gamma(\delta,s,\partial/\partial s) = \frac{1}{\kappa_0}\left\{-\frac{\delta^4}{45}\right\},$$

where $\delta = kh$ is the dimensionless thickness of the layer, s ks is the dimensionless arclength, $\chi = \varepsilon\,\mu$. The GIBC (1) is valid for an arbitrary cylinder when the curvature much more less then 1/h. The sum of operators $\alpha,\beta,\gamma$ in the right side of (1) is natural to name the generalized impedance . In the general case the generalized impedance is the formal asymptotic series of operators with tangent to S derivatives . We are restricted by the terms of the fourth order on $\delta$ . It ensures enough accuracy of the simulation of the layer .



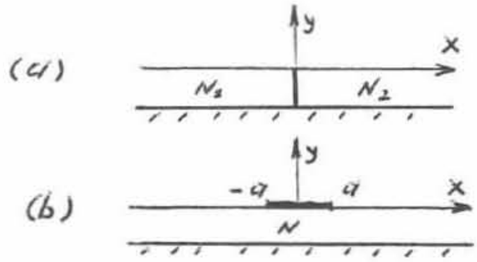




Fig.1.

Let us consider cylindrical domain G ( fig.1 ) . The smooth boundary S of the region G consists of two parts $S^+$ and $S^-$. We suppose , that the GIBC (1) with the coefficients $\alpha_+,\beta_+,\gamma_+$ are satisfied on $S^+$ and the same conditions with $\alpha_-,\beta_-,\gamma_-$ are satisfied on $S^-$ . In the points A and B of the GIBC's junction we compeled to fix the wave regim and to formulate the special contact conditions . To obtain the contact conditions one must take into account the law of energy conservation . We introduce the vectors $\mathbf{q}$ and $\mathbf{h} \in \mathbb{C}^3$:

$$(2)\quad \mathbf{q} = \begin{pmatrix} [\beta\,\frac{\partial U}{\partial s}] + [\frac{\partial}{\partial s}\;(\gamma\,\frac{\partial^2}{\partial s^2}\,U\,)] \\ [-\,\gamma\,\frac{\partial^2 U}{\partial s^2}\;] \\ \{-\,\gamma\,\frac{\partial^2 U}{\partial s^2}\;\} \end{pmatrix}\Bigg|_{A,B}, \qquad \mathbf{h} = \begin{pmatrix} U \\ \{\frac{\partial U}{\partial s}\} \\ [\frac{\partial U}{\partial s}] \end{pmatrix}\Bigg|_{A,B}$$

where $[f]|_{A\pm} = f(A+0) - f(A-0)$ and $\{f\} = (f(A+0) + f(A-0))/2$.

To fix the wave regim in the points A,B, to derive the type of junction of the GIBCs on $S^\pm$ we must set the correlation between q and h . It is evident , that this correlation must be linear

$$(3)\qquad \mathbf{q} = \mathbb{Z}\,\mathbf{h},$$

where $\mathbb{Z} = \{ z_{ik} \}$ is $3\times3$ matrix with the complex coefficients. In the general case to ensure the condition $\mathrm{Im}\langle \mathbb{Z}h,h\rangle \geq 0$ we need $\mathrm{Im}\ \mathbb{Z} > 0$ .The operator of the problem becomes dissipative . The conditions (2),(3) we name contact conditions and $z_{ik}$ are the contact impedances .

## 3. The factorization method and examples of contact conditions

Let us consider the diffraction of the plane wave $U^i = \exp(ikx\cos\phi - iky\sin\varphi)$ on the line $y = 0$ provided that $\frac{\partial U}{\partial y}|_{S^-} = l_-(x,\partial/\partial x)U|_{S^-}$, when $x < 0$ and $\frac{\partial U}{\partial y}|_{S^+} = l_-(x,\partial/\partial x)U|_{S^-}$ when $x > 0$, $\alpha_\pm,\beta_\pm,\gamma_\pm$ are constant . In the point (0,0) the wave regim is fixed by the contact conditions (3) . The contact impedances $z_{ik}$ are supposed to be known . We present the wave field as a sum of three terms

$$U = U^i + U^r + U^s,\ U^s(x,y) = B\frac{1}{2\pi i}\oint_{\mathbb{R}} \frac{\exp(ipx + i\sqrt{k^2-p^2}\ y)}{(p-k\cos\phi)\phi_1^+(p)\phi_2^-(p)}\,dp +$$

$$+ \frac{1}{2\pi i}\int_{\mathbb{R}} \frac{\exp(ipx + i\sqrt{k^2-p^2}\ y)}{\phi_1^+(p)\phi_2^-(p)}(-C_3p^2 + iC_2p + C_1)\,dp, \qquad (4)$$

$U^r = R_m^{\pm}\exp(ikx\cos\phi + iky\sin\phi)$ is the reflected by $S^-$ field,

$$R_m^{\pm} = \frac{ik\sin\phi + \alpha_\pm - \beta_\pm\cos^2\phi + \gamma_\pm\cos^4\phi}{ik\sin\phi - \alpha_\pm + \beta_\pm\cos^2\phi - \gamma_\pm\cos^4\phi},$$

$U^s$ is the scattered field , which we seek for. Let $U_2$ and $U_4^s$ be contineous . We have $A = (R_m^+ - R_m^-)(ik\sin\phi - \alpha_+ + \beta_+\cos^2\phi - \gamma_+\cos^4\phi)$. The constants $C_j$, $j = 1,2,3$ can be determined from the contact conditions (3) and $\phi_{1,2} = i\sqrt{k^2-p^2} - \alpha_\pm + \beta_\pm(p/k)^2 - \gamma_\pm(p/k)^4$, $\mathrm{Im}\sqrt{k^2-p^2} > 0$ , when $\mathrm{Im}\ (p) = 0$ ,$B = A\ \phi_1^+(k\cos\phi)/\phi_2^+(k\cos\phi)$. The factorization of $\phi$ can be carried out evidently [6].

Let us consider thin homogeneous layer which is simulated by the GIBCs with constant $\alpha,\beta,\gamma$ (3) . We suppose thin perfect conductor be placed in the layer and perpendicular to the metal back ( fig.1a ) .Provided the metal conductor separates two different coatings the contact conditions have the form

$$U(0,0)=0,\ \left\{ \gamma\frac{\partial^2 U}{\partial x^2} \right\}\Big|_{(0,0)} = 0\ ,\ \left[ \gamma\frac{\partial^2 U}{\partial x^2} \right]\Big|_{(0,0)} = 0. \qquad (5)$$

Using (6) we obtain linear algebraic system to determine $C_j$, $j = 1,2,3$ in (4). The impedance coating with an arbitrary metal defect in the point (0,0) can be simulated by the boundary condition

$$h \frac{\partial U}{\partial y}\Big|_{y=0} = (\alpha + \beta/k^2\, \partial^2/\partial x^2 + \gamma/k^4\, \partial^4/\partial x^4)U\Big|_{y=0} + C_1\,\delta(x) + C_2\,\delta'(x) + C_3\,\delta''(x),$$

where $\delta(x)$ is Dirac's delta-function . This condition must be interpreted by means of the generalized functions . The constants $C_1, C_2, C_3$ are the values $(-\beta)[U_x]/k^2+(-\gamma)[U_{xxx}]/k^4$, $(-\gamma)[U_{xx}]/k^4$, $(-\gamma)[U_x]/k^4$ respectively .

Another example of the contact conditions deals with the simulation of perfectly conducting strip placed on the boundary of the absorbing halfspace ( fig.1b ). The constants $C_j$ and contact impedances can be determined evidently [6]

$$C_1 = D_m(\varphi)\left[\frac{2\pi}{p_0-2\pi\theta}\right], C_2 = \varepsilon^2\,\frac{D_m(\varphi)}{k}\left[(i\pi)\cos\varphi\right], C_3 = \varepsilon^2\,\frac{D_m(\varphi)}{(k)^2}\left[\frac{(\pi/2)}{(p_0-2\pi\theta)}\right],$$

$\varepsilon = ka \ll 1$, $p_0 = \ln\left(\frac{\gamma\varepsilon}{4i}\right)$, $\ln\gamma = c$ is Euler's constant , $D_m(\phi)$ is refraction coefficient and $\theta$ is complex constant [6]. The formulae obtained in the paper can be used for different numerical experiments. The figure 2 presents the computational results for the scattered field $U^s$ of the diffraction problem by narrow strip placed on the absorbing layer when N=2+i3 , ka=0.5 , $\psi = 30^0(1)$; $60^0(2)$ .

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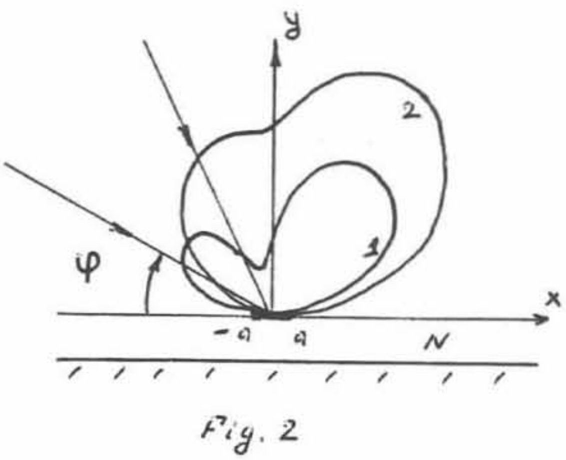


Fig. 2