ARTIFICIAL MEDIA WITH LOSSES ON VHF

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Creation of artificial materials with given electrodynamics parameters is of both theoretical and practical interest. The properties of artificial material with the features of anisotropic diamagnetic with dissipation built on the basis of nonmagnetic medium are discussed in this article. The first consideration of such mediums was made in [1,2].

Artificial diamagnetic is an organized structure of small conductive circuits, all of them lieing in parallel planes. The shape of the circuits is of no matter, but the flow of magnetic field through the contour of the circuits must be big enough.

Just for simplification of our analysis we'll consider circular contours. The geometry of the systems shown on fig.1.

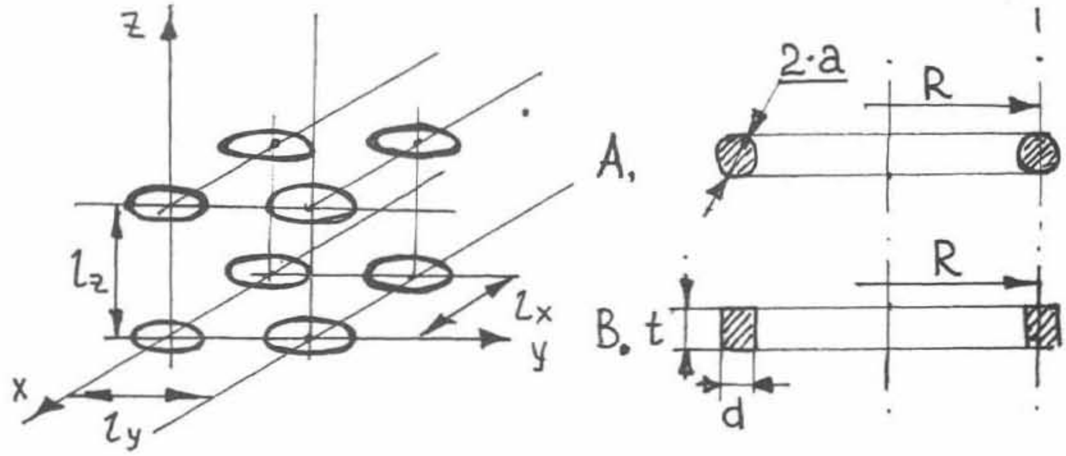

fig.1.

Description of properties of such medium by the local effective electric and magnetic permittivities is possible only if characteristic sizes of the system (R, l_x, l_y, l_z) which define the scale of electric and magnetic field configuration are much less than the scale of variation of average magnetic field [3], i.e. the wavelength, λ.

Presuming, that these conditions ($R \ll \lambda$; $l_x, l_y, l_z \ll \lambda$) are fulfilled, let's consider the case of the media with the relative electric and magnetic permittivities $\varepsilon = 1$, $\mu = 1$, in with the organized structure is allocated (fig. 1.).

The direction of vector of magnetic field form the angle θ with the Z-axis and it's dependence on the time is given by exp ($i\omega t$). Due to electric current, induced in each ring, there appears magnetic moment m:

$$m = I\cdot S = -\frac{j\cdot\omega\cdot\mu_0\cdot H\cdot\cos\theta\cdot\pi^2\cdot R^4}{r + j\cdot\omega\cdot L} = \langle\chi_0\rangle\cdot H \qquad (1)$$

where r - active resistivity of the ring, L - inductance, H - average local magnetic field, χ_0 - average magnetic susceptibility of single ring, $\mu_0 = 4\cdot\pi\cdot 10^{-7}$ [$H\cdot m^{-1}$].

In macroscopic approximation medium filled with magnetic moments, mentioned above is characterized with effective magnetic permittivity. If $\cos\theta \neq 0$, vector of electric field of wave has a component in the plate of the ring, wich is also an electrical dipole. So such structure has also effective dielectrical permittivity. In this article we will consider only magnetic properties of the media.

If currents in rings do not interact, the relative magnetic susceptibility of the media is as following $\langle\mu\rangle = \mu' - j\cdot\mu'' = 1 + \langle\chi\rangle$.

Average magnetic susceptibility an elementary volume of our material is given by $\langle\chi\rangle = \langle\chi_0\rangle\cdot N = \langle\chi_0\rangle\cdot R^{3}\cdot\alpha^{-3}$, where $\alpha = l/R \gg 2$, $l = l_x = l_y = l_z$, N - is density of rings.

As it was mentioned above, the shape of the contours is of no matter. So we'll analyze the rings made of round wire (case A) and plane rings (case B). If the conductor is thin enough (2a) and the conditions of quasi-statics (2b) are fulfilled so:

$$a \ll R, d \ll R, \qquad 2\cdot\pi\cdot R \ll \lambda \tag{2a,b}$$

the inductance of the ring is given by

$$L_a = \mu_0\cdot R\cdot[\mathrm{Ln}\,(8\cdot R/a)-1.75], \quad L_b = \mu_0\cdot R\cdot[\mathrm{Ln}(8\cdot R/d)-0.5] \tag{3a}$$

We'll disregard internal inductance because of (2a). Within the frequency range, defined by the condition of even destribution current density in the conductor

$$a \le \delta = [2/(\omega\cdot\mu_0\cdot\sigma)]^{1/2}, \quad t \le \delta \tag{4a,b}$$

the resistivity of the ring does not depend on the frequency (where t - the thickness of film layer, σ - conductivity).

$$r_a = 2\cdot R/(a^2\cdot\sigma), \qquad r_b = 2\cdot\pi\cdot R/(d\cdot t\cdot\sigma) \tag{5a,b}$$

Let's define border frequency according to conditions (4a,b):

$$\omega_a = 2/(a^2\cdot\mu_0\cdot\sigma), \qquad \omega_b = 2/(t^2\cdot\mu_0\cdot\sigma) \tag{6a,b}$$

Using (3a),(5a),(6a) we'll derive for the ring

$$\mu'_a = 1 - \frac{\Omega_a^2\cdot\pi^2\cdot\xi_a\cdot\cos\theta}{\alpha^3\cdot(1+\Omega_a^2\cdot\xi_a^2)}, \quad \mu''_a = 1 - \frac{\Omega_a\cdot\pi^2\cdot\cos\theta}{\alpha^3\cdot(1+\Omega_a^2\cdot\xi_a^2)} \tag{7a.b}$$

where $\xi_a = \mathrm{Ln}(8\cdot R/a)-1.75$, $\Omega_a = \omega/\omega_a \le 1$.

μ_a'' achieves it's maximum value $\mu''_{a\,max}$ when $\Omega_a = 1/\xi_a$ and it is equal $\mu''_{a\,max} = \pi^2/(2\cdot\alpha^3\cdot\xi_a) \le 0.25$ $(\theta=0)$.

It corresponds to maximum concentration of rings ($\pi^2\cdot\alpha^{-3} \simeq 1$) and minimum value of ξ_a ($\xi_a \simeq 2$). But in this case we have approximate conditions (2a) : $a/R \simeq 0.2$.

The same evaluations yield for circular dicks (fig.1b)

$$\mu'_b = 1 - \frac{A\cdot\Omega_b^2\cdot\pi^2\cdot\xi_b\cdot\cos\theta}{\alpha\cdot(\pi^2 + A^2\cdot\Omega_b^2\cdot\xi_b^2)}, \quad \mu''_b = 1 - \frac{A\cdot\Omega_b\cdot\pi^2\cdot\cos\theta}{\alpha^3\cdot(\pi^2 + A^2\cdot\Omega_b^2\cdot\xi_b^2)} \tag{8a,b}$$

where $\xi = \mathrm{Ln}(8\cdot R/(d+t)-0.5$, $A = d/t$, $\Omega = \omega/\omega \le 1$.

b b b

μ_b'' achieves it's maximum value $\mu_b''_{max}$ when $\Omega = \pi/(A\cdot\xi_a)$ and it is equal

$$\mu''_{b\,max} = \pi^2/(2\cdot\alpha^3\cdot\xi_b) \le 0.17,\ \xi_b \simeq 3,\ d/R \simeq 0.25 \quad (9)$$

If the skin-effect is strong ($a \gg \delta$, $t \gg \delta$, $\Omega_{a,b} \gg 1$) function $\mu(\Omega)$ for the ring of circular wire is given by

$$\mu_{as} = 1 - \frac{4\cdot\pi^2\cdot\Omega_a\cdot\xi_a\cdot\cos\theta}{\alpha^3\cdot(1 + 4\cdot\Omega_a\cdot\xi_a^2)} - j\cdot\frac{2\cdot\pi^2\cdot(\Omega_a)^{1/2}\cdot\cos\theta}{\alpha^3\cdot(1 + 4\cdot\Omega_a\cdot\xi_a^2)} \quad (10)$$

The maximum of μ'' is achieved when $\Omega_a = (2\ \xi_a)^{-2} < 1$. It contradicts the condition of strong skin-effect. In this case

$$\mu_{as} = 1 - \frac{\pi^2\cdot\cos\theta}{\alpha^3\cdot\xi_a} - j\cdot\frac{\pi^2\cdot\cos\theta}{2\cdot\alpha^3\cdot\xi_a^2\cdot(\Omega_a)^{1/2}}\ ,\ \Omega_a \gg 1 \quad (11a)$$

For the plane ring (fig.1.), we derive

$$\mu_{bs} = 1 - \frac{\pi^2\cdot\cos\theta}{\alpha^3\cdot\xi_b} - j\cdot\frac{\pi^2\cdot\cos\theta}{2\cdot\alpha^3\cdot A\cdot\xi_b^2\cdot(\Omega_b)^{1/2}}\ ,\ \Omega_b \gg 1 \quad (11b)$$

If we compare (11b) with (11a) we find,that $\mu_{bs}'' < \mu_{as}''$.

So, plane ring,which is more convenient to be done is worse than the circular ring both for weak and strong skin effect. Values of μ'', which can be achieved are much less for strong skin-effect. Fortunately, usage of such media when $a \gg \delta$, $t \gg \delta$ causes useless increase of weight of metal and density of media, because in the deep-metal of conductive rings there are no electric currents and this part of the rings does not take part in integral dissipation formation. From this point of view, the circular elements for which conditions (2a),(2b), (4a,b) are fulfilled should be used for given frequency range. Technological difficulties arise when realizing rings with the frequencies higher than 1 GHz, and it's expedient to use plane rings of the necessary thickness. From the other side, fulfillment of (4a) with the minimum ξ_a causes R to it's minimum value, maximum concentration of the rings grows up as R^{-3}.

Usage of plane rings with maximum radius with fulfillment the conditions of quasi-statics (2b) allows to minimize the number of rings in the unit of volume with the maximum density and provide fulfillment of condition (4b) by choosing thickness t.

Let's discuss the interaction of currents in rings. There are several ways of consideration of this effect : when the interaction is weak (Lorentz-Lorenz formula), and strong (Onzager formula). Moreover, when the condition of quasi-statics is fulfilled for limited volume , any level of interaction can be taken into account by finding the distribution of the currents induces in the rings. This can be done by solving the system of liner algebraic equations of form

$$Z_k\cdot I_k + j\cdot\omega\cdot\sum_n M_{nk}\cdot I_n = E_{int} = const \quad (12)$$

where I_k -electric current in the ring which number is k, $Z_k = r_k + j\omega L_k$-personal impedance of the ring, M_{nk}-mutual inductance. Such task is extremely complicated even if maximum advantage is taken of the symmetry of the structure. An attempt was made to estimate the effect of interaction in the limited volume of the media filled with small rings directly.

For this purpose we studied the the structure analogical to the one, shown on fig.1. To simplify the calculations we considered $N_x = N_y = 2 \cdot l + 1$ $(l \in N)$. We have made two different numerical experiments. Assuming N_x, N_y, N_z to be constants, we changed l_x, l_y, l_z ($l_x = l_y$ for simplification), and vise a versa, we changed $N_x = N_y$ and N_z with constants standing for l_x, l_y, l_z.

We compared effective relative permittivity with interaction μ_i and without interaction μ_i. Frequency dependence was smoothly and looked like (7b).

A. Increase of l_z when $N_x (= N_y)$ and N_z are fixed leads to decrease of μ_i. This is attributed to the decrease of electrical current density in the unity volume.

It's remarkable, that $\mu_i > \mu_a$ when $l_z > l_x = l_y$ and $\mu_i < \mu_a$ when $l_z \gg l_x$ i.e. the rings are closely packed. This can be attributed to the fact, that in more addendums in equation (12) coefficients Mnk are positive (M_{nk} are negative for the rings, which lie nearby the plane of the k - th ring). That's why the currents, derived from (12) are of less amplitude, than they were if no interaction was taken into account. If l_z is large enough, however the influence of addendums with positive M_{nk} in (12) is less due to decrease of M_{nk} and this leads to increase of electric currents and μ_i''.

It's interesting to note, that the growth μ_i, when l_z is increased is much less, than the decrease of μ_i due to decrease of electric current density in the unity volume. That's why μ_i depends on l_z monotonous.

B. The increase of N_z ($N_x = N_y$=const) with fixed l_x, l_y, l_z leads to the decrease of μ_i. This can be attributed to the increase of the number of positive addendums in the summa (12). Growth of μ_i when $N_x = N_y$ are increased (N_z=const) is due to the number of addendums with negative M_{nk}. This effect is however rather weak, μ_i changes less than when l_z is changed.

Using the artificial diamagnetic which is a cubic system of exclusive rings which do not interact enables obtaining visible magnetic dissipations in given frequency range ($\mu_i'' = 0.2$).

It is shown, that in most cases interaction leads to the decrease of μ_i''.

The values of μ_i can reach 1 in the systems of closely packed rings even when interaction is strong.

It's shown, that the optimum shape of element which is both technological and effective is plane ring.

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