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REFRACTION EFFECT IN THE EARTH-IONOSPHERE WAVEGUIDE FOR VLF RADIO WAVES

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In order to discuss the location of atmospherics by the direction finding technique, it would be better not to neglect the earth's magnetic field in the ionosphere. Suppose that the ionosphere consists of two regions which are characterised by the QT- and QL-approximation, respectively, and that radio waves propagate through the boundary between two regions. In this paper an approximate theory of refraction of waves propagating to great distances is presented assuming the flat earth and ionosphere. As shown in Fig. 1, each terrestrial wave-guide is denoted as region I and region II and a vertical electric dipole source exist in region I. Now, the waves from this source are received in region II, mode conversion at the boundary between region I and region II must be considered. Such a problem may, however, be imaged in the case of the propagation from south to north in the northern hemisphere.

Now, let suppose that no reflected waves at the bounrary, $X=0$, and that both walls are highly conducting, i.e. $R_g(C)\approx 1$ and $R_i(C)\approx\pm 1$. Then electric Hertz vector $\Pi_{1z}^{(e)}$ is represented as follows;

$$\Pi_{1z}^{(e)} \simeq \frac{i2\pi M}{h}\sqrt{\frac{2i}{\pi k S_1^{(1)}\rho}}\cos(kC_1^{(1)}z)e^{ikS_1^{(1)}X_0}$$

$$\cdot\exp[-ikS_1^{(1)}(X\cos\phi + y\sin\phi)],$$

$$\left(kS_1^{(1)}\rho = kS_1^{(1)}\sqrt{(X-X_0)^2+y^2}\gg 1\right),$$

where $M = \dfrac{I\,ds}{4\pi i\omega\varepsilon_0}$,

if $|n_i C| \gg 1$,

$$C_1^{(1)} \simeq \frac{\pi}{kh} + i\frac{\Delta}{\pi},$$

$$S_1^{(1)} \simeq \left[1-\left(\frac{\pi}{kh}\right)^2\right]^{1/2} - i\left[1-\left(\frac{\pi}{kh}\right)^2\right]^{-1/2}\frac{\Delta}{kh},$$

$$\Delta \simeq \sqrt{i}\left(\sqrt{\frac{\omega\nu}{\omega_0^2}} + \sqrt{\frac{\omega\varepsilon_0}{\sigma_g}}\right),$$

if $|n_i C| \ll 1$,

$$C_1^{(1)} \simeq \frac{\pi}{2kh} + \frac{\sqrt{i}}{kh}\left\{\left(\sqrt{\frac{\omega_0^2}{\omega\nu}} + i\sqrt{\frac{\omega\nu}{\omega_0^2}}\right)^2 \cdot\left(\frac{\pi}{2kh}\right)^2 + \sqrt{\frac{\omega\varepsilon_0}{\sigma_g}}\right\},$$

$$S_1^{(1)} \simeq 1 - \frac{1}{2}\left(\frac{\pi}{2kh}\right)^2 + \frac{\sqrt{i}}{kh}\left\{-\left(\sqrt{\frac{\omega_0^2}{\omega\nu}} + i\sqrt{\frac{\omega\nu}{\omega_0^2}}\right)\left(\frac{\pi}{2kh}\right)^2 + \sqrt{\frac{\omega\varepsilon_0}{\sigma_g}}\right\}.$$

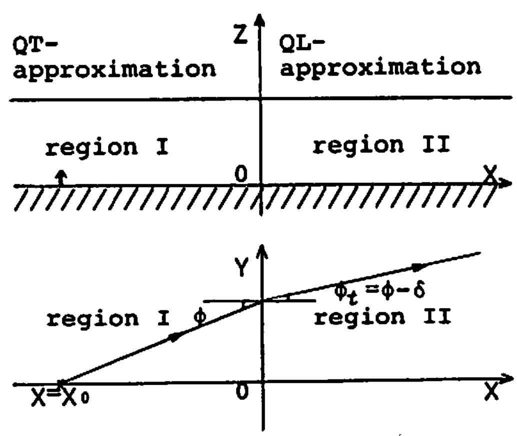

Fig. 1. Waveguide model and refraction at the boundary between region I and region II.

The ionosphere for which the QT-approximation is applicable may be regarded as the medium in which no earth's magnetic field is imposed and waves of higher order modes rapidly attenuate with the traversing distance in comparison with the one of the first order mode.

In the ionosphere characterised by the QL-approximation both ordinary and extraordinary waves exist and these waves are circularly polarised. So that four Hertz vectors, $\Pi_{oz}^{(e)}$, $\Pi_{oz}^{(m)}$, $\Pi_{ez}^{(e)}$ and $\Pi_{ez}^{(m)}$ are considered. Consequently two Hertz vectors, $\Pi_{2z}^{(e)}$ and $\Pi_{2z}^{(m)}$ must be supposed in region II. For simplification, waves transmit parpendicularly into the ionosphere, approximately, and that the reflection coefficients are independent of azimuthal angle ϕ and that $\omega_r/\omega >> 1$ and $|n_o n_e C^4| << 1$. Reflection coefficient $_{\parallel}R_{\parallel}$ and vector potential in region II can be represented as follws;

$$_{\parallel}R_{\parallel} \simeq -e^{-2\beta_{\parallel}C}$$

$$\Pi_{2z}^{(e)} \simeq \sum_{n=1}^{\infty} B_n^{(e)} \sqrt{\frac{2i}{\pi k S_n^{(2)} \rho}} \cos(k C_n^{(2)} z) \cdot \exp[-ikS_n^{(2)}\{(x-x_o)\cos\phi_{tn} + y\sin\phi_{tn}\}],$$

$$\beta_{\parallel} \simeq \frac{n_o n_e}{n_o + n_e} \frac{C_o + C_e}{C_o C_e} \ll 1,$$

$$S_n^{(2)} \simeq \bar{S}_n + \frac{1}{kh\bar{S}_n}\left[\sqrt{\frac{\omega_r}{\omega}} e^{-i3\pi/4} \cdot \sec\frac{\tau}{2} \cdot \left\{1 + \frac{\omega}{\omega_r} e^{i\pi/2} \cos^2\frac{\tau}{2}\left(1 - 2\tan^2\frac{\tau}{2}\right)\right\}\bar{C}_n^2 + \sqrt{\frac{\omega\varepsilon_o}{\sigma_g}} e^{-i\pi/4}\right],$$

$$C_n^{(2)} \simeq \bar{C}_n + \frac{1}{kh}\left[\sqrt{\frac{\omega_r}{\omega}} e^{i\pi/4} \cdot \sec\frac{\tau}{2} \cdot \left\{1 + \frac{\omega}{\omega_r} e^{i\pi/2} \cos^2\frac{\tau}{2}\left(1 - 2\tan^2\frac{\tau}{2}\right)\right\}\bar{C}_n + \frac{1}{\bar{C}_n}\sqrt{\frac{\omega\varepsilon_o}{\sigma_g}} e^{i3\pi/4}\right],$$

$$\bar{C}_n = \frac{(n - 1/2)\pi}{kh}, \quad \bar{S}_n = \sqrt{1 - \bar{C}_n^2},$$

$$|n_i C| \ll 1, \quad B_1^{(e)} \simeq \frac{i2\pi M}{h},$$

$$|n_i C| \gg 1,$$

$$B_n^{(e)} \simeq \frac{i2\pi M}{h} \sqrt{\frac{S_n^{(2)}}{S_1^{(1)}}} \left\{\frac{(-1)^n}{(n + 1/2)\pi} + \frac{(-1)^{n+1}}{(3/2 - n)\pi}\right\}.$$

As magnetic Hertz vector $\Pi_{2z}^{(m)}$ is sufficiently small in comparison with the electric Hertz vector $\Pi_{2z}^{(e)}$, the magnetic Hertz vector can be neglected for simplification. And if the wave is received at a great distance from the boundary, $x=0$, only the first order mode remains in region II. And Snell's law must hold at the boundary, $x=0$. Therefore,

$$S_1^{(1)} \sin\phi = S_1^{(2)} \sin(\phi - \delta)$$

Consequently, the refraction errors of the constant phases of radio waves propagating in daytime and nighttime can be estimated by the next equations, respectively. In the case of daytime,

$$R_e(\delta)_d \simeq \frac{\bar{C}_1^2}{\sqrt{2}\,\bar{S}_1 kh} \cdot \frac{\sqrt{\omega_o^2/\nu}\cos\frac{\tau}{2} - \sqrt{\omega_r}}{\sqrt{\omega\,\omega_o^2\omega_r/\nu}\cos\frac{\tau}{2}} \cdot \left(\omega\cos\frac{\tau}{2} + \sqrt{\omega_o^2\omega_r/\nu}\right)\tan\phi \ (\mathrm{rad.}),$$

and in the case of nighttime,

$$R_e(\delta)_n \simeq \frac{3}{8}\left(\frac{\pi}{kh}\right)^2 x \tan\phi \ (\mathrm{rad.}).$$

Now, if the following quantity to each parameter is assumed;
$f=10$ kHz, $h=70$ km, $N=1\times10^3$ cm^{-3}, $\nu=5\times10^6$ sec^{-1}, $B=0.4$ Gauss, then

$$\mathrm{Re}(\delta)_d = 1.8\times10^{-4}\times\tan\phi \ (\mathrm{rad.}),$$

$f=10$ kHz, $h=85$ km, $N=1\times10^4$ cm^{-3}, $\nu=3\times10^5$ sec^{-1}, $B=0.4$ Gauss, then

$$\mathrm{Re}(\delta)_n = 0.0112\times\tan\phi \ (\mathrm{rad.}),$$

or $\quad \mathrm{Re}(\delta)_n = 0.665\times\tan\phi \ (\mathrm{deg.}).$

As is shown above, the refraction error in daytime is vary small, but the one in nighttime becomes very great with increasing the incident angle ϕ. This refraction error is very important in the direction-finding of atmospherics which propagate over a few kilo meters.