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TEMPORAL FREQUENCY SPECTRA OF MULTIFREQUENCY WAVES IN A TURBULENT ATMOSPHERE

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In the past, temporal frequency spectra of a wave propagating in a turbulent atmosphere have been considered by Tatarski and others for the case of a plane wave. However, recent interest in remote probing and communication problems has created a need for more complete theoretical treatment of the wave propagation in a turbulent atmosphere. Specifically, at millimeter and optical wavelength, there is a need to extend the previous work to include the temporal frequency spectra of a beam wave at multifrequency observations.

This paper presents general formulations for temporal frequency spectra of the fluctuations of a plane, spherical and beam wave operating at two frequencies based on weak turbulence and frozen-in assumptions. The cross spectra and the coherence are obtained for the amplitude fluctuations, the phase fluctuations at two different frequencies, and the amplitude at one frequency and the phase at another frequency. They are expressed as an integral over the distance and the spectral wave number. The results are examined in detail for a plane wave case using the Kolmogorov spectrum. In the limit of large outer scales and small inner scales of turbulence, the temporal spectra are given in terms of the confluent hypergeometric function $\Psi(a,c;z)$ which is independent of the Kummer function. For the spectrum of the index of refraction κ^{-n}, the amplitude spectrum behaves as $k^{(5-n)/2}$ for $\omega \to 0$ and $k^2\omega^{1-n}$ for $\omega \to \infty$. The phase spectrum behaves as $k^2\omega^{1-n}$ for $\omega \to 0$ and $\omega \to \infty$ with different constant.

The amplitude coherence can be shown to approach

$$\left(\frac{k_2}{k_1}\right)^{\frac{n-1}{2}} \left\{ \left[\left(\frac{1+\frac{k_1}{k_2}}{2}\right)^{\frac{n-1}{2}} - \left(\frac{1-\frac{k_1}{k_2}}{2}\right)^{\frac{n-1}{2}} \right]^2 \right\}$$

as $\omega \to 0$ and the phase coherence approaches one as $\omega \to 0$.

The above results are compared with the experimental data obtained by Janes, Thompson, Smith and Kirkpatrick.[1] In their experiments, they used the radio frequencies of 9.6 GHz and 34.52 GHz and the path length L of 64.25 km. The theoretical curve for the ratio of the amplitude spectra approaches $(k_2/k_1)^{5-n/2} = 2.34$ as $\omega \to 0$ and $(k_2/k_1)^2 = 12.85$ as $\omega \to \infty$, and the ratio of the phase spectra shows a wavy variation near $\omega \sim V\sqrt{2\pi/\lambda L}$ where V is the wind velocity. These results show excellent agreement with the experimental data. Also, the amplitude coherence approaches $[0.69]^2$ and the phase coherence approaches one as $\omega \to 0$, which agree well with the experimental data.

The wind velocity is determined from the fluctuation data to be 15.6 knots. Of the data obtained from the National Oceanic and Atmospheric Administration, Hawaii shows that the wind velocity was 14 knots at the time of the experiment, showing close agreement.

The structure constant C_n is calculated from the amplitude fluctuation data to be $8.3 \times 10^{-7}\ m^{-1/3}$ and from the phase fluctuation to be $8.67 \times 10^{-7}\ m^{-1/3}$, showing also the general agreements. These agreements indicate the general validity of the theory for the K and X band over the distance as long as 60 km, and these theories may be used with confidence for cm and millimeter waves.

References

1. H. B. Janes, M. C. Thompson, Jr., D. Smith and A. W. Kirkpatrick, "Comparison of simultaneous line-of-sight signals at 9.6 and 34.52 GHz," IEEE Trans. on Antennas and Propagation, Vol. AP-18, No. 4, pp 447-451, July 1970.