A NEW TECHNIQUE OF POINT MATCHING METHOD FOR THE PROBLEMS OF SCATTERING BY STRIP GRATINGS

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1. INTRODUCTION

Strip gratings have been used in many devices,such as reflector antenna,spectrum analyzer,and selective surface for solar energy.

Several studies have been made on the analysis of plane wave scattering by planar periodic gratings [Agronovich et al.,1962 ; Neureuther and Zaki,1968 ; Itakura,1968 ; Luneburg and Westpfahl, 1971].However,most of them are mathematically too complicated for computer analysis.

We investigated the problem of scattering by the planar periodic gratings,using the point matching method (PMM) [Hinata and Hosono,1976;Yamasaki et al.,1978].In our previous papers,it was shown that (1) PMM gives correct results when it is used in the convergence domain of modal expansions,and that (2) PMM is quite efficient and has wide range of applicablity.

In the prsent paper,we propose an improved PMM (MPMM : Modified Point Matching Method) which reduces the order of the simultaneous equation at least by half under the condition of the same accuracy as usual PMM.

To check the accuracy of MPMM,the truncation errors of the transmission coefficients are evaluated for increasing the number of matching points. Numerical examples are given for the problems which seem to be quite difficult by other methods.

2. FORMULATION of MODIFIED POINT MATCHING METHOD (MPMM)

Consider a plane wave incident on the planar grating,which is uniform in the z direction and periodic in the x direction (p: period of the grating) as shown in Figure 1.The wave vector of the incident wave lies on the x-y plane and the angle of incidence is θ. The x'-y'plane is perpendicular to wave vector. Let γ be the angle between the incident electric vector and z'-axis.

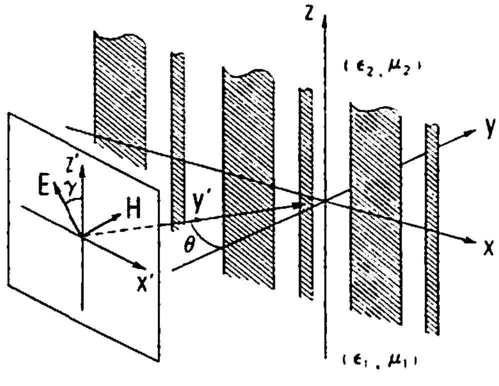

Fig.1 Geometry of the problem.

The z components of scattered waves in the regions y<0 and y>0 can be expanded as follows:

$$y<0; \quad E_z^{(1)}=(E_0\cos\gamma)\phi_0^{(1)}(x,y)+\sum_{n=-\infty}^{\infty} b_n^{(E)}\phi_n^{(1)}(x,-y)$$
$$H_z^{(1)}=(H_0\sin\gamma)\phi_0^{(1)}(x,y)+\sum_{n=-\infty}^{\infty} b_n^{(H)}\phi_n^{(1)}(x,-y) \qquad (1)$$

$$y>0; \quad E_z^{(2)}=\sum_{n=-\infty}^{\infty} a_n^{(E)}\phi_n^{(2)}(x,y),\ H_z^{(2)}=\sum_{n=-\infty}^{\infty} a_n^{(H)}\phi_n^{(2)}(x,y) \qquad (2)$$

,where $\phi_n^{(m)}(x,y) \triangleq \exp\{jk_1 x\sin\theta+jk_{mn}y+j(2\pi n/p)x\}$, $k_{mn}^2 \triangleq k_m^2-\{(2\pi n/p)+(k_1\sin\theta)\}^2$, $k_m \triangleq \omega(\mu_m\varepsilon_m)^{1/2}$, $m=1,2$, $\mathrm{Im}\{k_{mn}\}\geq 0$, $\mathrm{Re}\{k_{mn}\}\geq 0$, $H_0 \triangleq (\mu_1/\varepsilon_1)^{-1/2}E_0$.
The time dependence $\exp(-j\omega t)$ is omitted from the field components.The other tangential field components E_x and H_x can be obtained from Maxwell's equations.
In our MPMM,infinite sum in (1) and (2) are approximated by a finite sum . Furthermore,the approximate values $a_n^{(E)}(N)$,.. ..,$b_n^{(H)}(N)$ (N:trancation mode number) for $a_n^{(E)}$,....,$b_n^{(H)}$ are obtained from the boundary conditions at $(x_q,0)$,where $x_q \triangleq \{p/(2N+1)\}q$,(q=0,1,2,....,2N):

$$\Phi(x_q) \triangleq \sum_{n=-N}^{N}\{P_n c_n(N)u_n(x_q)\}=0, \quad x_q \in C=\{x_q \mid q=j_1,j_2,....,j_M\} \qquad (3)$$
$$\Psi(x_q) \triangleq \sum_{n=-N}^{N}\{Q_n c_n(N)-w_n\}u_n(x_q)=0, x_q \in \overline{C}=\{x_q \mid q=i_1,i_2,...,i_L\} \qquad (4)$$

,where $\{x_q \mid q=i_1,i_2,....,i_L\}$ lie on the gaps and $\{x_q \mid q=j_1,j_2,...,j_M\}$ on the strips (L+M=2N+1).
$c_n(N)$,$P_n(N)$ and so on in (3),(4) are defined as follows:

$$c_n(N) \triangleq \begin{pmatrix} a_n^{(E)}(N)/(2E_0\cos\gamma) \\ a_n^{(H)}(N)/(2H_0\cos\gamma) \end{pmatrix} \quad P_n \triangleq \begin{pmatrix} 1 \\ k_{2n} \end{pmatrix} \quad \delta_{mn} \triangleq 1 \ \ m=n, \ \ \triangleq 0 \ \ m\neq n$$
$$Q_n \triangleq \begin{pmatrix} (k_{1n}/k_{10})+(\mu_1/\mu_2)(k_{2n}/k_{10}) \\ 1+(\varepsilon_1/\varepsilon_2)(k_{2n}/k_{1n}) \end{pmatrix} \quad w_n \triangleq \delta_{n0} \quad u_n(x) \triangleq \exp(j2\pi nx/p)$$

Using the orthogonality relation $\sum_{n=-N}^{N} u_m(x_q)u_n^*(x_q)=(2N+1)\delta_{mn}$ (*:complex conjugate),and the boundary conditions of (3) and (4),we obtain the following equations:

$$\sum_{r=i_1}^{i_L} K_N(x_q-x_r)f_r=F(x_q), q=i_1,i_2,...,i_L \qquad (5)$$

,where $K_N(x) \triangleq \sum_{n=-N}^{N}(Q_n/P_n)u_n(x)$, $F(x) \triangleq \sum_{n=-N}^{N} w_n u_n(x)=u_0(x)$.

The L-dimensional simultaneous equation (5) is solved to obtain $\{f_r\}$,then the expansion coefficients $\{c_n(N)\}$ can be computed from

$$c_n(N)=\{\sum_{q=i_1}^{i_L} f_q u_n^*(x_q)\}/P_n \quad ,n=-N,-N+1,....,N \qquad (6)$$

In similar manner,we can readily derive the following M-dimentional simultaneous equation from (3) and (4):

$$\sum_{r=j_1}^{j_M} \overline{K}_N(x_q-x_r)g_r=G(x_q), q=j_1,j_2,...,j_M \qquad (7)$$

,where $\overline{K}_N(x) \triangleq \sum_{n=-N}^{N}(P_n/Q_n)u_n(x)$, $G(x) \triangleq -\sum_{n=-N}^{N}(P_n w_n/Q_n)u_n(x)$.

The unknown coefficients $\{c_n(N)\}$ are determined from

$$c_n(N)=\{\sum_{q=j_1}^{j_M} g_q u_n^*(x_q)+w_n\}/Q_n \quad ,n=-N,-N+1,\ldots,N \tag{8}$$

using the solutions $\{g_r\}$ of (7).
The normalized total transmitted power $\rho_t(N)$ and reflected power $\rho_r(N)$ are obtained by integrating the y component of the Poynting vector over a period:

$$\rho_t(N)=\sum_{n=-N}^{N}|T_n(N)|^2 \quad , \quad \rho_r(N)=\sum_{n=-N}^{N}|R_n(N)|^2 \tag{9}$$

Here, $|T_n(N)|^2$ and $|R_n(N)|^2$ are the normalized transmitted power and reflected power of n-th mode.
We can prove mathematically that the energy conservation relation $\rho_t(N)+\rho_r(N)=1$ always holds(therefore energy error $\triangleq$ $1-\rho_t(N)-\rho_r(N)=0$), which is a distinct feature of this method.

3. RESULTS

Figures 2 and 3 show the plots of $|R_0(N)|^2$ and $|T_{+1}(N)|^2$ as a function of $1/D_m$ (D_m:the dimention of simultanous equations) including the other results.

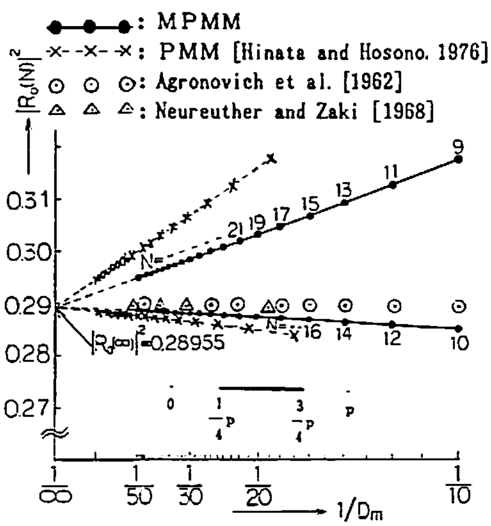

Fig.2 $|R_0(N)|^2$ vs.$1/D_m$.
$\gamma=\pi/2$(H-Polarization), $\theta=0°$
$\varepsilon_2/\varepsilon_1=\mu_2/\mu_1=1, p/\lambda=1.5$.

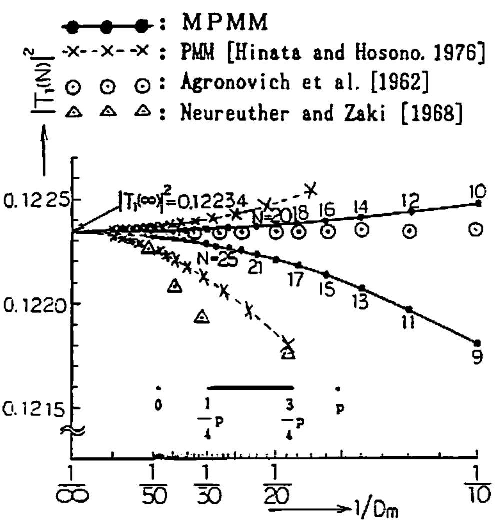

Fig.3 $|T_{+1}(N)|^2$ vs.$1/D_m$.
$\gamma=\pi/2$(H-Polarization), $\theta=0°$
$\varepsilon_2/\varepsilon_1=\mu_2/\mu_1=1, p/\lambda=1.5$.

In computing the scattering characteristics,we always estimate the relative errors from the plots like Figures 2 and 3.
The general results are:

1. The relative errors of $|R_0(N)|^2$ and $|T_{+1}(N)|^2$ by MPMM are less than 0.36% and 0.01% for $D_m \geqq 30$. While $D_m \geqq 60$ is necessary for PMM to attain the same accuracy .
2. The accuracy of MPMM is comparable to that of the algorithm

proposed by Agronovich et al.[1962],which is quite complicated to derive.

Figure 4 shows $\rho_t(N)$ versus $1/D_m$ for the grating with three strips of width p/8 in a period ,which seems intractable by Agronovich et al.[1962] method.In this case, the series of N-values which give less errors are {…,35,31,27,23,…} for $\gamma=0$ (E-polarization) and {…,36,32,28,24,…} for $\gamma=\pi/2$ (H-polarization).In Fig-ure 5, numerical examples of the normalized transmitted power ($\gamma=\pi/2$, $\theta=0$, $\varepsilon_2/\varepsilon_1=2$) are shown for the gratings with three stripes in a period, which also seems difficult to be analyzed by other methods. Wood anomalies occur at the wave number $k_1^{(n)} = |n| / \{(\varepsilon_2/\varepsilon_1)^{1/2} - (n/|n|)\sin\theta\}$ where n-th space harmonic begins to appear.Figure 5 shows that the structure in a period of the grating influnces greatly on the behavior of Wood anomalies.

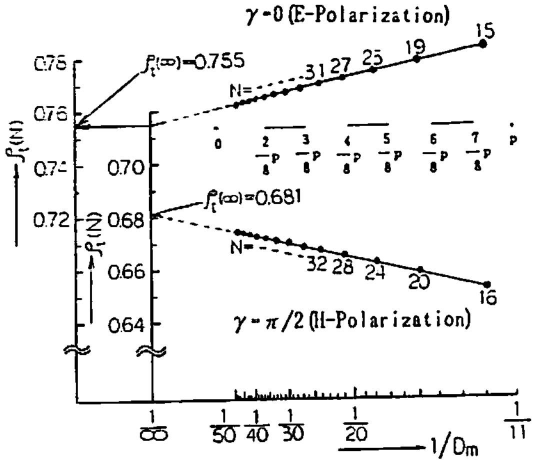

Fig.4 $\rho_t(N)$ vs.$1/D_m$.
$\theta=0°$,p/λ=3.5, $\varepsilon_2/\varepsilon_1=2$, $\mu_2/\mu_1=1$.

4. CONCLUSION

We proposed a modified point matching method (MPMM) and showed that it is quite useful for improving the computational acuuracy. This method can be applied to many other important problems such as the scattering from cylindrical lamellae.

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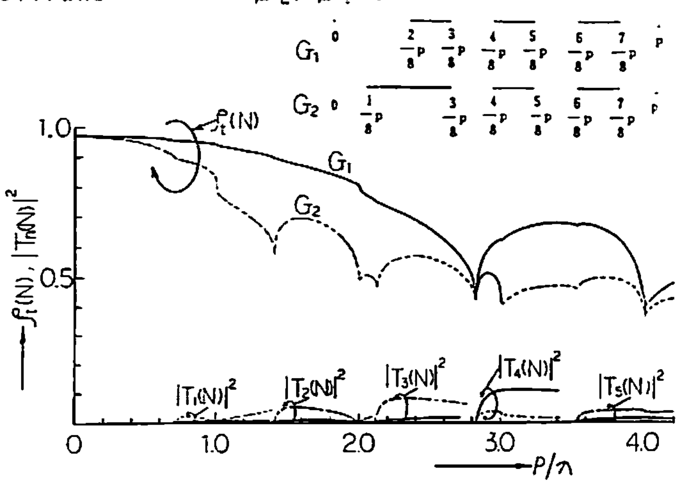

Fig.5 $\rho_t(N)$ and $|T_n(N)|^2$ vs.p/λ
$\gamma=\pi/2$ (H-Polarization), $\theta=0°$,
$\varepsilon_2/\varepsilon_1=2$, $\mu_2/\mu_1=1$.