AN APPROXIMATE SOLUTION OF SINGULAR INTEGRAL EQUATION FOR TWO DIMENSIONAL GRATINGS

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Introduction

A scattering analysis of two dimensional gratings is presented. Approximated formula for reflection coefficients are derived by the singular integral equation method. The numerical results for the normal incident TE wave are shown and are checked by the energy error.

Scattering Formula

Consider a thin conductive sheet that is periodically preforated with rectangular aperture. The periods along x and y axisis are a and b, the widths of aperture δ_1 and δ_2, as shown in Fig.1. An incident wave is the TE field with the propagation vector k. In Fig.2, θ is the angle between z-axis and k, angle ϕ is between x-axis and the projection of k on the x-y plane.

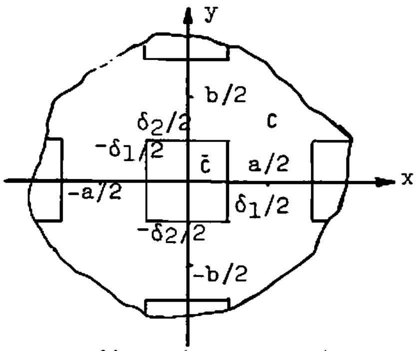

Fig.1 two dimensional gratings

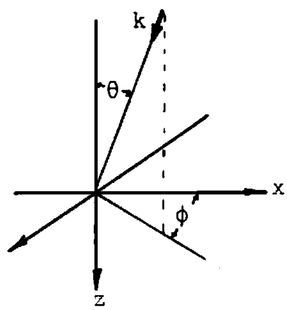

Fig.2 the propagation vector of the TE field

Since the structure is periodic, the Floquet condition must be satisfied. We can write the scalar mode function at z=0 plane as follows. (The $\exp(j\omega t)$ time dependence omitted)

$$g_{mn} = \exp\{-j(U_m x + V_n y)\} \quad \text{where: } U_m = k\sin\theta\cos\phi + \frac{2m\pi}{a} \quad V_n = k\sin\theta\sin\phi + \frac{2n\pi}{b} \qquad (1)$$

So, the incident TE wave on the surface can be expressed as,

$$E^i = (\sin\phi \, i_x - \cos\phi \, i_y) g_{mn} \qquad (2)$$

Because of symmetry of the structure, if we find the m,n-th mode reflection coefficients under the condition of the magnetic wall instead of the aperture, the two dimensional gratings reflection R_{mn} can be obtained by the following formula:

$$R_{mn} = \tfrac{1}{2}(C_{mn} - \delta_{m0}\delta_{n0})N^e_{mn}, \quad T_{mn} = \tfrac{1}{2}(C_{mn} + \delta_{m0}\delta_{n0})N^e_{mn} \qquad \text{for TE mode.}$$

$$R_{mn} = T_{mn} = \tfrac{1}{2} D_{mn} N^m_{mn} \qquad \text{for TM mode.} \quad (3)$$

Here, C_{mn} and D_{mn} are the reflection coefficients for TE wave and TM wave in the condition of magnetic wall. N^e_{mn} and N^m_{mn} are normalizing constants. If we write the scattering waves as E^r and H^r, the boundary condition are written as;

$$H_x = H_x^i + H_x^r = 0 \;,\quad H_y = H_y^i + H_y^r = 0 \qquad (x,y)\in \bar{C}$$
$$E_x = E_x^i + E_x^r = 0 \;,\quad E_y = E_y^i + E_y^r = 0 \qquad (x,y)\in C \tag{4}$$

Using the scalar mode functions the scattered field can be expressed as

$$H_y=\{\cos\theta\sin\phi(1-C_{00})-\frac{\cos\phi}{\cos\theta}D_{00}\}g_{00}-\Sigma\Sigma\left(\frac{V_nW_{mn}}{T_{mn}k}C_{mn}+\frac{U_mk}{T_{mn}W_{mn}}D_{mn}\right)g_{mn}\;,$$
$$H_x=\{\cos\theta\cos\phi(1-C_{00})+\frac{\sin\phi}{\cos\theta}D_{00}\}g_{00}-\Sigma\Sigma\left(\frac{V_nk}{T_{mn}W_{mn}}D_{mn}-\frac{U_nW_{mn}}{T_{mn}k}C_{mn}\right)g_{mn}\;, \tag{5}$$

where $T_{mn}^2=U_m^2+V_n^2 \qquad W_{mn}=\begin{cases}\sqrt{k^2-T_{mn}^2} & \text{if } k\geq T_{mn}\\ j\sqrt{T_{mn}^2-k^2} & \text{if } k\leq T_{mn}\end{cases}$.

Considering the amplitude of C_{mn} and D_{mn}, the convergence of series $\Sigma\frac{V_nW_{mn}}{T_{mn}k}C_{mn}$ and $\Sigma\frac{U_nW_{mn}}{T_{mn}k}C_{mn}$ is poor. We shall transform the infinite series of (0,n) and (m,0) modes into the singular integral equations. By an appropriate manipulation,we find the simultaneous singular integral equations for the scattered electric fields:

$$\begin{cases}\cos\theta\cos\phi(1-C_{00})+\frac{\sin\phi}{\cos\theta}D_{00}-\{\sin\theta\sin\phi\iint K_{11}(x,\xi)G_x(\xi,\eta)d\xi d\eta\\ -\frac{j}{\delta a}\iint K_{12}(x,\xi)\frac{\partial}{\partial\xi}G_y(\xi,\eta)d\xi d\eta\}+\Sigma'\Sigma'\frac{v_nk}{T_{mn}W_{mn}}D_{mn}g_{mn}g_{00}^*\\ -\Sigma'\Sigma'\frac{u_mW_{mn}}{T_{mn}k}D_{mn}g_{mn}g_{00}^*=0\\ \cos\theta\sin\phi(1-C_{00})-\frac{\cos\phi}{\cos\theta}D_{00}-\{j\frac{1}{\delta a}\iint K_{21}(y,\eta)\frac{\partial}{\partial\eta}G_x(\xi,\eta)d\xi d\eta-\cos\phi\sin\theta\iint\\ K_{22}(y,\eta)G_y(\xi,\eta)d\xi d\eta\}-\Sigma'\Sigma'\frac{v_nW_{mn}}{T_{mn}k}C_{mn}g_{mn}g_{00}^*-\Sigma'\Sigma'\frac{u_mk}{T_{mn}W_{mn}}D_{mn}g_{mn}g_{00}^*\\ =0\end{cases} \tag{6}$$

Where: $K_{11}(x,\xi)=f_{11}+\frac{1}{\delta_a(x-\xi)} \qquad f_{11}=\Sigma'\left(\frac{u_mW_{mn}}{T_{mn}^2}+\frac{jm}{m}\right)e^{-j2m\delta_a(x-\xi)}$

$$K_{12}(x,\xi)=f_{12}+\frac{\pi}{ka\delta_a(x-\xi)} \qquad f_{12}=\Sigma'\left(\frac{u_m^2W_{mn}}{2mkT_{mn}^2}-j\frac{m\pi}{ka\ m}\right)$$

If $x-y$, $m-n$, $n-m$, $\xi-\eta$, $\eta-\xi$ in k_{11} and k_{12}, we can get k_{22} and k_{21}. By the boundary conditions we assume that the equations (6)have solutions as follows.[1)]

$$G_x(x,y)=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}Q_{10} \qquad G_y(x,y)=\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}Q_{20} \tag{7}$$

Substituting (7) into (6) we obtain the equation about the undetermined constants Q_{10} and Q_{20}.

$$\begin{cases}\cos\theta\cos\phi(1-C_{00})+\frac{\sin\phi}{\cos\theta}D_{00}+F_1Q_{10}+\left(\frac{j\pi^3}{ka\delta_a^2}+F_2\right)Q_{20}=0\\ \sin\phi\cos\theta(1-C_{00})+\frac{\cos\phi}{\cos\theta}D_{00}-\left(\frac{j\pi^3}{kb\delta_b^2}+F_3\right)Q_{10}+F_4Q_{20}=0\end{cases} \tag{8}$$

where: the series $F_1\sim F_4$ can be evaluated by using Bessel functions.

Using (7), the C_{mn} and D_{mn} also can be expressed by Q_{10} and Q_{20}, so all constants can be determined.

If the TE wave is incident normally, the calculation become very simple. Consequently we get the reflection coefficient of (0,0) mode as following.

$$R_{00} = -4\left(j\frac{\pi^3}{ka\delta_a} + F_2\right)/\left\{2\pi^2+4\left(j\frac{\pi^3}{ka\delta_a} + F_2\right)\right\}^{-1} \qquad (9)$$

Numerical Evaluation

The constant series F_2 are converged quickly because of production of Bessel functions,so the series can be truncated at small number. Fig.3 and Fig.4 give us a comparision with analysis of the mode-matching method (M.M.M) and the singular integral equation (S.I.E). In the calculation of mode-matching method we can not take many mode functions for two dimensional problem, because unknown number would be too many to calculate practically by the computer. Below figure, the truncated number is 3, the energy error is too large. On the other hand, by the singular integral equation method we get answer quickly and correctly. The CPU time for geting the curve is reduced to 18 second that is needed 20 minute for the mode-matching method. From this analysis, we can find the singular phenomenon of scattering, that is called as Wood's anomalies[2]. Considering the balance of energy , the energy error is defined. By the energy error we also can say the analysis of the singular integral equation is better for two dimensional scattering problem.

Conclusion

A approach employing the singular integral equation is used to calculate the reflection coefficients for two dimensional gratings. The numerical examples were made for normal incidence. The calculation time was greatly reduced.

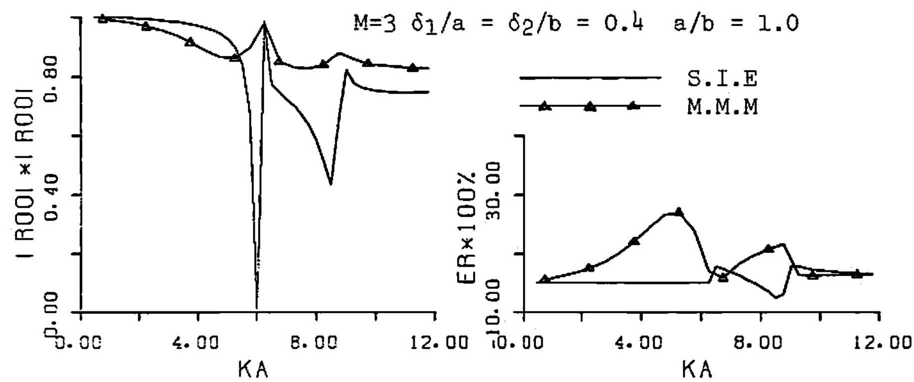

Fig.3 the reflection coefficient R_{00} for example 1.

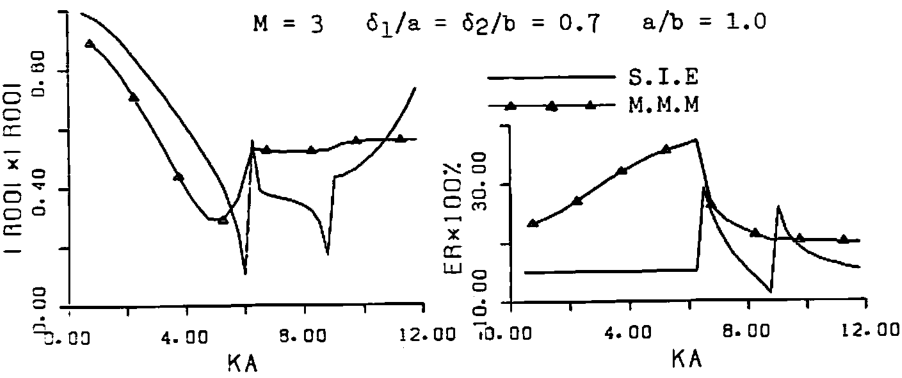

Fig.4 the reflection coefficient R_{00} for example 2.

References

1. L.Lewin: " On the Resolution of a Class of Waveguide Discontinuity Problems by the Use of Singular Integral Equation ",IEE trans., MTT-9, pp.321-332(1961).
2. Hessel and A. Oliner: " A New Theory of Wood's Anomalies on Optical Gratings ", Applied Optics, pp.1275-1297(1965.10).