THE RESPONSE TO A PULSED LINE SOURCE IN A GOOD CONDUCTOR

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Abstract In eddy current nondestructive testing an indication of the effect of a defect is needed and in this paper a theoretical approach to model the defect is outlined and applied to the case of a horizontal line defect inside the metal. The approach is presented in three steps. It is assumed that there is an impulsive (in time) current density having the orientation and geometry of the defect at the defect position and the fields in the air are solved for. Also it is assumed that there is an impulse field applied to the surface of the metal out of the aperture of a masked probe and a solution is made for the fields inside the metal. From the electric field and the conductivity of the metal, the current that flows along the boundary of the defect can be found. Convolution is then employed to determine the response of the defect to the applied pulsed magnetic field. This will be useful in gaining information on the size, location and orientation of the defect.

Introduction For some time eddy currents have been applied to the problems of nondestructive evaluation of materials as an alternative to the well-known methods of X-rays and ultrasonics. Eddy currents have the advantages of being less expensive than X-rays and can detect more defects in the case of two or more metals fastened together than ultrasonics can. Pulsed eddy currents offer advantages over the sinusoidal eddy currents in that there are less thermal effects and the resolution is better. A defect such as a void or crack would disturb the magnetic field pattern from that existing without the defect. The defect in the material then would act as a source (1,2) inside the materials. A pulsed point source (1) has been considered and this paper is to extend this work to a line source in the material.

Method Employed The first step calls for the determination of the fields produced by a current excitation which is impulsive in time and has orientation and direction of the defect. The solution can be put in an integral form which can be evaluated numerically. The second step calls for the determination of the fields at the defect positions due to a field applied to the metal surface. This step results in an integral form solution which can be evaluated numerically. The third step involves a convolution of the results of steps one and two.

Application of the Method Assume a line defect of length L existing inside a metal of conductivity σ, permeability μ and permittivity ε, and that the defect extends between the points $(0,y_1,-z_1)$ and $(L,y_1,-z_1)$ as shown in Fig. 1. For an $Hz=EXP(-r)Cos(\phi)\delta(t)$ at $t=0$ applied to the surface of the metal it is required to find the field (Hz) detected by a pick-up coil at (x,y,z).

1) Step 1. Assume that the current density

$$J = \delta(y-y_1)\delta(z+z_1)[U(x^1)-U(x^1-L)]\,\delta(t)\hat{x} \quad (1)$$

flows in the line (defect)

To satisfy the boundary conditions the Hertz vector potential must have the two components Π_x and Π_z, which satisfy

$$\nabla^2\Pi_{1x} - s\mu\sigma\hat{\Pi}_{1x} = -\frac{1}{\sigma}\delta(y-y_1)\delta(z+z_1)[U(x^1)- U(x^1-L)] \quad (2)$$

$$\nabla^2\Pi_{1z} - s\mu\sigma\hat{\Pi}_{1z} = 0 \quad (3)$$

in the metal and

$$\nabla^2\hat{\Pi}_{2x} - \mu_o\ \varepsilon_o s^2\hat{\Pi}_{2x} = 0 \quad (4)$$

$$\nabla^2\Pi_{2z} - \mu_o\varepsilon_o s^2\hat{\Pi}_{2z} = 0 \quad (5)$$

in the air.

The solution to equations (2) to (5) that satisfies the boundary conditions is

$$\hat{\Pi}_{2x}= \int_o^L\int_o^\infty \frac{1}{2\Pi\varepsilon_o s}\ \frac{\lambda\,\mathrm{Exp}(-z\sqrt{\lambda^2+\mu_o\varepsilon_o s^2} - z_1\sqrt{\lambda^2+\mu_o\sigma s})}{[\sqrt{\lambda^2+\mu_o\varepsilon_o s^2} + \sqrt{\lambda^2+\mu_o\sigma s}]}\ J_o(\lambda r^1)\,d\lambda dx^1 \quad (6)$$

from which

$$H_z(x_1y_1z_1s) = -\frac{1}{2\Pi}(y-y_1)\int_o^L\int_o^\infty \frac{\lambda^2}{\lambda+\mu_m}\ e^{-\lambda z-\mu_m z_1}\ \frac{J_1(\lambda r^1)}{r^1}\ d\lambda dx^1 \quad (7)$$

With the approximation $\sqrt{\lambda^2+\mu_o\varepsilon_o s^2} = \lambda$, and the use of Laplace transform tables

$$H_z(x,y,z,t) = -\frac{(y-y_1)}{2\Pi\sqrt{\mu\sigma}}\int_o^L\int_o^\infty \lambda^2 e^{-\lambda z-\lambda^2\frac{t}{\mu\sigma}}$$

$$\frac{J_1(\lambda r^1)}{r^1}\left[\frac{\sqrt{\Pi\sigma}}{\sqrt{\Pi t}}\ e^{-z_1^2\frac{\mu\sigma}{4t}} - e^{\lambda z_1+\lambda^2 t}\ \mathrm{erfc}\left(\frac{z_1\sqrt{\mu\sigma}}{2\sqrt{t}} + \frac{\lambda\sqrt{t}}{\sqrt{\mu\sigma}}\right)\right]d\lambda dx^1 \quad (8)$$

where

$$r^1 = [(x-x')^2 + (y-y^1)^2]^{1/2}$$

Also erfc(x) is the complementary error function and $J_1(\lambda r^1)$ is the first order Bessel function of the first kind.

2) Step 2. In this step under the assumptions that the metal does not have a defect, the displacement current density is small and there is no free charge density in the metal. Maxwell's equations are solved for the fields in the metal. Enforcement of the boundary condition on H_z at x=0 results in

$$E_x(x,y,z,t)=\frac{1}{\sigma}\int_0^{\infty}\frac{\lambda^2}{(1+\lambda^2)^{3/2}}\ J_o(\lambda r)\ \frac{|z|\sqrt{\mu_o\sigma}}{2\sqrt{\Pi}}\ \frac{e^{\left(-z_1^2\frac{\mu_o\sigma}{4t}-\frac{\lambda^2 t}{\mu\sigma}\right)}}{t^{3/2}}\sin\phi\cos\phi d\lambda \qquad (9)$$

from which the effective current density along the defect can be taken as

$$J_x(y_1,-z_1,t) = \int_0^L\int_0^{\infty}\frac{\lambda^3}{(1+\lambda^2)^{3/2}}\ J_o(\lambda r)\ \frac{|Z_1|\sqrt{\mu_o\sigma}}{2\sqrt{\Pi}}\ \frac{e^{\left(-z_1^2\frac{\mu_o\sigma}{4t}-\frac{\lambda^2 t}{\mu_o\sigma}\right)}}{t^{3/2}}\sin\phi\cos\phi d\lambda dx' \qquad (10)$$

where

$r[y1^2+x'^2]^{1/2}$; $\sin\phi = \frac{y_1}{r}$; $\cos\phi = \frac{x^1}{r}$ and

$J_o(\lambda r)$ is the zero order Bessel function of the first kind.

3) Step 3. The response at the pick-up coil due to the defect will be the convolution of equations (8) and (10).

Results and Conclusions Numerical values of the convolution of (15) and (16) were computed and are presented in Figs. 2 to 4. Figure 2 shows the variation in the magnitude of the peak magnetic field as a function of the defect length. Figure 3 presents the change in the peak magnetic field as a function of depth in the metal. Figure 4 shows the variation for a horizontal change in the position of the defect.

It is believed that this method will be useful in locating and estimating the size of metallic defects. The method may also be extended in the direction of defects having considerable area and volume configurations.

References

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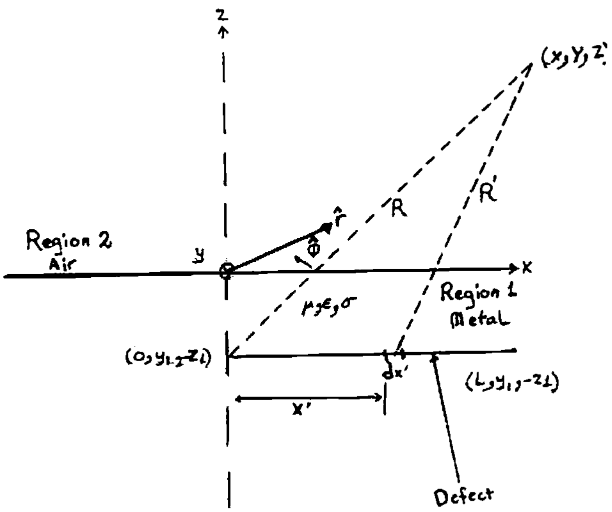

Fig. 1. The position of the defect in the metal and the pick-up coil in the air.

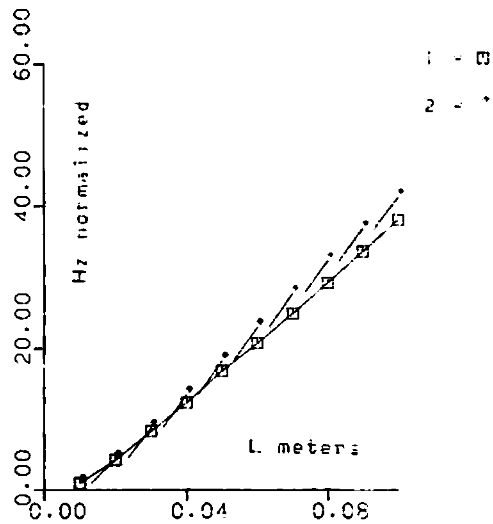

Fig. 2. The magnitude of the peaks of Hz as a function of the defect length. Hz is normalized to that for L=0.01 meter. The upper curve is for Nickel while the lower one is for Aluminum.

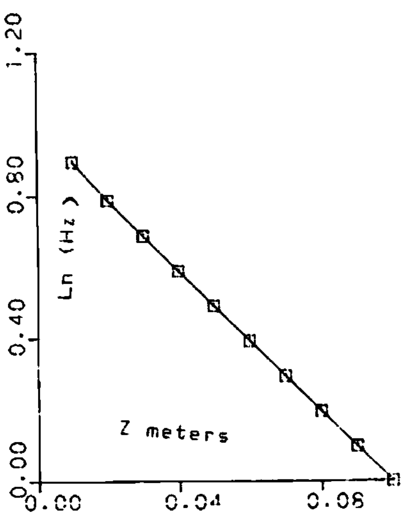

Fig. 3. The natural logarithm of the peaks of the magnitude of Hz as a function of z for a defect in Aluminum. Hz is normalized to that at z=0.01.

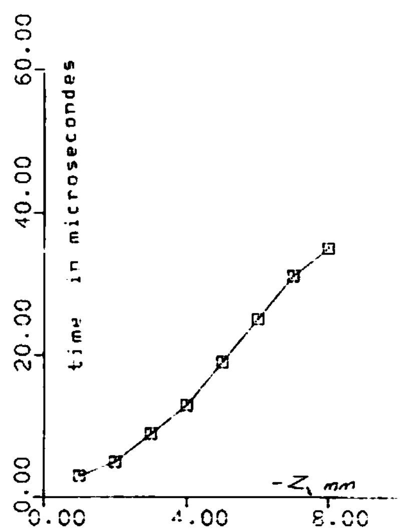

Fig. 4. Time of the peaks of Hz as a function of the defect depth for stainless steel.