NONLINEAR EFFECTS FOR FMS WAVES PROPAGATING IN MAGNETIZED PLASMA

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For the fast magneto-sonic (FMS) waves, which are excited in magnetized plasma with $4\pi nT/B^2 \ll 1$, and $\omega < \omega_B$ with $\omega_B = eB/Mc$, and with the conditions of $k\lambda_D \ll 1$, $k_x^2 \gg k_\perp^2$, and $v' \ll c_A$ being carried out, the dispersion relation can be written in the form of

$$\omega \approx c_A k_x \left[1 + k_\perp^2/k_x^2 + \chi(\theta)\, \lambda_D^2 k_x^2\right] \tag{1}$$

where $c_A = B^2/4\pi nM$ is Alfven velocity, v' is x-component of ionic velocity, $\lambda_D^2 = r_B^2/2 = T/2m\omega_B^2$, and θ is an angle between $\vec{k}_x$ and magnetic field $\vec{B}$. An insignificant rate of dispersion means that the principal process is a triwave one for the small amplitude waves. At this, the weak nonlinearity condition determines a small angle value between interacting waves. The FMS waves structure depends on the dispersion coefficient

$$\beta = -c_A\, \chi(\theta)\, \lambda_D^2 = c_A \frac{c^2}{2\,\omega_{oi}^2} \left(\frac{m}{M} - \mathrm{ctg}^2\theta\right)$$

sign which is defined by θ angle value, namely: for the near-to-transverse propagation the dispersion is negative at $|\pi/2 - \theta| \leqslant (m/M)^{1/2}$, and it is positive outside this cone.

To describe FMS waves of small amplitude having the close angular distribution the Kadomtsev-Petviashvili (KP) equation is correct, moreover, for $\beta < 0$, the self-focusing is observed [1]. However, near the cone of $\theta = \mathrm{arctg}(M/m)^{1/2}$ where $\beta \to 0$ the results obtained in [1] are needed in closer definition so far as relation (1) must be supplemented the dispersion term of the next order playing here a principal role [2,3]. This term has a form of γk_x^5 with

$$\gamma = c_A \frac{c^4}{8\omega_{oi}^4}\left[3\left(\frac{m}{M} - \mathrm{ctg}^2\theta\right)^2 - 4\,\mathrm{ctg}^4\theta\,\left(1 + \mathrm{ctg}^2\theta\right)\right].$$

In this case, the dispersion's character becomes more complicated and it is defined by correlation of signs of β and γ (see fig. 1). Thus, for $\beta > 0$, $\gamma < 0$ the case of negative dispersion takes a place (region B in fig. 1), and, for $\beta > 0$, $\gamma > 0$ (region A) and $\beta < 0$, $\gamma < 0$ (region C), the cases of "mixed" dispersion (when the dispersion sign is different for small and big k) are. At this, the propagation of the FMS waves of small amplitude having the close angular distribution will described by the KP equation generalization - Belashov-Karpman (BK) equation obtained in [2,3], which, for the nondissipative case, has a form of

$$\partial_x\left(\partial_t h + \alpha h \partial_x h + \beta \partial_x^3 h + \gamma \partial_x^5 h\right) = -(c_A/2)\,\Delta_\perp h \tag{2}$$

where $\alpha = (3/2)c_A \sin\theta$, and $h=B_\sim/B$ is the dimensionless FMS wave amplitude ($B_\sim$ is the wave's magnetic field). The nonlinear term's form as $\alpha h \partial_x h$ is a consequence of the sound velocity renormalization and reflects a little of probability of other nonlinear processes caused by the vector nonlinearity.

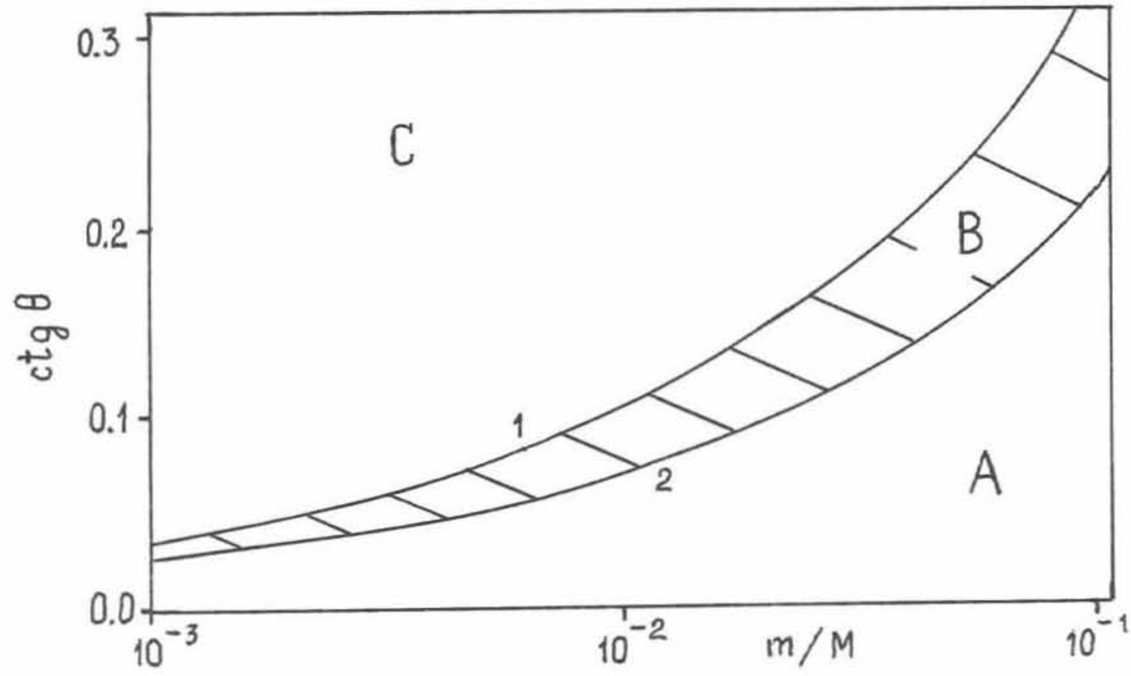

Fig. 1. Dispersion's character near to cone of $\theta = \mathrm{arctg}(M/m)^{1/2}$: 1 - $\beta=0$, 2 - $\gamma=0$.

Let us consider the three-dimensional stationary FMS waves' beam propagating in plasma to magnetic field $\vec{B}$ at angle θ near the cone of $\theta=\mathrm{arctg}(M/m)^{1/2}$. For such beam, using the transition to the new variables as $x \to -st$, $y \to -s\kappa^{1/2}y$, $z \to -s\kappa^{1/2}z$, $t \to sx$, $h \to -(6/\alpha)h$, $s=|\gamma|^{1/4}$, $\kappa = c_A/2$, we obtain from BK equation (2) the equation [4]

$$\partial_t(\partial_x h + 6h\partial_t h - \varepsilon\partial_t^3 h - \lambda\partial_t^5 h) = \Delta_\perp h \qquad (3)$$

describing the FMS waves beam propagation along the x axis from boundary x=0. Here $\varepsilon = \beta s^{-2}$, $\lambda=\mathrm{sign}(\gamma)$.

Equation (3) was being solved numerically using the methods developed in [5] in axial-symmetric geometry for $\Delta_\perp = \partial_\rho^2 + (1/\rho)\partial_\rho$, $\rho^2=y^2+z^2$ under the condition of the total absorption at the integration's region boundaries (with a due account of $\partial_\rho h|_{\rho=0} = 0$) and with the boundary condition in x=0 in the form of $h(0)=h(t,0,\rho)=\cos(1t)\exp(-\rho^2)$ localized in y,z plane and setting the time-periodic axial-symmetric FMS waves beam.

A series of numerical experiments on the FMS waves beam propagation's study at different values of the beam intensity at x=0 and different θ (cases A, B, C) enabled us to obtain the following results. In region A where $\lambda=1$, $\varepsilon > 0$, for any h(0) values the spatial evolution of FMS beam leads at first to the beam focusing, that is defined by dominant role of nonlinear processes at the beginning. At this, we observed (see fig. 2, curves 1,2) the beam compression along the ρ axis with propagation of one along the x axis: $l_\rho(x) \sim l_\rho(0) \cdot h(0)/h(x)$, with simultaneous fast increasing of beam intensity in axis:

$$h(x) \sim h(0)\left[1 + (3.59x)^{2.3}\right].$$

Further, in $x \sim 1$ (that depends on $\mathcal{E}$ value), the nonlinearity "saturation" mode is realized on account of decrising of l_ρ when the term proportional to $\partial_t^5 h$ becomes to play dominant role. That leads to the stopping of self-focusing. With further propagation the defocusing stage comes. It comes to its close by formation of stationary beam (i.e. $h_{max}(x)$=const, $l_\rho(x)$=const), that corresponds the results obtained in [6] analytically.

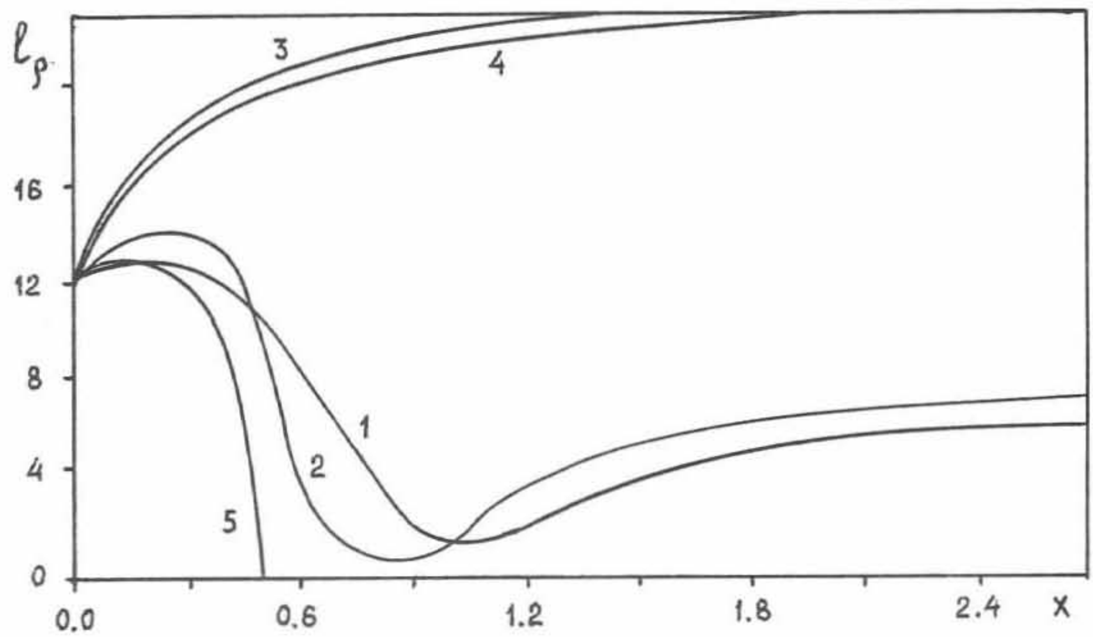

Fig. 2. Changing of l_ρ section of beam propagating along x axis: 1 - λ=1, $\mathcal{E}$=1.34; 2 - λ=1, $\mathcal{E}$=2.24; 3 - λ=-1, $\mathcal{E}$=1.34; 4 - λ=-1, $\mathcal{E}$=-1.34; 5 - λ=0, $\mathcal{E}$=-1.34.

In regions B and C answering the coefficients values λ=-1, $\mathcal{E} \gtrless 0$, we observed the sound scattering with propagation along x axis for any beam intensity h(0) (see curves 3,4 in fig. 2).

The test miscalculations for eq. (3) with λ=0 showed that the self-focusing is possible only for $\mathcal{E} < 0$ (see fig. 2, curve 5), that is in conformity with a result obtained in [1] for the KP equation.

Thus, our results show that, for the FMS waves beam propagating in plasma to field $\vec{B}$ at angles Θ near the cone of $\Theta = \mathrm{arctg}(M/m)^{1/2}$, the self-focusing phenomenon is not observed even if the dispersion for the small k is positive. At this, on a level with sound scattering the nonlinear stationary propagation may be observed. Let us note, that for $|\pi/2 - \Theta| \gg (m/M)^{1/2}$ considered eqs. (2),(3) must be supplemented the terms being proportional to the mixed derivatives because there is $k_\perp \gg k_x$ here, and proper terms being proportional to $k_x^i k_\perp^j$ with $i,j=1,2,\ldots$ appear in dispersion relation.

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References

1. Manin D.Yu. and Petviashvili V.I., Pisma v ZhETF, 1983, 38, 427.
2. Belashov V.Yu. and Karpman V.I., Preprint of IZMIRAN No.43. Moscow, 1990.
3. Karpman V.I. and Belashov V.I., Phys. Lett., 1991, 154A, 131.
4. Belashov V.Yu., In: IX Seminar po ONCh izlucheniyam. Tezisy Dokl. Moscow: IZMIRAN, 1991, 42.
5. Belashov V.Yu., Preprint of IKIR. Magadan, 1989. See also: Belashov V.Yu., J. Comp. Phys., 1992 (in press).
6. Belashov V.Yu., Dokl. Acad. Nauk SSSR, 1991, 320, 85.