Linear Constrained CMA Approach to Robust Adaptive Beamforming

Xin Song, Jinkuan Wang, Yanbo Xue and Yinghua Han
School of Information Science and Engineering
Northeastern University, Shenyang, 110004, China
E-mail: sxin78916@mail.neuq.edu.cn

1. Introduction

Adaptive beamforming is used for enhancing a desired signal while suppressing noise and interference at the output of an array of sensors. Adaptive beamforming has applications in fields such as radar, sonar, seismology, radio astronomy, and wireless communications [1]-[5]. In particular, the development of robust adaptive beamforming spans over two decades.

When adaptive arrays are applied to practical problems, it is well known that adaptive beamforming algorithms can suffer significant performance degradation because the array response vector for the desired signal is not known exactly [6], [7]. In fact, the performances of the existing adaptive array algorithms are known to degrade substantially in the presence of even slight mismatches between the actual and presumed array responses to the desired signal. Similar types of degradation can take place when the signal array response is known precisely but the training sample size is small. Therefore, robust approaches to adaptive beamforming appear to be of primary importance. There are several efficient approaches that are known to provide an improved robustness against some types of mismatches, such as the linearly constrained minimum variance (LCMV) beamformer [8], the diagonal loading of the sample covariance matrix [9]. But these methods cannot be expected to provide sufficient robustness improvements.

Linear constrained constant modulus algorithm (LC-CMA) is an effective solution to the problem of interfere capture in constant modulus algorithm (CMA). But in practical, the performance of the LC-CMA degrades in the presence of even slight mismatches between the actual and presumed array responses to the desired signal. In this paper,on the basis of LC-CMA, we develop a novel robust LC-CMA (RLC-CMA). Our RLC-CMA provides excellent robustness against the signal steering vector mismatches and small training sample size, offers faster convergence rate. Computer simulations demonstrate a visible performance gain of the proposed RLC-CMA over other traditional and robust adaptive beamforming techniques.

2. Problem Formulation

2..1 Mathematical Model

Consider a uniform linear array (ULA) with M omnidirectional sensors spaced by the distance d and D narrow-band incoherent plane waves, impinging from directions $\{\theta_0, \theta_1, \cdots, \theta_{D-1}\}$. The structure of adaptive beamforming is shown in Fig. 1. The observation vector is given by

$$\begin{aligned}\mathbf{X}(k) &= \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k) \\ &= s_0(k)\mathbf{a} + \mathbf{i}(k) + \mathbf{n}(k)\end{aligned} \tag{1}$$

where $\mathbf{X}(k) = [x_1(k), x_2(k), ..., x_M(k)]^T$ is the complex vector of array observations, $s_0(k)$ is the signal waveform, $\mathbf{a}(k)$ is the signal steering vector, $\mathbf{i}(k)$ and $\mathbf{n}(k)$ are the interference and noise components, respectively. The output of a narrowband beamformer is given by

$$y(k) = \mathbf{W}^H\mathbf{X}(k) \tag{2}$$

where $\mathbf{W} = [w_1, w_2, ..., w_M]^T$ is the complex vector of beamformer weights, M is the number of array sensors and $(\cdot)^T$ and $(\cdot)^H$ stand for the transpose and Hermitian transpose, respectively. The signal-to-interference-plus-noise ratio (SINR) has the following form:

$$\mathbf{SINR} = \frac{\mathbf{W}^H\mathbf{R_s}\mathbf{W}}{\mathbf{W}^H\mathbf{R_{i+n}}\mathbf{W}} \tag{3}$$

where

$$\mathbf{R_s} = E\{\mathbf{s}(k)\mathbf{s}^H(k)\} \tag{4}$$

$$\mathbf{R_{i+n}} = E\{(\mathbf{i}(k) + \mathbf{n}(k))(\mathbf{i}(k) + \mathbf{n}(k))^H\} \tag{5}$$

ISBN: 89-86522-77-2 94460©KEES

are the $M \times M$ signal and interference-plus-noise covariance matrices, respectively, and $E\{\cdot\}$ denotes the statistical expectation.

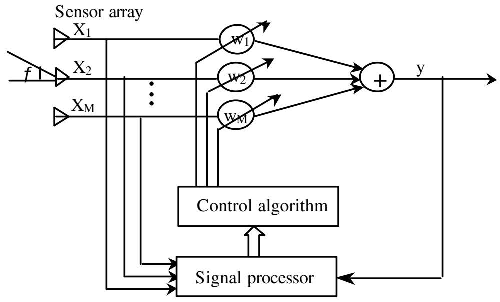

Fig. 1: The structure of adaptive beamforming.

2..2 Linear Constrained Constant Modulus Algorithm (LC-CMA)

The constant modulus algorithm (CMA) is a blind algorithm. CMA is a gradient-based algorithm that works on the premise that the existence of an interference causes fluctuation in the amplitude of the array output, while otherwise has a constant modulus. It updates the weights by minimizing the cost function [10].

$$J(k) = \frac{1}{2}E[(|y(k)|^2 - y_0^2)^2] \tag{6}$$

Using the following equation:

$$\mathbf{W}(k+1) = \mathbf{W}(k) - \mu\mathbf{g}(\mathbf{W}(k)) \tag{7}$$

where $y(k) = \mathbf{W}^H(k)\mathbf{X}(k+1)$ is the array output after the nth iteration, y_0 is the desired amplitude in the absence of interference, and $\mathbf{g}(\mathbf{W}(k))$ denotes an estimate of the gradient of the cost function.

$$\mathbf{g}(\mathbf{W}(k)) = 2\mu\gamma(k)\mathbf{X}(k+1) \tag{8}$$

where

$$\gamma(k) = (|y(k)|^2 - y_0^2)y(k) \tag{9}$$

The weight update equation for this case becomes

$$\mathbf{W}(k+1) = \mathbf{W}(k) - 2\mu\gamma(k)\mathbf{X}(k+1) \tag{10}$$

CMA is computationally less complex than other algorithms. But when SINR is low and constant modulus of interference sources are existed, the performances of CMA are known to degrade severely. Linear constrained constant modulus algorithm (LC-CMA) is an effective solution to the problem of interference capture in CMA.

LC-CMA is a real-time constrained algorithm for determining the optimal weight vector. The optimal weight vector is the solution of the following optimization problem:

$$\min_{\mathbf{W}} E[(|y(k-1)|^2 - |y(k)|^2)^2] \quad \text{subject to} \quad \mathbf{W}^H\mathbf{a} = 1 \tag{11}$$

Optimization technique used to find $\mathbf{W}$ will use Lagrange multiplier method, thus, the expression for $\mathbf{W}$ becomes

$$\mathbf{W}(k+1) = \mathbf{P}[\mathbf{W}(k) + \mu e^H(k)\mathbf{X}(k)] + \mathbf{F} \tag{12}$$

where

$$\begin{aligned} \mathbf{F} &= \mathbf{a}[\mathbf{a}^H\mathbf{a}]^{-1} \\ \mathbf{P} &= \mathbf{I} - \mathbf{a}[\mathbf{a}^H\mathbf{a}]^{-1}\mathbf{a}^H \\ e(k) &= (|y(k-1)|^2 - |y(k)|^2)y(k) \end{aligned} \tag{13}$$

LC-CMA requires the knowledge of the direction-of-arrival (DOA) of the desired signal, but in practical application, the performance degradation of LC-CMA may become evident because some of underlying assumptions on the environment, sources, or sensor array can be violated and this may cause a mismatch between the presumed and actual signal steering vectors.

3. Robust Linear Constrained Constant Modulus Algorithm (RLC-CMA)

We develop a novel approach to robust adaptive beamforming that provides an improved robustness against the signal steering vector mismatches and small training sample size.

In practical applications, we assume that the norm of the steering vector distortion $\mathbf{\Delta}$ can be bounded by some known constant $\varepsilon > 0$, $||\mathbf{\Delta}|| \leq \varepsilon$. Then, the actual signal steering vector belongs to the set,

$$\mathbf{\Phi}(\varepsilon) = \{\mathbf{b}|\mathbf{b} = \mathbf{a} + \mathbf{\Delta}, \quad ||\mathbf{\Delta}|| \leq \varepsilon\} \tag{14}$$

The array response should not be smaller than one, i.e.,

$$|\mathbf{W}^H\mathbf{b}| \geq 1 \quad \text{for all} \quad \mathbf{b} \in \mathbf{\Phi}(\varepsilon) \tag{15}$$

According to [11],we can rewrite the constraint as

$$\mathbf{W}^H\mathbf{a} = \varepsilon||\mathbf{W}|| + 1 \tag{16}$$

The optimal problem (11) can be written as the following problem

$$\min_{\mathbf{W}} E[(|y(k-1)|^2 - |y(k)|^2)^2] \quad \text{subject to} \quad \mathbf{W}^H\mathbf{a} = \varepsilon||\mathbf{W}|| + 1 \tag{17}$$

Use Lagrange multiplier method to conclude the weight vector by minimizing the following function

$$H(\mathbf{W}, \lambda) = (|y(k-1)|^2 - |y(k)|^2)^2 + \lambda(\varepsilon^2\mathbf{W}^H\mathbf{W} + \mathbf{W}^H\mathbf{a} + \mathbf{a}^H\mathbf{W} - \mathbf{W}^H\mathbf{a}\mathbf{a}^H\mathbf{W} - 1) \tag{18}$$

where λ is a Lagrange multiplier. Take the gradient of $H(\mathbf{W}, \lambda)$

$$\mathbf{\Gamma}(\mathbf{W}, \lambda) = -e^H(k)\mathbf{X} + (\lambda\varepsilon^2\mathbf{I} - \lambda\mathbf{a}\mathbf{a}^H)\mathbf{W} + \lambda\mathbf{a} \tag{19}$$

The weight update equation for RCL-CMA becomes

$$\begin{aligned} \mathbf{W}(k+1) &= \mathbf{W}(k) - \mu\mathbf{\Gamma}(\mathbf{W}, \lambda) \\ &= \mathbf{W}(k) + \mu[e^H(k)\mathbf{X}(k) - (\lambda\varepsilon^2\mathbf{I} - \lambda\mathbf{a}\mathbf{a}^H)\mathbf{W}(k) - \lambda\mathbf{a}] \end{aligned} \tag{20}$$

The weight vector in (20) satisfies the constraint in (16) at every iteration.

To summarize, our proposed robust linear constrained-CMA (RLC-CMA) consists of the following steps.

Step 1) Initialize $\mathbf{W}(0)$, $y(0)$

Step 2) Compute the desired signal steering $\mathbf{a}$ and let $i = 1$.

Step 3) Compute λ by using (20) and the constraint in (16).

Step 4) Use λ obtained in (19) to obtain the gradient $\mathbf{\Gamma}(\mathbf{W}, \lambda)$.

Step 5) Use $\mathbf{\Gamma}(\mathbf{W}, \lambda)$ obtained in (20) to update the weight vectors.

Step 6) If $i = n$, stop. Otherwise, set $i = i + 1$, and go to step 3.

4. Simulation Results

In this section, we present some simulations to justify the performance of proposed RLC-CMA. We assume a uniform linear array with $M = 10$ omnidirectional sensors spaced half a wavelength apart. For each scenario, 100 simulation runs are used to obtain each simulated point. In all examples, we assume two interfering sources with plane wavefronts and the directions of arrival (DOAs) 30° and 50°, respectively.

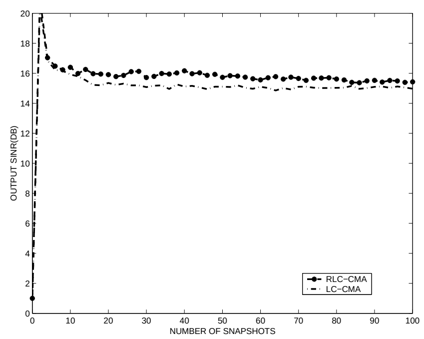

Fig. 2: Output SINR versus N.

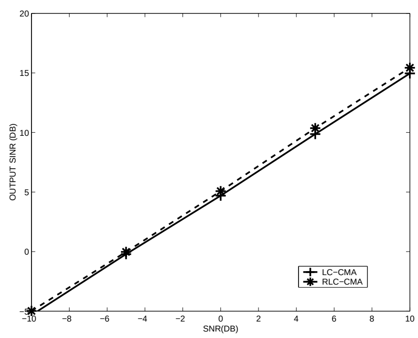

Fig. 3: Output SINR versus SNR.

In the first example, the plane-wave signal is assumed to impinge on the array from $\theta = 3°$. Fig. 2 displays the performance of the two methods tested versus the number of snapshots for the fixed $SNR = 10dB$ and $\varepsilon = 1.5$. Fig. 3 shows the performance of these algorithms versus the SNR for the fixed training data size $N = 100$. The example demonstrates that the performance of RLC-CMA can be seen to outperform that of LC-CMA.

In the second example, a scenario with the signal look direction mismatch is considered. We assume that both the presumed and actual signal spatial signatures are plane waves impinging from the DOAs 3° and 6°, respectively. This corresponds to a 3° mismatch in the signal look direction. Fig. 4 displays the performance of the two methods tested versus the number of snapshots for $SNR = 10dB$ and $\varepsilon = 1.5$. The performance of these algorithms versus the SNR for the fixed training data size $N = 100$ is shown in Fig. 5. In this example, LC-CMA is

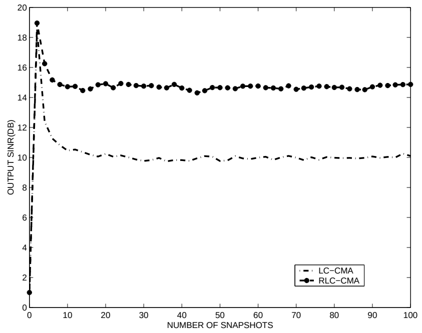

Fig. 4: Output SINR versus N.

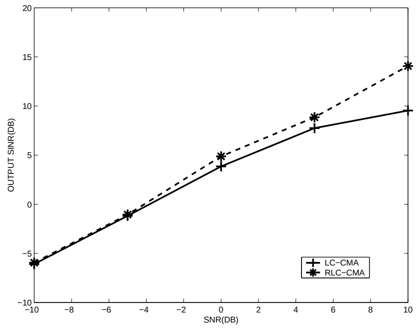

Fig. 5: Output SINR versus SNR.

very sensitive even to slight mismatches that can easily occur in practical situations. Moreover, LC-CMA shows poor performance at all values of the SNR and N.

5. Conclusions

The proposed RLC-CMA in this paper is based on classical LC-CMA. The proposed RLC-CMA is an effective solution to the problem of interference capture in CMA. RLC-CMA offers faster convergence rate and provides excellent robustness against the desired signal mismatches and small training sample size. Moreover, the mean output SINR of RLC-CMA is better than that of LC-CMA in a wide range of SNR and N. Our simulation figures clearly demonstrate that in all examples, the proposed RLC-CMA is shown to consistently enjoy a significantly improved performance as compared with LC-CMA.

References

[1] L. E. Brennan, J. D. Mallet, and I. S. Reed, "Adaptive arrays in airborne MTI radar," IEEE Trans. Antennas Propagation, vol. 24, pp. 607-615, Sept. 1976.

[2] J. L. Krolik, "The performance of matched-field beamformers with Mediterranean vertical array data," IEEE Trans. Signal Processing, vol. 44, pp. 2605-2611, Oct. 1996.

[3] E. Y. Gorodetskaya, A. I. Malekhanov, A. G. Sazontov, and N. K. Vdovicheva, "Deep-water acoustic coherence at long ranges: Theoretical prediction and effects on large-array signal processing," IEEE J. Ocean.Eng., vol. 24, pp. 156-171, Apr. 1999.

[4] S. Shahbazpanahi, A. B. Gershman, and Z. Q. Luo, "Robust adaptive beamforming for general-rank signal models" IEEE Trans. Signal Processing, vol. 51, pp. 2257-2269, Sept. 2003.

[5] T. S. Rapapport, Ed., Smart Antennas: Adaptive Arrays, Algorithms, and wireless Position Location. Piscataway, NJ: IEEE, 1998.

[6] C. L. Zahm, "Effects of errors in the direction of incidence on the performance of an adaptive array," Proc. IEEE, vol. 60, pp. 1068-1069, Aug. 1972.

[7] D. D. Feldman and L. J. Griffiths, "A projection approach for robust adaptive beamforming," IEEE Trans. Signal Processing, vol. 42, pp. 867-876, Apr. 1994.

[8] R. A. Monzingo and T. W. Miller, Introduction to Adaptive Arrays. New York: Wiley, 1980.

[9] B. D. Carlson, "Covariance matrix estimation errors and diagonal loading in adaptive arrays," IEEE Trans. Aerosp. Electron. Syst., vol. 24, pp. 397-401, July 1988.

[10] L.C. Godara, "Application of antenna arrays to mobile communication, Part II: Beam-forming and direction-of-arrival considerations," Proc. IEEE, vol.85, pp. 1218-1219, Aug. 1997.

[11] S. A. Vorobyov, A. B. Gershman, and Z.Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem," IEEE Trans. Signal Processing, vol. 51, pp. 313-323, Feb. 2003.