DIFFRACTION OF A PLANE WAVE BY AN INCLINED PARALLEL PLATE GRATING —— THE CASE OF TM INCIDENCE ——

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1. Introduction

Diffraction problems concerned with infinite plane gratings have been widely investigated so far both theoretically and experimentally by many research workers [1][2]. From a technical point of view, it is suggested that gratings with parallel plate geometry are useful for design of frequency selective surfaces. In fact, one of the authors has previously analyzed the diffraction by a parallel plate grating with and without dielectric loading and clarified by numerical computation that gratings of this type have characteristics of filters or resonators [3][4]. In the present paper, we shall generalize a grating geometry treated in [3] and analyze the problem of diffraction by an inclined parallel plate grating via rigorous procedure based on the Wiener-Hopf technique. This problem is somewhat more difficult than the case when the inclined angle is zero (see Fig. 1). An incident field is chosen to be a TM-polarized plane wave. The case of TE incidence has already been analyzed by one of the authors with a similar procedure to that employed here [5]. The time factor is assumed to be $\exp(-i\omega t)$ and omitted throughout this paper.

2. Formulation of the problem

Let us consider the problem of diffraction by a parallel plate grating with an inclined angle θ as shown in Fig. 1. The grating is assumed to be uniform in y-direction. Then, this problem reduces to the two dimensional one. For simplicity, we assume that $2a > d$ and $0 < \theta < \pi/2$. Let the total field $\psi^t(x,z)$ be

$$\psi^t(x,z) = \psi^i(x,z) + \psi(x,z), \tag{1}$$

where

$$\psi^i(x,z) = e^{-ik(x\sin\theta_0 + z\cos\theta_0)} \tag{2}$$

for $|\theta_0 - \theta| < \pi/2$ and $k\ (=\omega\sqrt{\mu_0\varepsilon_0})$ is a free space wave number. Since

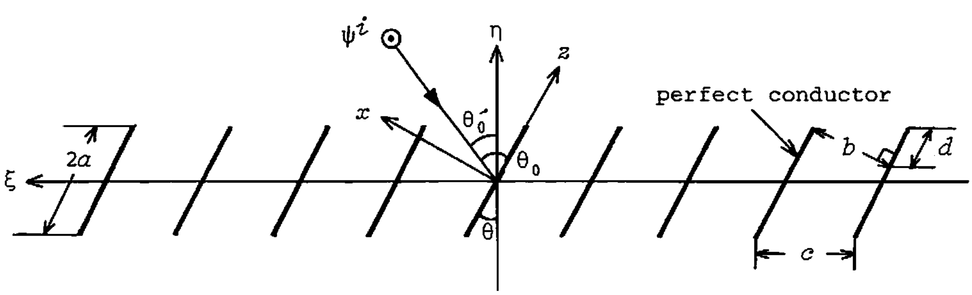

Fig. 1. Geometry of the problem (period : c).

$\psi(x,z)$ clearly satisfies the reduced wave equation

$$(\partial^2/\partial x^2 + \partial^2/\partial z^2 + k^2)\psi(x,z) = 0, \tag{3}$$

hence our problem at hand is to determine the solution of (3) subject to appropriate boundary conditions, radiation and edge conditions. Nonzero components of electromagnetic fields can be obtained from

$$(H_y, E_x, E_z) = \left(\psi, \frac{1}{i\omega\varepsilon_0}\frac{\partial\psi}{\partial z}, \frac{i}{\omega\varepsilon_0}\frac{\partial\psi}{\partial x}\right). \tag{4}$$

As is usual in the Wiener-Hopf analysis, we assume that the medium is slightly lossy, i.e.,

$$k = k_1 + ik_2, \qquad 0 < k_2 \ll k_1. \tag{5}$$

Furthermore, define the Fourier transform of $\psi(x,z)$ with respect to z as

$$\Psi(x,\alpha) = (2\pi)^{-1/2}\int_{-\infty}^{\infty}\psi(x,z)\,e^{i\alpha z}\,dz, \qquad \alpha = \sigma + i\tau. \tag{6}$$

Then by taking account of (5) and the radiation condition, we can show that $\Psi(x,\alpha)$ is regular in the strip $-k_2\cos(\theta_0 - 2\theta) < \tau < k_2\cos\theta_0$. Since the grating has a periodic structure of a period c, we shall restrict the following analysis to the unit cell $0 \leq x \leq b$.

Taking the Fourier transform of (3) and solving the resultant transformed wave equation, it turns out that $\Psi(x,\alpha)$ can be expressed as

$$\Psi(x,\alpha) = \frac{\cosh\gamma x\, e^{-i(\alpha d + u)} - \cosh\gamma(x-b)}{\gamma\sinh\gamma b}\{e^{-i\alpha a}U_-(\alpha) + e^{i\alpha a}U_{(+)}(\alpha)\},$$

$$u = kc\sin\theta_0', \tag{7}$$

where

$$U_{\binom{-}{(+)}}(\alpha) = \Psi'_{\mp}(0,\alpha) \pm \begin{Bmatrix} A \\ B \end{Bmatrix}\frac{1}{\alpha - k\cos\theta_0}, \tag{8}$$

$$A = -\frac{k\sin\theta_0}{(2\pi)^{1/2}}e^{ika\cos\theta_0}, \qquad B = -\frac{k\sin\theta_0}{(2\pi)^{1/2}}e^{-ika\cos\theta_0}, \tag{9}$$

$$\Psi_{\pm}(x,\alpha) = \pm(2\pi)^{-1/2}\int_{\pm a}^{\pm\infty}\psi(x,z)\,e^{i\alpha(z\mp a)}\,dz \tag{10}$$

and γ is defined by $(\alpha^2 - k^2)^{1/2}$ whose proper branch is chosen to be $\mathrm{Re}\,\gamma > 0$. In (8), the prime denotes a derivative with respect to x. It is obvious that $\Psi_+(x,\alpha)$ and $\Psi_-(x,\alpha)$ defined by (10) are regular in the upper ($\tau > -k_2\cos(\theta_0 - 2\theta)$) and lower ($\tau < k_2\cos\theta_0$) halves of the α-plane, respectively. After some manipulations with the aid of (7), we have

$$G(\alpha)J_1(0,\alpha) = e^{-i\alpha a}U_-(\alpha) + e^{i\alpha a}U_{(+)}(\alpha), \quad -k_2\cos(\theta_0 - 2\theta) < \tau < k_2\cos\theta_0, \tag{11}$$

where

$$J_1(0,\alpha) = \Psi(+0,\alpha) - \Psi(-0,\alpha), \tag{12}$$

$$G(\alpha) = \frac{\gamma \sinh \gamma b}{2\{\cos(\alpha d + u) - \cosh \gamma b\}} . \tag{13}$$

It may be shown by taking account of the boundary condition that $J_1(0,\alpha)$ is an entire function and is proportional to the surface current induced on the plate at $x=0$. Equation (11) is the desired Wiener-Hopf equation satisfied by unknown functions $U_-(\alpha)$ and $U_{(+)}(\alpha)$.

3. Solution of the Wiener-Hopf equation

Applying a similar procedure to that employed in [5], we get the formal solution to the Wiener-Hopf equation (11) as follows :

$$U_-(\alpha) = -G_-(\alpha,\theta) \sum_{n=0}^{\infty} \delta_n c_n^- \frac{e^{-2a\gamma_n} G_+(i\gamma_n,\theta)\, u_n^+}{\alpha - i\gamma_n} , \tag{14}$$

$$U_{(+)}(\alpha) = -G_+(\alpha,\theta) \left[\sum_{n=0}^{\infty} \delta_n c_n^+ \frac{e^{-2a\gamma_n} G_-(-i\gamma_n,\theta)\, u_n^-}{\alpha + i\gamma_n} + \frac{B}{G_+(k\cos\theta_0,\, \theta)(\alpha - k\cos\theta_0)} \right], \tag{15}$$

where

$$\gamma_0 = -ik, \qquad \gamma_n = \{(n\pi/b)^2 - k^2\}^{1/2} \quad \text{for} \quad n \geqq 1, \tag{16}$$

$$u_n^+ = U_{(+)}(i\gamma_n), \qquad u_n^- = U_-(-i\gamma_n), \tag{17}$$

$$\delta_0 = 1/2, \qquad \delta_n = 1 \quad \text{for} \quad n \geqq 1, \tag{18}$$

$$c_n^{\pm} = \pm 2(bi\gamma_n)^{-1} \{1 - (-)^n \cos(u \mp di\gamma_n)\} \tag{19}$$

and $G_\pm(\alpha,\theta)$ denote the split functions [5] of the Wiener-Hopf kernel $G(\alpha)$. In (14) and (15), $u_n^\pm$ are unknowns which can be determined by solving an infinite system of linear equations. These equations are derived by substituting $\alpha = -i\gamma_m$ and $i\gamma_m$ for $m = 0, 1, 2, \cdots$ into (14) and (15), respectively.

Scattered field representations can be obtained by substituting (14) and (15) into (7) and taking the Fourier inverse according to the formula

$$\psi(x,z) = (2\pi)^{-1/2} \int_{-\infty+i\tau_0}^{\infty+i\tau_0} \Psi(x,\alpha)\, e^{-i\alpha z}\, d\alpha, \quad -k_2\cos(\theta_0 - 2\theta) < \tau_0 < k_2 \cos\theta_0 . \tag{20}$$

Since singularities of $\Psi(x,\alpha)$ in the α-plane are only simple poles, the integral can be evaluated by computing the residues. This gives,

$$\begin{aligned} \psi &= \sum_{n=-\infty}^{\infty} R_n\, e^{-i\alpha_n \xi - \beta_n(\eta - a\cos\theta)} && \text{for} \quad \eta > a\cos\theta, \\ &= -\psi^i + \sum_{n=0}^{\infty} \{A_n\, e^{-\gamma_n(z+a)} + B_n\, e^{\gamma_n(z-a)}\} \cos\frac{n\pi}{b}x && \text{for} \quad -a < z < a - d, \\ &= -\psi^i + \sum_{n=-\infty}^{\infty} T_n\, e^{-i\alpha_n \xi + \beta_n(\eta + a\cos\theta)} && \text{for} \quad \eta < -a\cos\theta, \end{aligned} \tag{21}$$

where

$$\alpha_0 = k\sin\theta_0', \qquad \alpha_n = (2n\pi + u)/c \quad \text{for} \quad n \neq 0, \tag{22}$$

$$\beta_0 = -ik\cos\theta_0', \qquad \beta_n = (\alpha_n^2 - k^2)^{1/2} \quad \text{for} \quad n \neq 0. \tag{23}$$

In derivation of (21), the new coordinate (ξ, η) has been introduced. R_n, T_n and A_n, B_n are scattering coefficients of the Floquet's mode and the TM waveguide mode, respectively. Since these coefficients are expressed by unknowns $u_n^{\pm}$ appearing in (14) and (15), the solution thus obtained are eventually formal. However, if the width $2a$ of each grating plate is large compared with the spacing b between plates, the solution can be considerably simplified.

Let us consider the case when only the dominant TEM mode propagates in the waveguide region $-a < z < a - d$. Then all terms of the infinite series in (14) and (15) can be ignored except $n = 0$. This will lead to the approximate solution. After some arrangements, finally we obtain approximate expressions for R_n and T_n as follows :

$$R_n \approx \frac{(2\pi)^{1/2} i\, e^{-i\alpha_n a\sin\theta}}{\alpha_n\cos\theta - i\beta_n\sin\theta}\, \mathrm{Res}[G_+(\alpha,\theta)]_{\alpha = -i\beta_n^+}$$

$$\cdot\left[\frac{c_0^+ C_- D}{2(1 + C_+ C_-)}\, \frac{e^{2ika}\, G_-(-k,\theta)}{\beta_n^+ + ik} - \frac{B}{G_+(k\cos\theta_0,\, \theta)\,(\beta_n^+ - ik\cos\theta_0)}\right], \tag{24}$$

$$T_n \approx \frac{(2\pi)^{1/2} i\, e^{i\alpha_n a\sin\theta}}{\alpha_n\cos\theta + i\beta_n\sin\theta}\, \mathrm{Res}[G_-(\alpha,\theta)]_{\alpha = i\beta_n^-}$$

$$\cdot\, \frac{c_0^- D}{2(1 + C_+ C_-)}\, \frac{e^{2ika}\, G_+(k,\theta)}{\beta_n^- - ik}, \tag{25}$$

where $\beta_n^{\pm} = \beta_n\cos\theta \mp i\alpha_n\sin\theta$ and

$$C_\pm = c_0^{\pm}\, e^{2ika}\, \frac{G_+(k,\theta)\, G_-(-k,\theta)}{4k}, \qquad D = \frac{B\, G_+(k,\theta)}{k(1 - \cos\theta_0)\, G_+(k\cos\theta_0,\, \theta)}. \tag{26}$$

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