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# BOUNDARY-CONTACT PROBLEMS IN ELECTROMAGNETIC DIFFRACTION THEORY

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## 1.Introduction

The idea of usage of the impedance boundary conditions (IBC) in e.m. diffraction theory has maintained in the 40'th years of the century . The IBCs allow to simulate scattering body with the high degree of accuracy and to exclude the investigation of complex wave processes inside the scatterer . These ideas are succesfully applied in radiophysics, and the corresponding IBCs were named Stchukin-Leontovich's boundary conditions The necessity of usage of more accurate models for the scattering surfaces has arised in the last few years due to development of new technologies . The corresponding boundary conditions were named the generalized impedance boundary conditions ( GIBC ) . The applications of the GIBCs we link with the following directions of the diffraction theory : the scattering of optic radiation by metallic bodies [1,2] and the scattering of radio waves by thin multylayered coatings [3,4,5] . The GIBCs in the scalar diffraction problem contain the tangent derivatives of high order (the fourth order in our case), beside the wave field and their normal derivative .

The formulation of the problem becomes more complicated when the coefficients in the GIBCs have jump in the fixed point ( for the 2-dimensional problem ) or on the curve ( for the 3-dimensional problem ). In this case we are forced to formulate the additional conditions in these points (the contact conditions ) . The constants , included in the contact conditions , we name the contact impedances . The contact impedances are the local characteristics of the point of junction for the different material coatings . The contact impedances are supposed to be known and included in formulation of the problem . The appropriate choice of the contact impedances is the essential step to formulate the diffraction problem .

2. The formulation of boundary-contact problems

Let us consider cylindrical e.m. diffraction problem . The wave field U ( where U=E<sub>z</sub> for TE polarization , U=H<sub>z</sub> for TM polarization ) satisfies the Helmholtz's equation beyond the absorbing body or dielectric metal backed layer  $\Omega$  and the boundary condition on the surface S of  $\Omega$ 

 $\frac{\partial U}{\partial n} \Big|_{S} = \left\{ (\alpha(s,\delta) + \frac{\partial}{\partial s}\beta(s,\delta)\frac{\partial}{\partial s} + \frac{\partial^{2}}{\partial s^{2}}\gamma(s,\delta)\frac{\partial^{2}}{\partial s^{2}} )U \Big|_{S} + O(\delta^{4}) \right\}$ (1)

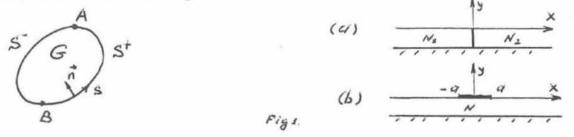
U is the sum of known incident and unknown scattered field . The

solution must satisfy the principle of absorbtion. The coefficients  $\alpha, \beta, \gamma$  in the operator  $l(s, \partial/\partial s)$  (1) have the form

$$\alpha(\delta, s) = \frac{1}{\kappa_0} \left[ 1 - \delta^2(\chi/3) - \delta^4(\chi^2/45) \right]$$

$$\beta(\delta, \mathbf{s}, \partial/\partial \mathbf{s}) = \frac{1}{\kappa_0} \left[ -(\delta^2/3) \right], \quad \gamma(\delta, \mathbf{s}, \partial/\partial \mathbf{s}) = \frac{1}{\kappa_0} \left\{ -\frac{\delta^4}{45} \right\},$$

where  $\delta = kh$  is the dimensionless thickness of the layer, s ks is the dimensionless arclength,  $\chi = \varepsilon \mu$ . The GIBC (1) is valid for an arbitrary cylinder when the curvature much more less then 1/h. The sum of operators  $\alpha, \beta, \gamma$  in the right side of (1) is natural to name the generalized impedance . In the general case the generalized impedance is the formal asymptotic series of operators with tangent to S derivatives . We are restricted by the terms of the fourth order on  $\delta$ . It ensures enough accuracy of the simulation of the layer .



Let us consider cylindrical domain G (fig.1). The smooth boundary S of the region G consists of two parts S' and S. We suppose, that the GIBC (1) with the coefficients  $\alpha_+, \beta_+, \gamma_+$  are satisfied on S<sup>+</sup> and the same conditions with  $\alpha_-, \beta_-, \gamma_-$  are satisfied on S<sup>-</sup>. In the points A and B of the GIBC's junction we compeled to fix the wave regim and to formulate the special contact conditions. To obtain the contact conditions one must take into account the law of energy conservation. We introduce the vectors q and  $h \in \mathbb{C}^3$ :

$$(2)^{\mathbf{q}} = \begin{pmatrix} \begin{bmatrix} \beta & \frac{\partial U}{\partial s} \end{bmatrix} + \begin{bmatrix} \frac{\partial}{\partial s} & (\gamma & \frac{\partial^2}{\partial s^2} & U & ) \end{bmatrix} \\ \begin{bmatrix} -\gamma & \frac{\partial^2 U}{\partial s^2} \end{bmatrix} \\ \frac{\partial s^2}{\partial s^2} & \frac{\partial^2 U}{\partial s^2} \end{bmatrix} , \quad \mathbf{h} = \begin{pmatrix} U \\ \frac{\partial U}{\partial s} \\ \frac{\partial s}{\partial s} \\ \frac{\partial U}{\partial s} \end{bmatrix} \Big|_{\mathbf{A}, \mathbf{B}}$$

where  $[f]|_{A^{\pm}} = f(A+0) - f(A-0)$  and  $\{f\} = (f(A+0) + f(A-0))/2$ . To fix the wave regim in the points A,B, to derive the type of junction of the GIBCs on S<sup>±</sup> we must set the correlation between q and h. It is evident, that this correlation must be linear

 $q = \mathbb{Z} h$ ,

(3)

where  $\mathbb{Z} = \{ z_{1k} \}$  is  $3 \times 3$  matrix with the complex coefficients. In the general case to ensure the condition  $\text{Im}(\mathbb{Z}h,h) \ge 0$  we need Im  $\mathbb{Z} > 0$ . The operator of the problem becomes dissipative. The conditions (2),(3) we name contact conditions and  $z_{1k}$  are the contact impedances.

# 3. The factorization method and examples of contact conditions

# Let us consider the diffraction of the plane wave $U^{1}$ =

$$\begin{split} &\exp(ikx\,\cos\phi\ -\ iky\,\sin\phi\ ) \ \text{on the line }y\ =\ 0\ \text{provided that }\frac{\partial U}{\partial y}|_{S}\ =\ 1_{(x,\partial/\partial x)}U|_{S}\ , \ \text{when }x\ <\ 0\ \ \text{and }\ \frac{\partial U}{\partial y}|_{S}\ +\ =\ 1_{(x,\partial/\partial x)}U|_{S}\ , \ \text{when }x\ >\ 0,\ \alpha_{\pm},\beta_{\pm},\gamma_{\pm}\ \text{are constant}\ .\ \text{In the point }(0,0)\ \text{the wave regim is fixed by the contact conditions }(3)\ .\ \text{The contact impedances }z_{ik}\ \text{are supposed to be known}\ .\ \text{We present the wave field as a sum of three terms} \end{split}$$

$$U = U^{i} + U^{r} + U^{s}, \quad U^{s}(x, y) = B \frac{1}{2\pi i} \oint_{\mathbb{R}} \frac{exp(ipx + i\sqrt{k^{2} - p^{2}} y)}{(p - kcos\phi)\phi_{1}^{*}(p)\phi_{2}^{-}(p)} dp + \frac{1}{2\pi i} \int_{\mathbb{R}} \frac{exp(ipx + i\sqrt{k^{2} - p^{2}} y)}{\phi_{1}^{*}(p)\phi_{2}^{-}(p)} (-C_{3}p^{2} + iC_{2}p + C_{1})dp, \quad (4)$$

 $U^{r} = R_{\overline{m}}^{+} exp(ikx \cos\phi + iky \sin\phi)$  is the reflected by S<sup>-</sup> field, iksin $\phi + \alpha = \beta \cos^{2}\phi + \alpha \cos^{4}\phi$ 

$$R_{m}^{+} = \frac{1 \times \sin \phi + \alpha_{+} - \beta_{+} \cos \phi + \gamma_{+} \cos \phi}{1 \times \sin \phi - \alpha_{+} + \beta_{+} \cos^{2} \phi - \gamma_{+} \cos^{4} \phi},$$

U<sup>s</sup> is the scattered field , which we seek for. Let U and U<sup>s</sup> be contineous . We have A =  $(R_m^+ - R_m^-)$  (  $iksin\phi - \alpha_{\pm} + \beta_{\pm}cos^2\phi - \gamma_{\pm}cos^4\phi$ ). The constants C<sub>j</sub>, j = 1,2,3 can be determined from the contact conditions (3) and  $\phi_{1,2} = i\sqrt{k^2 - p^2} - \alpha_{\pm} + \beta_{\pm}(p/k)^2 - \gamma_{\pm}(p/k)^4$ ,

 $\operatorname{Im}\sqrt{k^2-p^2} > 0$ , when  $\operatorname{Im}(p) = 0$ ,  $B = A \phi_1^*(k\cos\phi)/\phi_2^*(k\cos\phi)$ . The factorization of  $\phi$  can be carried out evidently [6].

Let us consider thin homogeneous layer which is simulated by the GIBCs with constant  $\alpha, \beta, \gamma$  (3). We suppose thin perfect conductor be placed in the layer and perpendicular to the metal back (fig.la).Provided the metal conductor separates two different coatings the contact conditions have the form

$$U(0,0)=0, \left\{ \left. \gamma \frac{\partial^2 U}{\partial x^2} \right\} \right|_{(0,0)} = 0, \left[ \left. \gamma \frac{\partial^2 U}{\partial x^2} \right] \right|_{(0,0)} = 0.$$
(5)

Using (6) we obtain linear algebraic system to determine  $C_j$ , j = 1,2,3 in (4). The impedance coating with an arbitrary metal defect in the point (0,0) can be simulated by the boundary condition

$$h \frac{\partial U}{\partial y|_{y=0}} = (\alpha + \beta/k^2 \partial^2/\partial x^2 + \gamma/k^4 \partial^4/\partial x^4)U| + C \delta(x) + C \delta'(x) + C \delta'(x) + C \delta'(x)$$

where  $\delta(x)$  is Dirac's delta-function . This condition must be interpreted by means of the generalized functions . The constants  $C_1, C_2, C_3$  are the values  $(-\beta) [U_x]/k^2 + (-\gamma) [U_{xxx}]/k^4$ ,  $(-\gamma) [U_{xx}]/k^4$ ,  $(-\gamma)[U_{..}]/k^4$  respectively .

Another example of the contact conditions deals with the simulation of perfectly conducting strip placed on the boundary of the absorbing halfspace ( fig.1b ). The constants C, and contact impedances can be determined evidently [6]

$$\begin{split} &C_1 = D_m(\varphi) \left[ \frac{2\pi}{p_0 - 2\pi\theta} \right], C_2 = \epsilon^2 \; \frac{D_m(\varphi)}{k} \left[ (i\pi)\cos\varphi \right], C_3 = \epsilon^2 \; \frac{D_m(\varphi)}{(k)^2} \left[ \; \frac{(\pi/2)}{(p_0 - 2\pi\theta)} \right], \\ &\epsilon = ka \ll 1, \quad p_0 = \ln\left(\frac{\gamma\epsilon}{41}\right), \; \ln\gamma = c \; \text{is Euler's constant} \;, \; D_m(\varphi) \; \text{is refraction coefficient and } \theta \; \text{is complex constant [6]. The formulae obtained in the paper can be used for different numerical experiments. The figure 2 presents the computational results for the scattered field US of the diffraction problem by narrow strip placed on the absorbing layer when N=2+i3 , ka=0.5 , &\psi = 30^{\circ}(1); \\ & 60^{\circ}(2) \; . \end{split}$$

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