High Resolution Timing Vector for FPGA Implementation of a Phased Array DBF

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Abstract

Efficient digital phased array beams require a high resolution timing vector. The timing coefficients cause speed bottleneck for FPGA implementation of beam forming in the HF frequency range. In this paper we present polyphase interpolator to overcome the speed limitation of FPGA implementation of digital beamforming (DBF) for ionospheric radar. The radar requires a constant phasing vector to steer the beam over a specified region of the ionosphere. Sixteen digital beams are derived from the vector with beam resolution of approximately three degrees. The phasing weights can be employed either in the time domain or in the frequency domain. Comparison of phase delay and time delay methods is presented for broadband frequencies of the radar. Performance of the proposed DBF system is presented using clock efficiency and FPGA implementation.

Keywords

Field Prgorammable Gate Array (FPGA), Digital Beamforming (DBF), Tasman International Geospace Environment Radar (TIGER).

1 Introduction

With the introduction of fast reconfigureable devices and technology, it is now possible to realize beamforming techniques in a Field Prgorammable Gate Array (FPGA) with much accuracy and ease. Digital Beamforming (DBF) has many advantages over conventional analog phasing including flexible antenna patterns, multiple beams without any degradation, and closely spaced beams [4]. To add this flexibility to the radar we investigate hardware efficient algorithms and evaluate their implementation.

Tasman International Geospace Environment Radar (TIGER) is one component of the SuperDARN network that analyzes the space weather in southern hemisphere. The phased array pulses are modulated using analog heterodyne operations with limitations of the analog inflexibility. Currently we are investigating different methods to implement this radar onto reconfigurable devices. Most of the radar would be in the digital domain and configuration could be changed on the fly.

In section 2 TIGER interferometer is described where fixed beams are produced with a predefined direction. The radar phasing is based on constant time delay. This is because beam direction is fixed even with variable frequency range. A comparison of time and phase delay beamforming is addressed in section 3. A high resolution timing vector is introduced in section 4 using oversampling filter. The timing resolution is directly proportional to the oversampling ratio and requires reduced FPGA clock. This performance measure is described in section 4.1. FPGA implementation of the proposed method is discussed using word serial bit parallel method. Performance of the proposed method is addressed using transmitted angle resolution in section 5. In the last section conclusions are drawn.

2 Antenna Geometry for TIGER

The phasing of signals extends the transmission or detection range of a transmitter or receiver. Phased array antennas are used to steer a narrow beam over an arc from a fixed antenna geometry. For each transmitter or receiver the direction is adjusted with systematic phase delays to each of the antennas in the array. In the TIGER antenna array, the antenna elements are arranged with uniform spacing, as shown in Figure 1, where d is inter-element spacing, θ is angle normal to modulated wave. It is obvious that the modulated wave by element N will be delayed by a differential distance of $d\sin\theta$ compared with a wave at element N+1. If we consider the phase of the transmitted signal is zero at the origin, then the phase lead at element N to that element at origin is Nkd sin θ , where $k = 2\pi/\lambda$ is a space constant. The operational frequency of the HF radar is 8 to 20 MHz, therefore the wavelength varies using the relationship $c = f\lambda$, and c is the velocity of light in the vacuum. The number of array elements and a space between them determine the beamwidth and size of sidelobes [5].

The angle resolution of the TIGER phasing array can be calculated using antenna geometry shown in Figure 1. The time delay required to introduce a half beam step of 1.5 degrees off center for the first beam is 1.3nsec for our 15.24 meter antenna spacing. Successive antennas in the array must be stepped by the same 1.3nsec for this beam. The second beam will be a full beam step of 4.5 degrees (the initial 1.5 degrees plus the 3 degrees beam step size) and antennas will

need a time delay of 3.9nsec with 3.9nsec time step between successive antennas.



Figure 1 (a) A generic linear phased array (b) constant differential distance in the TIGER antenna geometry.

The phase delay could be implemented in the frequency domain or in the time domain. In the following section, we compare both techniques.

3 Comparison of Phase and Time Delay

Beamforming

In the literature, beamforming methods are broadly categorized as weighting coefficients in the time domain and in the frequency domain. In this section we demonstrate that the time domain method is more suitable for constant beam span of the radar [3]. Simulation results were obtained using the geometry displayed in Figure 1 and based on isotropic radiators. In the current system uniform distribution is used for transmission; therefore results are presented for this type of pulsed transmission.

3.1 Phase Delay Beamforming

We start with the frequency domain methods, where a phase delay is inserted for beam steering. The array factor for the phased array geometry can be written as [4]

Array Factor =
$$\sum_{n=0}^{N-1} \exp jn\left(\frac{2\pi d}{\lambda}\sin\theta + \alpha\right)$$
(1)

and phase delay α for transmitted frequency f_0 can be defined as

$$\alpha = \frac{2\pi d}{\lambda_0} \sin \theta_0 \tag{2}$$

The electrical spacing d/λ increases with frequency and therefore beamwidth becomes narrower at higher frequencies. This effect is shown in Figure 2 when the operational frequency is varied from 8MHz to 20MHz. For a signal modulated at 8MHz, the first null beamwidth (FNBW) is 5.5 degrees and at 20MHz this becomes 2 degrees. In the frequency dependent method, the phase delay introduces overlap at lower and higher frequencies as the phase delay that produces the correct beam at lower frequencies will not work for high frequencies. In order to produce equivalent beams at different frequencies, the phase delay must be varied with frequency.



Figure 2 Phase delay beam steering (a) First Null Beamwidth (FNBW) is 5.5 degrees at 8MHz, and (b) FNBW is 2.0 degrees at 20MHz.

A close view of the above beam patterns reveals that with the phase delay technique, direction of the beams is changed at different frequencies. A larger picture of the above results is illustrated in Figure 3 comparing second beam from the bore sight. For constant phase weights, change in the frequency causes squint in the beam direction.



Figure 3 Effect of phase delay beam steering

From above discussion it is concluded that the phase delay beamforming is not suitable for broadband application such as TIGER. In the next section we discuss phasing vector using time delay weights.

3.2 Time Delay Beamforming

For time delay beam steering, a constant delay is used for a broad range of frequencies. The array factor can be written as

Array Factor =
$$\sum_{n=0}^{N-1} \exp jn\left(\frac{2\pi d}{\lambda}\sin\theta + \omega\Delta t\right)$$
(3)
where time delay is
$$\Delta t = \frac{d}{c}\sin\theta_0$$
(4)

An example is shown in Figure 4 demonstrating the time delay beamforming at lower and higher frequencies. The output beam span is of similar form to Figure 2 with the exception of two factors. The first is less overlap error at higher frequencies at the output of the antennas [3]. Secondly, comparison of Figure 2 and 4 reveals that the time domain method has a constant phase span at lower and at higher operational frequencies. The time delay technique introduces a frequency dependent phase delay so that at any particular frequency, identical results are produced.

For the time domain approach, sets of time delays are required for each beam direction and these sets of delays are constant for all frequencies. This means that for a sixteen element array with sixteen beam directions, sixteen sets of sixteen time delays are required. It should be noted that left/right symmetry of the beam scanning can be included so that delays can be shared for beams left and right of the bore sight. In this way eight pairs of eight delays can generate all sixteen beams for the sixteen antennas.



Figure 4 Time delay beam steering (a) FNBW is 5.5 degrees at 8MHz, and (b) FNBW is 2.0 degrees at 20MHz.

Now similar to the frequency dependent method, a simulated result is shown in Figure 5 comparing beam direction at lower and higher frequencies. Since frequency is inversely proportional to the wavelength therefore beam becomes narrower at high frequency. However direction of the beam is identical to lower frequency beam.



Figure 5 Effect of time delay beam steering

Hence our discussion concludes that the time domain beam steering is better approach for the radar operation. In the next section a new hardware efficient method is proposed to generate phasing signals for the radar in the digital domain.

4 Proposed DBF Method for High Resolution

Timing Vector

In the simplest form, the DBF can be introduced using fixed clock cycles for each channel. In this way, a time delayed version of the signal is readily available using a set of ripple latches which are sequentially clocked [1]. The drawback of this technique is higher clock requirement which is not feasible in current FPGAs. To overcome the clock rate limitations of the latch method, multirate filters can be used. Multirate filters provide higher sampling rate than the input clock.

A proposed method is illustrated in Figure 6 employing a polyphase interpolator to distribute the digital delay in each channel. The phase delay is also dependent on the fixed phased array antenna geometry shown in Figure 1, the antenna spacing is 50 feet or 15.24 meters. The synthesized RF signal is divided into parallel channels. The effect of digital quantization can significantly be reduced using an analog reconstruction filter (ARF) after the D to A [2]. The phased RF signal is passed through the transmitter/receiver switch and sent to the antenna for transmission.



Figure 6 proposed DBF network using polyphase interpolator.

A polyphase filter is concurrent realization of FIR filters providing higher clock rate and also retaining the linear phase response of the FIR models. The proposed method exhibits a number of advantages. Firstly this method is computationally efficient since reduced processing is required for inserting zeros. Secondly a higher sampling rate means that the time delay can be added with higher accuracy. Thirdly the polyphase filter is an equivalent realization of padding zeros and digital filtering, thus more computations can be saved. Since the polyphase interpolator is based on symmetric FIR filters, some efficiency can also be achieved by exploiting its symmetric property.

4.1 Performance of the Proposed Method

The performance of the proposed method can be calculated using angle resolution and required clock speed. The beam resolution is dependent on the input clock rate and clock requirement is considerably higher. For example, using a latch method the clock must run with a 1.3nsec period (or approximately 800MHz) to introduce precision of less than 1.5 degrees. A graph is plotted in Figure 7, showing the required clock speed against beam resolution. In comparison, the required clock speed is considerable reduced when timing weights are introduced using polyphase filters. The clock speed is inversely proportional to the number of subfilters in the polyphase network. From the graph, it is evident that the resolution is almost twice for five subfilters network than of a two subfilter realization. The clock speed is based on input clock signal of 100MHz to the polyphase network and multiplexing delay of sixteen clock cycles. For FIR filters of 100 taps, the clock delay is fifty cycles exploiting symmetric properties. The delay is reduced to twenty five cycles when the FIR is realized as polyphase network of two subfilters. Similarly for five subfilters the clock skew is ten clock cycles.

For the latch method a clock doubler is required to introduce an average resolution of 1.5 degrees with the Virtex II device, where maximum clock speed is 420MHz. Using the proposed high resolution method, the beam resolution of less than a degree can easily be achieved with a standard clock speed of an FPGA.



Figure 7 clock speed required in the proposed method. 5 FPGA Implementation using Word Serial Bit

Parallel (WSBP) Method

DBF can be synthesized onto an FPGA to introduce the flexibility of beam direction and beam resolution. A proposed DBF system is presented in the last section, now we will discuss simulation and synthesis of the phased array system. First we discuss word serial bit parallel method (WSBP).

In a serial arithmetic method, data and filter coefficients are processed through single shared resources. The computations are performed in integer form dealing with each input sample as a block of bits; another way of representing this scheme is by the use of the WSBP method. The output sample rate depends on computational duration of a signal path. For conventional symmetric applications the length of the signal path is half compared with non-symmetric samples.

As stated earlier, an interpolation filter is a better option for synthesizing the time vector. In this section we study a performance of serial arithmetic methods for a DBF system with sampling clock of 100MHz. The performance of serial polyphase interpolator can be compared using sampling delays added to the actual phase delay for beam steering. For an increased sampling rate the angular error is reduced since a greater number of samples is used for a given beamdwidth. From equation (2) we can write the differential delay of the wave front from one antenna element to second element as

$$\sin \theta = \frac{\lambda}{d} = \frac{delay in wavelength}{antenna spacing}$$
(5)

For a digitally sampled signal, a sample delay can be included as [6]

$$\sin \theta = \frac{\lambda}{d} = \frac{(sample period + delay)in wavelength}{antenna spacing}$$
(6)

A sampling effect can be derived from the above two expressions and utilizing $c = f\lambda$

sampling effect =
$$\frac{c}{f_s d}$$
 (7)

From this expression it is concluded that sampling effects on beamforming can be reduced employing a faster sampling clock, so accurate control of the delay may be established.

Performance of the single and parallel polyphase filters was simulated for transmitted beam angles as shown in Figure 8. The angular error is calculated for a sample delay and multiplexing delay. The multiplexing delay is result of converting serial RF signal into parallel antenna array. The graph is drawn for a polyphase structure using two subfilters. The input sampling rate to the beamforming system is 100MHz and spacing between the antennas is 15.24m. The angular error caused by the single polyphase model is almost eight times than that of the parallel polyphase filters.



Figure 8 performance using serial arithmetic polyphase filters

When synthesized onto Virtex II, single polyphase interpolation timing vector requires 200 slices, since each component of the 100 taps FIR realization is implemented concurrently. For parallel polyphase network containing two subfilters, almost two fold logic resources could be required compared with the single polyphase model.

6 Conclusions

In this paper we have presented a constant time delay vector to scan ionospheric irregularities. The phase delay for sixteen channels can be introduced in the time domain or frequency domain. However time domain approach is more useful for TIGER operation. The constant time vector generates almost constant beam span at lower and higher frequencies and beam direction is also constant. A reconfigurable DBF is proposed using a high resolution timing coefficients. The timing vector is based on concurrent filter structure and can produce high clock efficiency with higher number of subfilters.

7 References

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