ANALYSIS OF STRIP GRATINGS BY NUMERICAL SOLUTION OF SINGULAR INTEGRAL EQUATION

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The problem of plane wave diffraction by perfectly conducting strip gratings is analyzed by a singular integral equation formulation followed by Galerkin's method. The numerical results shows that the proposed method gives rather accurate solutions even when the matrix size is relatively small.

Fig. 1 shows the cross section of the grating consisting of perfectly conducting strips of negligible thickness placed on a dielectric slab. Let the incident plane wave be H-polarized, then the total magnetic field is written, using the Floquet expansion, as

$$H_y(x,z)=\begin{cases} e^{i(\kappa_0 z + k_0 x)} + \sum_{n=-\infty}^{\infty} a_n e^{i(\kappa_n z - k_n x)}, & 0 < x \\ \sum_{n=-\infty}^{\infty} (b_n e^{iK_n x} + c_n e^{-iK_n x}) e^{i\kappa_n z}, & -b < x < 0 \\ \sum_{n=-\infty}^{\infty} d_n e^{i[\kappa_n z + k_n(x+b)]}, & x < -b \end{cases} \quad (1)$$

where the time variation $\exp(i\omega t)$ is suppressed and

$$\kappa_n = k \sin\theta_i + 2n\pi/a\,,\ k = 2\pi/\lambda \quad (2)$$

$$\left.\begin{aligned} k_n^2 &= k^2 - \kappa_n^2 \\ K_n^2 &= k^2\varepsilon_r - \kappa_n^2 \end{aligned}\right\}. \quad (3)$$

k and ε_r are the free space wavenumber and the dielectric constant of the slab, respectively.

Imposing the boundary conditions at $x = -b$ (continuity of H_y and E_z), we can express the coefficients b_n and c_n in terms of d_n. The boundary conditions at $x = 0$ (continuity of E_z, vanishment of E_z on the strip, and continuity of H_y at the aperture) yield the integral representation of the coefficients

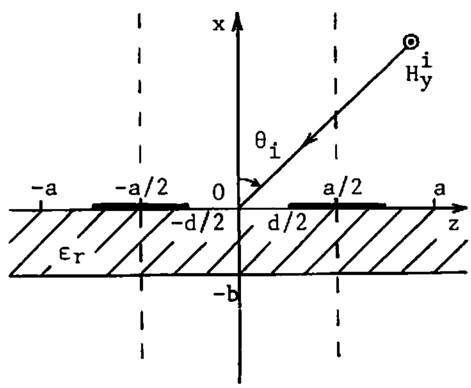

Fig. 1

$$\delta_{n0} - a_n = d_n F_{cn} = \frac{1}{ik_n a}\int_{-d/2}^{d/2} e^{-i2n\pi\eta/a}\, g(\eta)\, d\eta \quad (4)$$

and the integral equation

$$0 = 1 + \int_{-d/2}^{d/2} \sum_{n=-\infty}^{\infty} \frac{-(1+F_{sn}/F_{cn})/2}{ik_n a} e^{i2n\pi(z-\eta)/a} g(\eta)d\eta, \quad |z| < d/2 \tag{5}$$

where

$$g(\eta) = e^{-i\kappa_0\eta} \partial H_y(x,\eta)/\partial x|_{x=0} \tag{6}$$

$$F_{\substack{sn\\cn}} = \cos(K_n b) + i (k_n\varepsilon_r/K_n)^{\pm 1}\sin(K_n b) \tag{7}$$

The singular integral equation is derived as follows. Making use of the change of variables $\theta = 2\pi z/a$, $\phi = 2\pi\eta/a$, and $\bar{g}(\phi)d\phi = g(\eta)d\eta$ to (5) and decomposing the kernel function into the singular part of logarithmic order and the regular part, we obtain

$$0 = 1 + \int_{-\theta_0}^{\theta_0} \left[-\frac{1+\varepsilon_r}{4\pi} \log\frac{1}{2 - 2\cos(\theta-\phi)} + \sum_{n=-\infty}^{\infty} \nu_n e^{in(\theta-\phi)} \right] \bar{g}(\phi)d\phi, \quad |\theta| < \theta_0 \tag{8}$$

where

$$\theta_0 = d\pi/a \tag{9}$$

$$\nu_n = -F_0/(ik_0 a),\ n = 0; \quad -F_n/(ik_n a) + (1+\varepsilon_r)/(4|n|\pi),\ n \neq 0 \tag{10}$$

and $\nu_n = O(|n|^{-2})$ as $|n| \to \infty$. By differentiating (8) with respect to θ and the further change of variables $\xi = \exp(i\theta)$, $\zeta = \exp(i\phi)$, and $G(\zeta)d\zeta/(i\zeta) = \bar{g}(\phi)d\phi$, the following singular integral equation is derived:

$$\frac{1}{\pi i}\int_L \left(\frac{1}{\zeta - \xi} + \sum_{n=-\infty}^{\infty} \sigma_n \xi^n \zeta^{-n-1} \right) G(\zeta)d\zeta = 0, \qquad \xi \in L \tag{11}$$

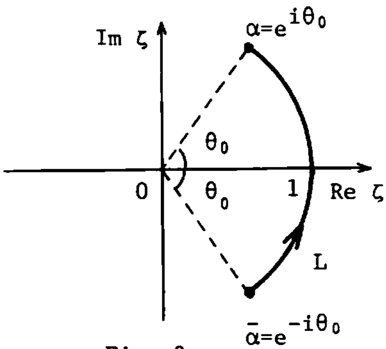

Fig. 2

where the contour L is the arc of a unit circle shown in Fig. 2 and

$$\sigma_n = \begin{cases} -1/2, & n = 0 \\ 2n\pi F_n/\{(1+\varepsilon_r)(ik_n a)\} - \mathrm{sgn}(n)/2, & n \neq 0. \end{cases} \tag{12}$$

An additional condition is necessary because (11) is derived by differentiating the original integral equation. It is obtained by substituting in (5) the specific value of θ, say, $\theta = 0$. This condition is transformed, by the change of variables mentioned above, to

$$\frac{1}{\pi i}\int_L \sum_{n=-\infty}^{\infty} \tau_n \zeta^{-n-1} G(\zeta)d\zeta = 1 \tag{13}$$

where

$$\tau_n = F_n/(ik_n a). \tag{14}$$

We solve the integral equations (11) and (13) by Galerkin's method. Referring the the edge condition, it is convenient to expand $G(\zeta)$ in the form

$$G(\zeta) = \sum_{m=1}^{M} G_m \Psi_m(\zeta) / \sqrt{(\zeta-\alpha)(\zeta-\bar{\alpha})} \tag{15}$$

where the function $\Psi_m(\zeta)$ is the m-order polynomial defined by

$$\Psi_m(\zeta) = \sum_{k=-[(m-1)/2]}^{[m/2]} \psi_k^{(m)} \zeta^k \tag{16}$$

and $\psi_k^{(m)}$'s are orthonormal functions on L determined by the Schmidt's method. We substitute (15) and (16) in (11) and (13). Then multiplying both sides of (11) by $(\pi i)^{-1}\Psi_\ell(\xi)\sqrt{(\xi-\alpha)(\xi-\bar{\alpha})}$ ($\ell = 1,2,\cdots,M-1$) and integrating with respect to ξ over L, we have the linear equations of M unknowns:

$$\left.\begin{aligned} &\sum_{m=1}^{M} (V_\ell^m + S_\ell^m)\, G_m = 0, \quad \ell = 1,2,\cdots,M-1 \\ &\sum_{m=1}^{M} T_m G_m = 1 \end{aligned}\right\}. \tag{17}$$

The matrix elements V_ℓ^m (S_ℓ^m amd T_m) are the finite (infinite) sums and contain the definite integrals which can be evaluated analytically.

Table 1 shows the convergence of the proposed method, compared with the results of the numerical techniques (the point matching method[3] and the mode matching method[4]) as well as the exact solution[1]. Clearly our method provides accurate results even when the matrix size is relatively small, while in other techniques the satisfaction of conservation of power does not mean the exact power distribution.

Fig. 3 shows the reflection power obtained by the present method and the iterative approximation[2]. The latter, reported by one of the authors, takes only the n=1 anomaly (at $k_1a = 0$) into consideration and fails to explain the occurrence of the n=-2 anomaly (at $a/\lambda \doteqdot 1.59$).

Table 1 ($a/\lambda = 1.2$, $d/a = 0.5$, $\varepsilon_r = 1$, $\theta_i = 0$)

Matrix rank	Power Zero ref.	Power Zero trans.	Power First Order	Power Total
5 (Present method)	0.364 491	0.175 660	0.116 253	1.008 904
7 (Present method)	0.370 839	0.170 787	0.114 414	1.000 716
9 (Present method)	0.370 329	0.171 139	0.114 652	1.000 077
11 (Present method)	0.370 381	0.171 103	0.114 628	1.000 004
78 (Ref.3)	0.377 476	0.163 947	0.114 649	1.000 012
41 (ref.4)	0.395 71	0.147 84	0.115 10	1.003 9
Exact (Ref.1)	0.370 309	0.171 169	0.114 631	

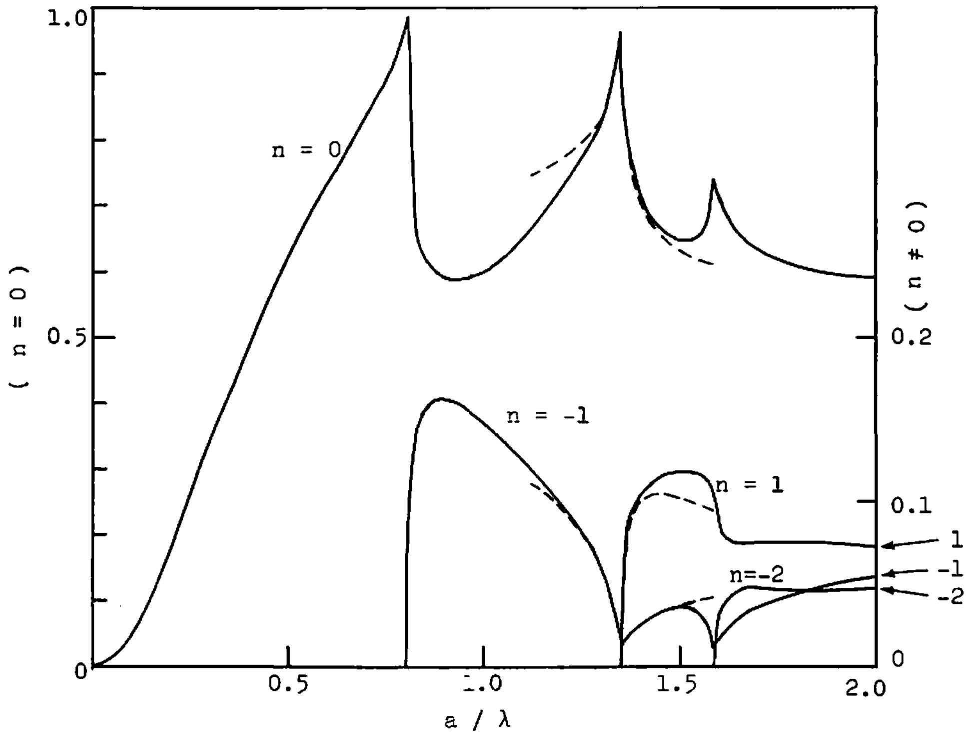

Fig. 3 Reflection Power for d/a = 0.2, $\varepsilon_r = 1$, $\theta_i = 15°$

(—— : present method, - - -:iterative approximation[2]).

References

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