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FOCUSING OF HIGHLY FLUCTUATING WAVES

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In many problems of radiophysics one has to consider focusing of the radiation whose phase and/or amplitude are random. Here we discuss the case of strong phase fluctuations ($\langle\varphi^2\rangle \gg 1$, the angular brackets $\langle\,\rangle$ denote the ensemble average). Unlike the prior treatments*) the focusing structures can possess arbitrarily great aberrations. Such a situation is met, for instance, when focusing occurs in natural media like the Earth's ionosphere[5], the solar corona[6], etc.

If the observation point $\vec{r}(x,y,z)$ is far enough from the focusing structure we can write for the field $u(\vec{r})$ (using the Huygens principle):

$$u(\vec{r})=\frac{k}{2\pi i}\int u_0 \exp\left[-ik\zeta+i\varphi+ik|\vec{r}-\vec{r}'|\right]\frac{d\vec{r}'}{|\vec{r}-\vec{r}'|} \quad (1)$$

Here k is the wavenumber, $u_0(\vec{r}')$ the field amplitude in the aperture of the focusing structure (over which the integration is carried out), $k\zeta(\vec{r}')$ and $\varphi(\vec{r}')$ stand respectively for the regular and the random phase in the aperture, $\langle\varphi\rangle=0$. We aim at deriving simple expressions for the average intensity of the radiation $I(\vec{r})=\langle u(\vec{r})u^*(\vec{r})\rangle$ (the asterisk* means the complex conjugate) and the field correlation function $K_u(\vec{r}_1,\vec{r}_2)=\langle u(\vec{r}_1)u^*(\vec{r}_2)\rangle$.

In the Fresnel approximation we obtain from (1)

$$I(\vec{r})=\frac{k^2}{4\pi^2 z^2}\int u_0(\vec{r}')u_0(\vec{r}'')\exp\Big\{-\langle\varphi^2\rangle[1-K_\varphi(\vec{r}',\vec{r}'')]-ik\Big[\zeta(\vec{r}')-\zeta(\vec{r}'')-\frac{(x'-x)^2+(y'-y)^2-(x''-x)^2-(y''-y)^2}{2z}\Big]\Big\}d\vec{r}'d\vec{r}'' \quad (2)$$

The random phase is assumed normally distributed and uniform, so that $K_\varphi(\vec{r}',\vec{r}'')$ depends on $\vec{\rho}=\vec{r}'-\vec{r}''$ only. Since the magnitude of $\langle\varphi^2\rangle$ is great, K_φ and ζ can be expanded in powers of ρ_x and ρ_y to quadratic terms. To neglect the higher-order terms we need $\langle\theta^2_{x,y}\rangle \gg \theta_0^2$, with $\langle\theta^2_{x,y}\rangle=\frac{1}{2k^2}\frac{\partial^2 K_\varphi(0,0)}{\partial\rho^2_{x,y}}$ being the mean squares of the arrival angles and θ_0 - the diffraction angle. If the most rapidly changing factor remained after the expansion in the integrand of Eq.2 is $\exp[-k^2/2(\langle\theta_x^2\rangle\rho_x^2+\langle\theta_y^2\rangle\rho_y^2)]$, then the limits for ρ_x, ρ_y can be extended to ∞ and the integration yields

$$I(\vec{r})=\frac{1}{\pi z^2\langle\theta^2\rangle}\int u_0^2(p,q)\exp\left[-\frac{\psi^2(x,y;p,q)}{\langle\theta^2\rangle}\right]dp\,dq \quad (3)$$

For simplicity's sake we set $\langle\theta_x^2\rangle=\langle\theta_y^2\rangle=\langle\theta^2\rangle/2$. Equation 3 is a formulation of the Huygens principle for the incoherent radiation. It implies that the observed intensity is made up of contributions from all the aperture elements weighed by $\exp(-\psi^2/\langle\theta^2\rangle)$.

The angle $\psi=\sqrt{\left[\zeta'_p(p,q)+\frac{x-p}{z}\right]^2+\left[\zeta'_q(p,q)+\frac{y-q}{z}\right]^2}$ is that between the average wavefront normal at (p,q) (average ray) and the line of sight from $\vec{r}(x,y,z)$ to (p,q). Such a description corresponds to geometrical optics.

Equation 3 provides for finite values of $I(\vec{r})$ at any observation point, including caustic surfaces or foci. Diffractive spread of the focal spot is replaced by that due to fluctua-

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*)There exists vast literature dedicated to statistical properties of the wave field in the focal plane of a lense (cf.[1,2]), statistical antenna theory[3,4] and other similar problems.

tions. However, the limits of validity of Eq.3 still need consideration. Retaining terms of the order $\exp(-\langle\varphi^2\rangle)$ in the expansion of Eq.2, one can obtain

$$I_\Sigma = I(\vec{r}) + \frac{k^2\exp(-\langle\varphi^2\rangle)}{4\pi^2}\left|\int u(\vec{r}\,')\frac{\exp ik|\vec{r}-\vec{r}\,'|}{|\vec{r}-\vec{r}\,'|}d\vec{r}\,'\right|^2$$

The second term is a "pure diffractive" intensity distribution i.e. the coherent part thereof. Now, the condition for the validity of Eq.3 is $I_{coh.} \ll I_{incoh.}$, which, together with $\langle\theta^2\rangle \gg \theta_0^2$, leads to the double inequality $1 \ll \langle\theta^2\rangle/\theta_0^2 \ll \exp\langle\varphi^2\rangle$.

The correlation function $K(\vec{r}_1,\vec{r}_2)$ can be calculated in a similar manner. For example, the transverse correlation (i.e. $z_1 = z_2 = \text{const}$) is described by

$$K_u^\perp(\vec{r}_{1\perp},z;\vec{r}_{2\perp},z) = \frac{\exp\left[-i\frac{k}{z}(x_0\Delta x + y_0\Delta y)\right]}{\pi z^2\langle\theta^2\rangle}\times \tag{4}$$

$$\int u_0^2(p,q)\exp\left[-\frac{\Psi^2(x_0,y_0;p,q)}{\langle\theta^2\rangle} + i\frac{k}{z}(p\Delta x + q\Delta y)\right]dp\,dq$$

with $\vec{r}_0(x_0,y_0) = \frac{\vec{r}_1+\vec{r}_2}{2}$; $\Delta\vec{r}(\Delta x,\Delta y) = \vec{r}_1 - \vec{r}_2$. The sense of Ψ is the same as in Eq.3. Examination of the correlation function (4) in dependence of two pairs of variables – x_0, y_0 and $\Delta x, \Delta y$ – gives information separately on the "pure" diffraction (despite the great phase fluctuations) and on the scattering. This can be easily seen in case of the error-free lense for which we have $\zeta(x',y') = \frac{x'^2+y'^2}{2F}$,

$$K_u^\perp(\vec{r}_{1\perp},F;\vec{r}_{1\perp},F) = \frac{\exp\left[-\frac{x_0^2+y_0^2}{F^2\langle\theta^2\rangle} - i\frac{k}{F}(x_0\Delta x + y_0\Delta y)\right]}{\pi F^2\langle\theta^2\rangle}$$

$$\times\int u_0^2(p,q)\exp\left[i\frac{k}{F}(p\Delta x + q\Delta y)\right]dp\,dq$$

The integral is essentially the Fraunhofer pattern of the aperture. In optics this result is known as the van Cittert-Zernike theorem[7].

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