1-IV C4

MODE CONVERSION OF A GAUSSIAN LIGHT BEAM PROPAGATING THROUGH THE ATMOSPHERE

Masaaki Imai

Research Institute of Applied Electricity, Hokkaido University

Sapporo, Japan

Recently, many studies have been made on the light beam wave propagation through a turbulent medium such as the atmosphere or the air between optical lenses constituting an optical waveguide. Fluctuations of the mean square log-amplitude and phase of a Gaussian beam have been analyzed by means of the Rytov approximation, assuming a Gaussian autocorrelation function or the Kolmogorov theory for the refractive index inhomogeneities of the medium. The fluctuation properties can also be investigated by use of the mode conversion method which shows how the original beam mode is converted from the basic to the higher mode and, consequently , how the Gaussian distribution of the irradiance is deformed. The present paper discusses the mode conversion of a Gaussian light beam propagated through the atmosphere. Computational results show the dependence of the mode conversion on propagation length, beam spot size, and medium turbulence.

Let a Gaussian beam with spot size s and wave front curvature ℓ be radiated from the source: This beam takes

$$\phi_0(\mathbf{r}) = \frac{1}{(\pi s^2)^{1/2}}\,\frac{\exp\left[-\dfrac{\rho^2}{2}\,\dfrac{\frac{1}{s^2}-i\frac{k}{\ell}}{1-\frac{z}{\ell}-i\frac{z}{ks^2}}\right]}{1-\dfrac{z}{\ell}-i\dfrac{z}{ks^2}} \tag{1}$$

where k is the wave number of light in free space, $\rho^2 = x^2+y^2$. The perturbed field is assumed to originate from $\phi_0(\mathbf{r})$ propagating through a turbulent medium. The field can be obtained by use of the first perturbation technique[1],

$$\delta\phi(\mathbf{r}) = k^2\int_0^z dz_1 \iint_{-\infty}^{\infty} dx_1\,dy_1\,\delta\varepsilon(\mathbf{r}_1)\,\phi_0(\mathbf{r}_1)\,\frac{\exp\left[-ik\dfrac{|\rho-\rho_1|^2}{2(z-z_1)}\right]}{4\pi(z-z_1)} \tag{2}$$

Here, $\delta\varepsilon(\mathbf{r})$ is the random component of the dielectric constant, whose mean value $\langle\delta\varepsilon(\mathbf{r})\rangle$ equals zero. Expanding $\delta\phi(\mathbf{r})$ in terms of the higher modes corresponding to $\phi_0(\mathbf{r})$, we obtain the expansion coefficients which represent the occupation of a (p,q) mode in the total perturbed field at the distance z. The expectation value of the mode conversion is

$$\begin{aligned}\langle|C_{pq}(z)|^2\rangle &= \pi^2 k^2(0.033\,C_\varepsilon^2)\,\Gamma\left(p+q-\tfrac{5}{6}\right)\\ &\times\frac{(2p-1)!!(2q-1)!!}{p!\,q!\,(p+q)!!}\int_0^z dz_1\left[\frac{1}{\kappa_m^2}+\frac{s^2}{2}\left(\left(1-\frac{z_1}{\ell}\right)^2+\left(\frac{z_1}{ks^2}\right)^2\right)\right]^{\frac{5}{6}}\\ &\times\left[\frac{\left(1-\frac{z_1}{\ell}\right)^2+\left(\frac{z_1}{ks^2}\right)^2}{\frac{2}{s^2\kappa_m^2}+\left(1-\frac{z_1}{\ell}\right)^2+\left(\frac{z_1}{ks^2}\right)^2}\right]^{p+q}\end{aligned} \tag{3}$$

where $\kappa_m = 5.91/\ell_0$, ℓ_0 being the inner scale of the medium turbulence. C_ε is the structure constant of the dielectric constant whose value varies from 8×10^{-9} to 5×10^{-7} for the atmospheric turbulence. Note that for isotropic turbulence groups of the coefficients with equal propagation constants are meaningful physically. Replacing p+q by r and summing coefficients with same values of r, we obtain the reasonable mode conversion coefficient $\langle|C_r|^2\rangle$.

In Fig. 1, the respective coefficient and total one which are normalized by $\pi^2k^3(0.033\,C_\varepsilon^2)\,\kappa_m^{-11/6}\,\Gamma(1/6)$ are shown for ℓ =10 to 1000 m and s=1.8 to 18 cm provided that the practical values of ℓ_0=1 mm and 10 mm are chosen. The mode conversion for r=1 mode is the greatest of all, i.e., P_1 is larger by one or more order than P_2 or other curves. The total mode conversion $P_t=\sum_{r=1} P_r$ which is defined by summing P_r from r=1 to $r=\infty$, therefore, is similar to P_1. As the beam propagates the mode conversion increases. At a short distance of $z\kappa_m^2/k \ll 10$, i.e., in the ranges of $z \ll \ell$, it increases proportionally to z, and to 8/3 powers of z for $z > \ell$, In Fig. 2, the dependence of the mode conversion on beam spot is shown. As the beam spot size grows the conversion ratio increases with rate of 5/3 powers of s at a short distance. When $z\kappa_m^2/k > 1.0$ the mode conversion decreases regardless of the larger spot size. The critical point is seen at the focal plane of the focused beam as anticipated in the case of a Gaussian autocorrelation model for the random medium[1]. In the nighborhood of $z=\ell$, the conversion ratio drops off rapidly when the spot size is large, whereas the curves for small spot sizes increase monotonically. At a short distance the mode conversion of the Gaussian beam increases like that of a plane wave whose tendency is obtained by use of a Gaussian covariance under the assumtions of the radomly homogeneous medium[1].

The author wishes to thank Dr. T. Asakura for his encouragement and for his critical reading of the manuscript.

Reference

[1] M. Imai, S. Kikuchi, and T. Matsumoto, "Mode conversion of Gaussian light beams propagating through a random medium" Radio Science 5, 1009 (1970).

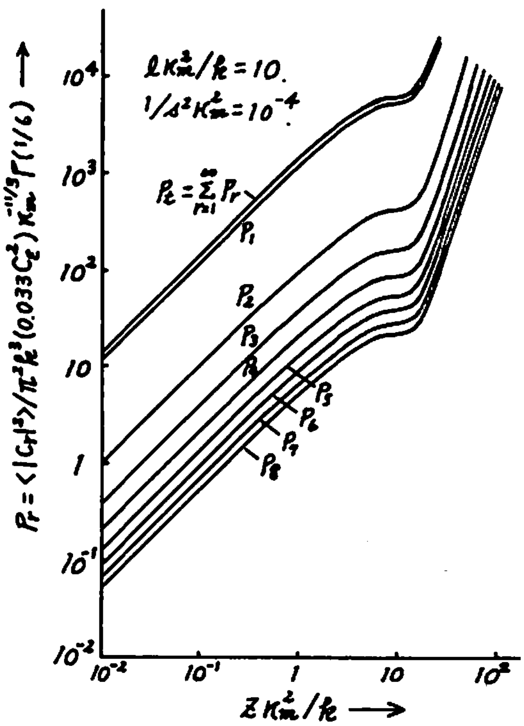

Fig. 1 Respective mode conversion coefficients for wave front curvature of $\ell\kappa_m^2/k=10.0$ and beam spot size of $1/s^2\kappa_m^2=10^{-4}$.

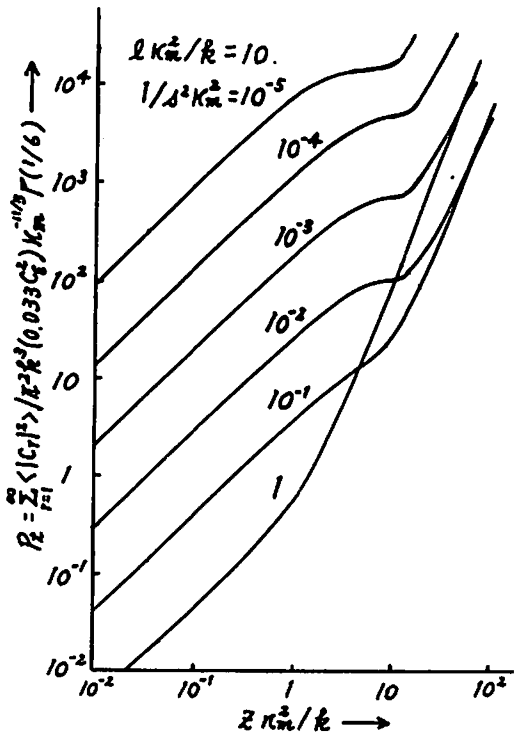

Fig. 2 Total mode conversion coefficients for the wave front curvature parameter of $\ell\kappa_m^2/k=10.0$ and the spot size parameters of $1/s^2\kappa_m^2=10^{-4}$ to 1.