Broadband Antenna Array Pattern Synthesis with specified Nulls Using Eigenvector Constraints

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ABSTRACT

This paper presents and compares the performance of two broadband antenna array pattern synthesis techniques with the capabilities of allowing a designer to specify the frequency response of the processor in the look direction and to place a controlled null anywhere in the pattern. Both techniques are based on an integrated power constraint over the specified null and have the potential for parallel processing to increase the computational speed in real time.

1 INTRODUCTION

The capability of placing a null in an array pattern is useful in suppressing non-stationary unwanted directional interferences whose directions of arrival are moving over time or not known exactly. A number of techniques are available for synthesizing such a pattern for narrowband antenna arrays. Information for these may be found in [1, 2, 3] and references therein.

This paper presents and compares the performance of two techniques to synthesize the antenna pattern of a broadband array. In both cases an array designer is able to place a broad null anywhere in the pattern along with a specified frequency response of the processor in the look direction without using steering delays. The steering delays are normally used to pre-steer an array in the look direction [4]. Both techniques employ linear constraints using eigenvector of a matrix derived from integrating the power response over the specified nulling sector and extend the results presented in [1] for narrowband arrays to the broadband case using the discrete Fourier transform method discussed in [5].

A comparison of the power patterns and the frequency response plots of the beamformer in the look direction as well as in the null direction is carried to show the performance of the two methods.

2 Problem Formulation

Consider a time domain element space broadband beamformer structure consisting of L omnidirectional elements and a tap delay line (TDL) section with N taps and tap spacing T without using steering delays in front of each element [4].

Let $\underline{x}(t-(m-1)T)$ and $\underline{w}_m$, $m=1,\cdots,N$ respectively denote a column of induced signals and array weights at the mth tap. Thus the array output is given by

$$y(t)=\underline{W}^T\underline{X}(t) \tag{1}$$

where

$$\underline{W}=[\ \underline{w}_1^T,\ \ \underline{w}_2^T,\ \ \cdots,\ \ \underline{w}_N^T\ ]^T \tag{2}$$

and

$$\underline{X}(t)=[\underline{x}^T(t),\underline{x}^T(t-T),\cdots,\underline{x}^T(t-(N-1)T)]^T. \tag{3}$$

Two problems are considered.

Problem 1: Find a minimum norm weight vector such that the processor has a specified frequency response in the look direction and the mean-square null depth over the spatial region of interest is below specified level [1].

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Problem 2: Find a weight vector matching to a predetermined weight vector

$$\underline{W}_0 = [\ \underline{w}_{01}^T,\quad \underline{w}_{02}^T,\quad \cdots,\quad \underline{w}_{0N}^T\ ]^T \tag{4}$$

in the least square sense such that the mean-square null depth over the spatial region of interest is below specified level [1].

Algorithm to estimate the weights are presented in the next section.

3 Weight Estimation Algorithms

Assume that the broadband signals are transformed to narrowband signals using DFT, that is,

$$\tilde{\underline{x}}(k) = \text{DFT}\{\underline{x}(nT-(N-1)T)\cdots,\underline{x}(nT-T),\underline{x}(nT)\}\ \ k=0,1,\cdots,(N-1) \tag{5}$$

and processed using narrowband processor structure with weights $\underline{h}(k)$, $k=0,1,\cdots,N-1$ producing output $\tilde{y}(k)=\underline{h}^H(k)\tilde{\underline{x}}(k)$. It can be established [5] that if the time domain output of this structure given by IDFT{ $\tilde{y}(k)$ } is equal to that given by (1) then the following relationship hold:

$$\underline{h}^*(k) = DFT\{\underline{w}_m\} \qquad \begin{array}{l} k=0,1,\cdots,N-1,\\ m=1,2,\cdots,N, \end{array} \tag{6}$$

and for odd N

$$\underline{h}(N-k) = \underline{h}^*(k) \qquad k=0,1,\cdots,\tfrac{N-1}{2}. \tag{7}$$

and

$$\underline{h}^H(k)\underline{S}_0(k) = \tilde{f}_k \qquad k=0,1,\cdots,\tfrac{N-1}{2} \tag{8}$$

where $\underline{S}_0(k)$ is the steering vector in the look direction for the kth bin, $\tilde{f}_k = DFT\{f_m\}$, $k=0,\cdots,N-1$, and f_m, $m=1,\cdots,N$ specify the frequency response of the broadband beamformer in the look direction.

A technique to synthesize a narrowband array pattern with a specified null using eigenvector constraints derived from the integrated power response is presented in [1]. Modifying this techniques for the present case satisfying (8) to obtain $\hat{\underline{h}}(k)$ and transforming these using inverse DFT to obtain $\underline{w}_m$, $m=1,\cdots,N$ results the following algorithm to estimate the solution of the first problem. This is referred to as the minimum norm algorithm.

I. Minimum Norm Algorithm

Assume that frequency bins $k=k_1,k_1+1\cdots,k_2$ correspond to the desired frequency band.

Step1: Calculate $\hat{\underline{h}}(k)$ $k=0,\cdots,\frac{N-1}{2}$ using

$$\hat{\underline{h}}(k) = \begin{cases} \dfrac{\underline{S}_0(k) - C(k)\underline{P}(k)}{\beta(k)} & k=k_1,k_1+1\cdots,k_2 \\[2ex] \dfrac{\underline{S}_0(k)\tilde{f}_k^*}{L} & \text{otherwise} \end{cases} \tag{9}$$

where

$$\beta(k) = L - \underline{P}^H(k)\underline{P}(k), \tag{10}$$

$$\underline{P}(k) = \left[\underline{E}_1^H(k)\underline{S}_0(k), \underline{E}_2^H(k)\underline{S}_0(k), \cdots, \underline{E}_{n0}^H(k)\underline{S}_0(k)\right]^T, \tag{11}$$

$$C(k) = \left[\underline{E}_1(k), \underline{E}_2(k), \cdots, \underline{E}_{n0}(k)\right], \tag{12}$$

$\underline{E}_i(k)$, $i=1,\cdots,n_0$ are eigenvectors associated with ordered eigenvalues $\lambda_i(k)$, $k=1,2,\cdots,n_0$ of matrix $Q(k)$ given by

$$Q(k) = \frac{1}{\Delta\theta}\int_{\theta_\ell}^{\theta_u} \underline{S}(k,\theta)\underline{S}^H(k,\theta)d\theta \tag{13}$$

$\Delta\theta = \theta_u - \theta_\ell$ defines the spatial region of interest $[\theta_\ell,\ \theta_u]$ where a broad null is required, and n_0 is the smallest integer such that the percentage trace of the $Q(k)$ matrix defined by

$$\%trace(k) = \left[\frac{\sum_{i=1}^{n_0}\lambda_i(k)}{\sum_{i=1}^{L}\lambda_i(k)} \times 100\%\right] \tag{14}$$

is greater than or equal some threshold value.

Step2: Calculate
$$\underline{\hat{h}}(N-k) = \underline{\hat{h}}^*(k) \qquad k = \tfrac{N+1}{2},\cdots,N-1 \tag{15}$$

Step3: Calculate the weights of the broadband beamformer $\underline{\hat{W}}$ using
$$\underline{\hat{w}}_m = IDFT\left\{\underline{\hat{h}}^*(k)\right\} \qquad \begin{array}{l} m = 1, 2, \cdots, N \\ k = 0, 1, \cdots, N-1. \end{array} \tag{16}$$

Now an algorithm, referred to as least square algorithm, to estimate the solution of the second problem is presented. It follows from (4) and (6) that the predetermined weight vector for narrowband processor at the kth bin is given by
$$\underline{h}_0^*(k) = DFT\left\{\underline{w}_{0m}\right\} \qquad \begin{array}{l} k = 0, 1, \cdots, N-1, \\ m = 1, 2, \cdots, N, \end{array} \tag{17}$$

Modifying the results presented in [1] to find $\underline{\hat{h}}(k)$ and transforming these to obtain $\underline{\hat{w}}_0$ results the following algorithm.

II. Least Square Algorithm

Step1: Calculate $\underline{\hat{h}}_0(k)$ $k = 0, \cdots, \frac{N-1}{2}$ using
$$\underline{\hat{h}}_0(k) = \begin{cases} \left[I - \underline{C}(k)\underline{C}^H(k)\right]\underline{h}_0(k) & k = k_1, k_1+1\cdots, k_2 \\ \underline{h}_0(k) & \text{elsewhere} \end{cases} \tag{18}$$

where $c(k)$ is given by (12).

Step2: Calculate
$$\underline{\hat{h}}_0(N-k) = \underline{\hat{h}}_0^*(k) \qquad k = \tfrac{N+1}{2},\cdots,N-1 \tag{19}$$

Step3: Calculate
$$\underline{\hat{w}}_{0m} = IDFT\left\{\underline{\hat{h}}_0^*(k)\right\} \qquad \begin{array}{l} m = 1, 2, \cdots, N \\ k = 0, 1, \cdots, N-1. \end{array} \tag{20}$$

4 EXAMPLE AND DISCUSSION

A 20 element linear array with inter element spacing of a half wavelength at the highest desired frequency is taken. A tap delay line filter of 125 taps with tap spacing T is used. The desired normalized frequency is assumed from 0.26 to 0.42 corresponding to frequency bins $33 \leq k \leq 54$. The normalization is carried out with the sampling frequency $f_s = \frac{1}{T}$. The look direction makes an angle of $40°$ with the line of array. The desired frequency response of the filter in the look direction is shown in Figures 1. The nulling sector is chosen to be $[120° - 140°]$ and the value of $\%trace(k)$ is taken to be $= 99.99\%$ for $k = 33, 34, \cdots, 54$.

The conventional weights are used to select the predetermined weight vector $\underline{W}_0$ such that the processor has a specified frequency response in the look direction as shown in Figures 1. The corresponding predetermined weights for the kth bin are given by $\underline{h}_0(k) = \frac{S_0(k)}{L}\tilde{f}_k^*$.

Figure 2 shows the power pattern of the broadband beamformer using weights estimated by the two methods along with that using the predetermined weights. One observes from the figure that the power pattern of the beamformer using the weights estimated by least square algorithm matches closely to that given by the predetermined weights and has a deep null over the spatial region of $[120° - 140°]$. Comparing the power patterns obtained using the two methods one observes that the power pattern obtained using the least square algorithm has a deeper null compared to that obtained using the minimum norm method. A comparison of the power patterns obtained using the two methods in the look direction is shown in Figure 3 and indicates that the processor has a slightly lower gain in the look direction using the least square method compared to that of the minimum norm method.

Figure 1 shows the frequency response of the beamformer in the look direction for both the methods along with the desired response and indicates that in both the cases the frequency response of the beamformer over the band of interest closely matches to the desired response.

Figure 4 shows the frequency response in the direction of $(130°)$ which is in the middle of nulling sector. The reduced response of the beamformer over all the frequency band of interest is clearly visible in the figure.

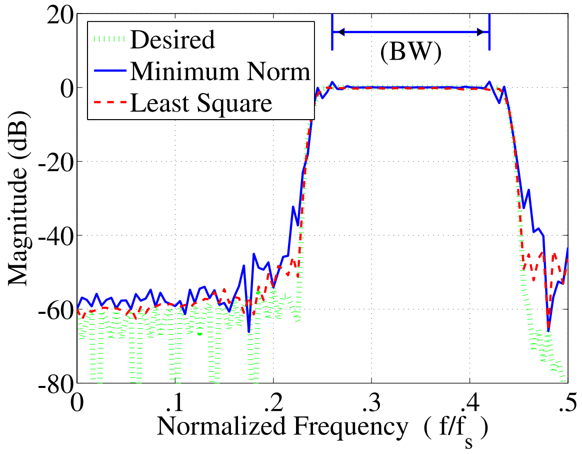

Figure 1: Frequency Response of the beamformer in the look direction.

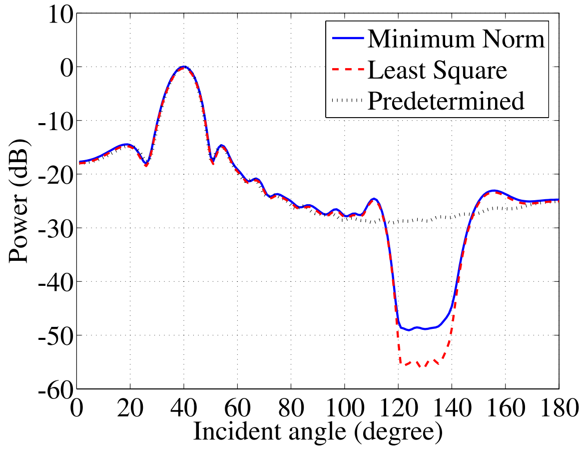

Figure 2: Power pattern of the beamformer.

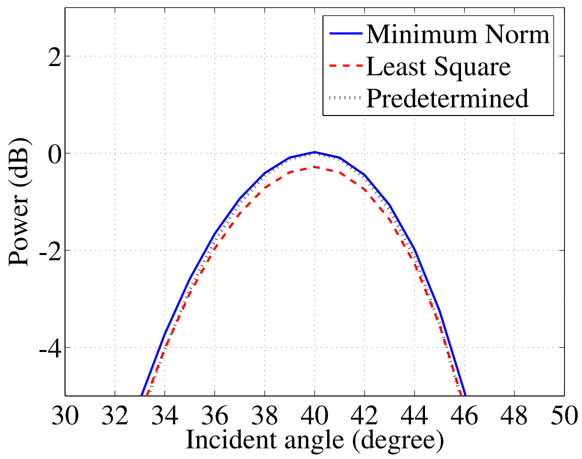

Figure 3: Close Up of Power pattern of the beamformer in the look direction.

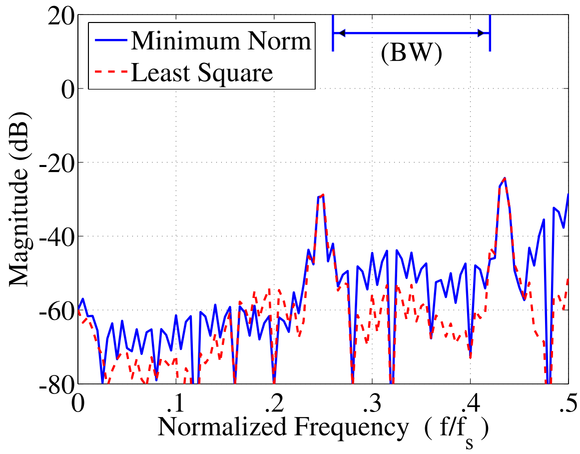

Figure 4: Frequency Response of the beamformer in the 130°.

References

[1] M. H. Er, "Synthesis of antenna array pattern with broad null constraints," *Journal of electrical and electronics engineering, Australia,–IE Aust. & IREE Aust.*, vol. 10, no. 2, pp. 136–145, June 1990.

[2] R.A. Shore, "A sector nulling technique revisited," *IEEE Transactions on Antennas and Propagation*, vol. 49, no. 5, pp. 843–844, May 2001.

[3] D.J. Shpak, "A method for the optimal pattern synthesis of linear arrays with prescribed nulls," *IEEE Transactions on Antennas and Propagation*, vol. 44, no. 3, pp. 286–294, March 1996.

[4] O.L. Frost, "An algorithm for linearly constrained adaptive array processing," *Proceedings of the IEEE*, vol. 60, no. 8, pp. 926–935, August 1972.

[5] L.C. Godara, "Application of the fast fourier transform to broadband beamforming," *Journal of the Acoustical Society of America*, vol. 98, no. 1, pp. 230–240, July 1995.