

Application of Divided-fitting Method Based on Coons Surface in Reflector Antenna Analysis

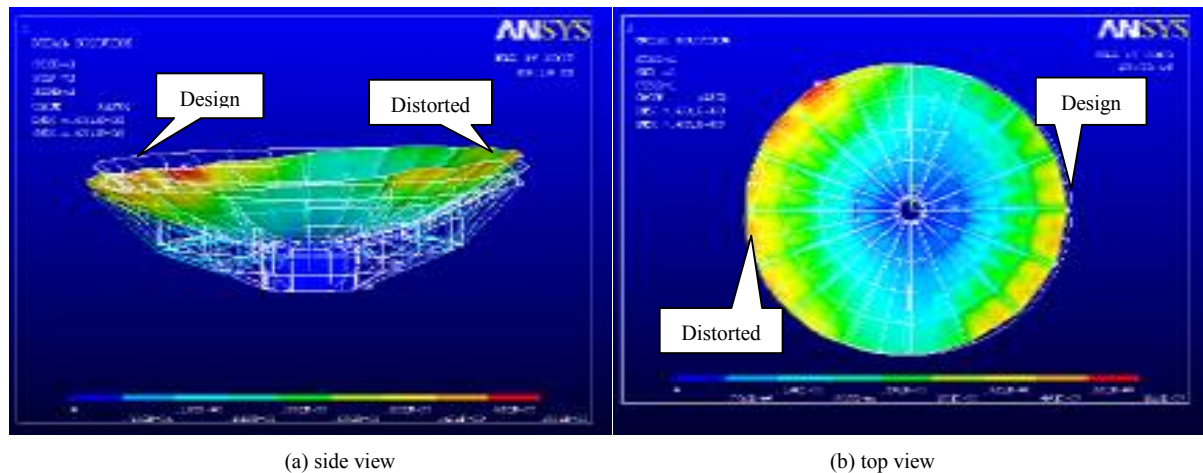
C. S. Wang, B. Y. Duan and Y. Y. Qiu

School of Electromechanical Engineering, Xidian University, Xi'an 710071, China

E-mail: congsiwang@tom.com

1. Introduction

Working under various conditions, antenna is affected by not only its self-weight but also temperature and wind load, which leads to the distortion of antenna reflector [1]. A comparison between the design and distorted parabolic antenna with its focus axis vertical to the horizon is shown in Fig. 1, from which the obviousness of the distortion of reflector caused by self-weight and wind load can be seen. In engineering, the high surface precision of antenna reflector is required because it has a significant effect on antenna electrical performances, such as antenna's efficiency, radiation pattern, boresight gain, and sidelobe level [2].



(a) side view

(b) top view

Figure 1. Comparison between design and distorted parabolic antenna.

The traditional antenna design method is mainly grouped into two parts. One is mechanical structural design offering surface rms error or probability distribution of the tolerance of the reflector; the other is electrical design analyzing the electrical performances of antenna [3]. However, the information of surface rms error cannot represent the full information of distorted reflector because rms error is always obtained by analyzing the best-fit paraboloid. Therefore, the fundamental theory of the best-fit paraboloid is introduced firstly. Then a new divided-fitting approach of distorted reflector is discussed in detail, with which the full surface errors of the distorted reflector can be accurately analyzed by using the hoop trigonometric function fitting and quadratic radial function fitting methods and adopting Coons surface fitting. This method can better describe the whole and partial messages of distorted antenna reflector. Finally, the application of divided-fitting approach to a 7.3-m reflector antenna design is given with right and useful results.

2. Distorted Reflector Fitting

The reflector sometimes is not identical to design paraboloid in antenna manufacturing and installing. Besides, reflector distortion will occur under various loads, thus causing the displacement of each node of back-up structure. As a result, the antenna reflector will not be a smooth surface. Generally, through the analysis of the antenna structure and node displacements, the mathematical equations for distorted antenna reflector can be determined by using surface fitting, and boundary conditions of electromagnetic field can be also decided [4].

The tolerance often used to analyze performances of reflector antennas is determined by the contour deviation of the reflector. In fact, the contour deviation is just the normal deviation ε_n or axial deviation ε_z or radial deviation ε_p of nodes in the reflector. However, the surface rms error determined by the contour deviation and calculated by best-fit paraboloid method cannot represent all the information of distorted reflector.

2.1 Best-fit Paraboloid

The basic idea to determine the best-fit paraboloid is to divide the displacement of each node in the reflector into two parts. One is the rigid motion (slide and rotation) of the whole paraboloid, and the change of antenna focal length. The other is the deviation of all nodes from the new paraboloid. Then make all the partial

derivatives of the rms axial error $\sqrt{\sum_{i=1}^n \delta_i^2} / n$ (n is the total number of sampling nodes) to six parameters equal to zero including the displacements of the vertex of parabolic reflector Δx , Δy and Δz , directions of its focus axis ϕ_x and ϕ_y , and the numerical change of the focal length Δf .

Let the coordinate of a sampling node locating in the surface of reflector antenna be (x_0, y_0, z_0) , and $(x_0 + u, y_0 + v, z_0 + w)$ be the corresponding position of the same node in the distorted reflector. What can be obtained are the normal equations for the best-fit paraboloid.

$$\mathbf{S}\boldsymbol{\beta} = \mathbf{H} \quad (1)$$

Where, $\boldsymbol{\beta} = [\Delta x \ \Delta y \ \Delta z \ \phi_x \ \phi_y \ \Delta f]^T$

$$\mathbf{S} = \begin{bmatrix} \sum Tf^2 x_0^2 & 0 & 0 & 0 & 0 & \sum Tfx_0^2(2f+z_0) \\ 0 & \sum Tf^2 y_0^2 & 0 & 0 & -\sum Tfy_0^2(2f+z_0) & 0 \\ 0 & 0 & -2\sum Tf^4 & 2\sum Tf^3 z_0 & 0 & 0 \\ 0 & \sum Tfy_0^2(2f+z_0) & 0 & 0 & -\sum Ty_0^2(2f+z_0)^2 & 0 \\ \sum Tfx_0^2(2f+z_0) & 0 & 0 & 0 & 0 & \sum Tx_0^2(2f+z_0)^2 \\ 0 & 0 & -2\sum Tf^3 z_0 & 2\sum Tf^2 z_0^2 & 0 & 0 \end{bmatrix}$$

$$\mathbf{H} = f^2 [\sum TKfx_0 \ \sum TKfy_0 \ \sum TKf^2 \ \sum TKy_0(2f+z_0) \ \sum TKx_0(2f+z_0) \ \sum TKfz_0]^T$$

Where, $T = \frac{f}{4(1+z_0)}$, $K = \frac{1}{f^2}(x_0 u + y_0 v - 2fw)$

The method determining the best-fit paraboloid for surface rms error cannot offer all the information of distorted reflector because of its neglect of the special effect of partial great distortion on antenna electrical performances. What follows is a new fitting method well solving the problems above.

2.2 Divided-fitting Method

Its principle is to integrate hoop trigonometric function fitting with quadratic radial function fitting, and apply Coons surface fitting to fit every zone separately. Therefore, the analysis of whole distorted reflector becomes possible, and the phase error of the far field intensity caused by distortion can be also obtained.

The expression of hoop trigonometric function fitting method is given as follows.

$$y = a_0 + \sum_{i=1}^m (a_i \cos ix + b_i \sin ix) = f(x; \mathbf{X}) \quad (2)$$

Where, a_0 , a_i and b_i ($i = 1, 2, \dots, m$) are the undetermined coefficients. According to the simulation analysis, m should be equal to 4 or 5. The variance of all the hoop nodes is shown in the following expression.

$$P = \sum_{k=1}^n [f(x_k; \mathbf{X}) - y_k]^2 = \sum_{k=1}^n \left[a_0 + \sum_{i=1}^m (a_i \cos ix_k + b_i \sin ix_k) - y_k \right]^2 \quad (3)$$

Where, (x_k, y_k) is the coordinate of the k th sampling node; n is the total number of sampling nodes.

By applying the way used in best-fit paraboloid method, the normal equations for hoop trigonometric function fitting method can be obtained.

$$\mathbf{A}\mathbf{X} = \mathbf{C} \quad (4)$$

The coefficient matrix is $\mathbf{A}_{(2m+1) \times (2m+1)} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1,2m+1} \\ a_{21} & a_{22} & \cdots & a_{2,2m+1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{2m,1} & a_{2m,2} & \cdots & a_{2m,2m+1} \\ a_{2m+1,1} & a_{2m+1,2} & \cdots & a_{2m+1,2m+1} \end{bmatrix} \quad (a_{ij} = a_{ji})$

Where, $a_{1j} = \sum_{k=1}^n \cos \left[\frac{j}{2} \right] x_k$, $a_{ij} = \begin{cases} \sum_{k=1}^n \cos \left[\frac{j}{2} \right] x_k \cdot \cos \left[\frac{i}{2} \right] x_k & (i \text{ is an even number}) \\ \sum_{k=1}^n \sin \left[\frac{j}{2} \right] x_k \cdot \sin \left[\frac{i}{2} \right] x_k & (i \text{ is an odd number}) \end{cases}$, $i \in (2, \dots, 2m+1)$.

In the expressions above, the symbol $[\dots]$ means to draw the integer part.

The constant terms are $C_{(2m+1) \times 1} = \left[\sum_{k=1}^n y_k \quad \sum y_k \cos x_k \quad \sum y_k \sin x_k \quad \dots \quad \sum y_k \cos mx_k \quad \sum y_k \sin mx_k \right]^T$

The following is the sequence of the unknown variables. $X = [a_0 \ a_1 \ b_1 \ \dots \ a_m \ b_m]^T$

What follows is the mathematical expression of quadratic radial function fitting method.

$$y = \sum_{k=0}^m a_k x^k \quad (5)$$

Where, a_0 , a_1 and a_2 are the undetermined coefficients (m is equal to 2).

According to the same process above, the corresponding normal equations can be obtained.

$$\begin{cases} s_{00}a_0 + s_{01}a_1 + \dots + s_{0m}a_m = t_{0y} \\ s_{10}a_0 + s_{11}a_1 + \dots + s_{1m}a_m = t_{1y} \\ \dots \\ s_{m0}a_0 + s_{m1}a_1 + \dots + s_{mm}a_m = t_{my} \end{cases} \quad (6)$$

Where $s_{jk} = \sum_{i=1}^n x_i^{j+k}$, $t_{jy} = \sum_{i=1}^n x_i^j y_i$ ($j = 0, 1, \dots, m$).

The coefficient matrix $S = [s_{ij}]$ is a positively definite symmetrical square matrix of $(m+1)$ order.

The weight coefficient d_i ($i = 1, 2, \dots, n$) (n is the total number of sampling nodes) may be introduced according to the different reliabilities of the fitting nodes when the distorted surface is being fitted. The higher the reliability of the sampling node is, the greater the value of d_i .

The Coons surface of second kind should be adopted to ensure that two Coons surfaces are continuous and smooth at the joint when the divided zone is fitted [5] (See Fig. 2). The surface of this kind is determined by four boundary curves $u0$, $u1$, $0w$ and $1w$, and their direction derivatives $u0_w$, $u1_w$, $0w_u$ and $1w_u$ (The intersections of two adjoining boundaries are marked as 00, 01, 10 and 11).

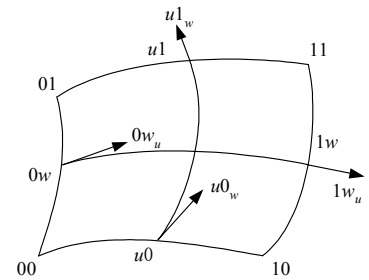


Figure 2. Coons surface unit of second kind.

What follows are the symbol expressions used in the formulae.

$$u0_u = \left. \frac{\partial(uw)}{\partial u} \right|_{w=0} \quad u0_w = \left. \frac{\partial(uw)}{\partial w} \right|_{w=0}$$

The specific formula of Coons surface fitting method is

$$uw = (u0 \ u1 \ u0_w \ u1_w) \begin{pmatrix} F_0(w) \\ F_1(w) \\ G_0(w) \\ G_1(w) \end{pmatrix} + (F_0(u) \ F_1(u) \ G_0(u) \ G_1(u)) \begin{pmatrix} 0w \\ 1w \\ 0w_u \\ 1w_u \end{pmatrix} - (F_0(u) \ F_1(u) \ G_0(u) \ G_1(u)) \begin{pmatrix} 00 & 01 & 00_w & 01_w \\ 10 & 11 & 10_w & 11_w \\ 00_u & 01_u & 00_{uw} & 01_{uw} \\ 10_u & 11_u & 10_{uw} & 11_{uw} \end{pmatrix} \begin{pmatrix} F_0(w) \\ F_1(w) \\ G_0(w) \\ G_1(w) \end{pmatrix} \quad (7)$$

Where F_0 , F_1 , G_0 and G_1 are the blending functions.

After fitting the distorted surface, the optical path error and the corresponding phase error of each sampling node can be calculated to analyze the electrical performances [6]. The phase error caused by nodes displacement can be determined with the availability of corresponding optical path error of nodes by using the normal deviation ε_n or radial deviation ε_ρ or axial deviation ε_z (Fig. 3).

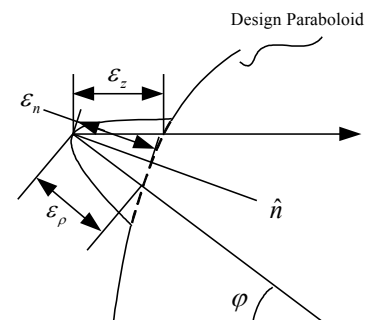


Figure 3. The deviation of node in reflector.

3. Simulation Results

A simulation analysis is made of a 7.3-m circular parabolic antenna with 2.5335926-m focal length. Its emission frequency bands are Ku and C; CCIR sidelobe envelope is CCIR.580-2; operating and survival wind speeds are 20 m/s and 55 m/s respectively; the range of working temperature is from -45° to 60° . The surface precision index is 0.5 mm in the normal direction of antenna reflector. Antenna reflector is a type of plate, and the back-up structure is a type of truss. The whole antenna has 16 radiate beams with uniform distribution along central body and 48 circular beams. The 16 fan-shaped reflectors are a typical shell structure regarded as shell unit, which is selected as "Shell63" in the finite element software ANSYS. The other finite element modeling parameters is given detailedly in Table 1.

Table 1 The parameter of finite element model of antenna structure.

Antenna Structure	Unit	Unit Type	Unit Num	Material
Reflector	Shell	Shell63	6400	Aluminum plate
Z-typed aluminium strengthening bars	Beam	Beam4	2560	Aluminum section bar
Upper and lower border	Beam	Beam4	176	Rectangular steel tube
Partial lower border	Bar	Link8	16	Rectangular steel tube
Webmember	Bar	Link8	80	Rectangular steel tube
Outer margin of upper border	Beam	Beam4	16	Steel plate
Circular beam	Bar	Link8	48	Steel tube

The simulation results are shown in Table 2 of the surface errors in the normal, radial and axial direction with different wind speeds and elevations, which adopted the above divided-fitting method based on Coons surface. Here, the wind load exerts the influence on antenna along the horizontal direction, and the elevation denotes the angle between the focal axis and the horizon. From the data shown in the Table 2, we found that the rms error of distorted reflector increases with the bigger elevation, and the more high wind speed is, more the surface error increases. Of course, the distorted surface error of antenna reflector in the radial direction is very close to one in the axial direction in three aspects that is maximum value, minimum value and rms error, which is consistent to the content in Fig.3.

Table 2 The surface precision of distorted reflector under different condition (mm).

Wind Speed (m/s)	Elevation ($^{\circ}$)	Normal Error			Radial Error			Axial Error		
		Max	Min	rms	Max	Min	rms	Max	Min	rms
30	0	0.476213	0.000006	0.129132	0.587026	-0.552741	0.142780	0.587025	-0.552740	0.142779
	45	0.650580	0.000035	0.147042	0.801832	-0.689342	0.164295	0.801834	-0.689347	0.164296
	90	0.724715	0.000002	0.148009	0.820450	-0.768235	0.166119	0.820455	-0.768231	0.166120
50	0	1.237070	0.000030	0.334563	1.505865	-1.411490	0.370751	1.505868	-1.411495	0.370750
	45	1.726801	0.000015	0.401891	2.118042	-2.051522	0.449635	2.118040	-2.051529	0.449637
	90	1.744561	0.000041	0.411790	2.150926	-2.123371	0.456516	2.150927	-2.123377	0.456515

4. Conclusion

Antenna structural design is an interdisciplinary analysis and design closely combining electromagnetic with mechanical structural technologies. In this paper, a novel divided-fitting method based on Coons surface for the distorted antenna reflector is submitted. The fitted surface is more close to the practical distorted surface than BFP because its foundation is to analyze the real shape of the distorted reflector. After the determination of each divided surface unit, the three reflector deviations and the corresponding optical path error can be calculated. Then according to antenna aperture theory, the electrical performance of antenna may be given and judged if it satisfies the requirement. Furthermore, the reasonableness of antenna mechanical structural design can be identified. Analysis results of the surface error of a 7.3-m reflector antenna verify the correctness of the analysis theory, approach and programs discussed above. All the research achievements have an extreme important role to the design and simulation predicted analysis of antenna virtual prototype, and analysis of electromechanical coupling performance of antenna providing a theoretical guidance and an effective method for improving the efficiency of antenna design simultaneously.

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