BISPECTRUM ANALYSIS OF NONLINEAR INTERACTIONS BETWEEN VLF TRANSMITTER SIGNALS AND ELF EMISSIONS

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Abstract

Nonlinear interaction between signals from a ground-based VLF transmitter and ELF emissions in the auroral ionosphere is studied by means of the bispectrum analysis. A bicoherence analysis has indicated that the sideband structure around the Siple transmitter signal received onboard the ISIS satellite is due to the nonlinear interaction between the Siple VLF signal and the pre-existing ELF emission.

1. Introduction

A spectral analysis of the second order (power spectrum) loses the phase information among the different Fourier components, while the bispectrum (third order) and/or the bicoherence (normalized bispectrum) retain this information.

The use of bicoherence analysis has appeared to be very successful in the studies of nonlinear wave-wave interaction (and turbulent) phenomena in different fields, such as harmonic generation of ocean surface gravity waves (Elgar and Guza, 1985), brain wave emissions (Huber et al., 1971), nonlinear transfer of energy in turbulent flows (Lii et al., 1976; Van Atta, 1979), and nonlinear interaction between density fluctuation modes in laboratory plasmas (Kim and Powers, 1979). The higher-order spectral analysis appears to be unique in attempts to answer the question of the presence of multi-wave interaction processes, because the phase information is not lost in it.

The nonlinear wave-wave interaction process has been proposed to explain the generation mechanisms of some kinds of space plasma waves and recently there have been published the first evidence on this in the field of space plasma waves (Tanaka et al., 1987 ; Lagoutte et al., 1989). In order to extend this study, we carried out the collaborative work on the ISIS satellite reception of Siple VLF transmitter signals in the auroral ionosphere in cooperation with Stanford University(USA) and Communications Research Laboratory(Tokyo). The experiment was carried in the winter of the year 1987/88, and the purpose of this paper is to apply the bicoherence analysis to the wave data obtained from this campaign in order to have further evidence of the presence of nonlinear wave-wave interaction in the auroral ionosphere.

2. Definitions of bispectrum and bicoherence

Let x(t) be a real and stationary signal with zero mean value and X(f) its Fourier transform. Hereafter, X_k, the Fourier component at the frequency f_k is noted as

$$X = \int_0^T x(t)\exp(-2\pi j f_k t)dt \tag{1}$$

The definitions of the third moment of the time series are concerned with the autocorrelation and power spectrum. The third moment for the record (i) at the time lags τ_1 and τ_2 is defined by

$$R(\tau_1,\tau_2)=E\{\frac{1}{T}\int_0^T x^{(i)}(t)x^{(i)}(t+\tau_1)x^{(i)}(t+\tau_2)dt\} \tag{2}$$

where the average E{ } is taken over i=1,...M successive data records. By definition, the bispectrum at the frequencies f_k and f_l is the two-dimensional Fourier transform of $R(\tau_1, \tau_2)$:

$$B(k,l)=B(f_k,f_l)=\int_{-T}^{T}\int_{-T}^{T}R(\tau_1,\tau_2)\exp(-2\pi j(f_k\tau_1+f_l\tau_2))d\tau_1 d\tau_2 \tag{3}$$

Now, the bispectrum can be defined as a product of three Fourier components (Kim and Powers, 1979):

$$B(k,l)=\lim_{T\to\infty}\frac{1}{T}E\{X_k^{(i)}X_l^{(i)}X_{k+l}^{(i)*}\} \tag{4}$$

where X_{k+l}^* is the complex conjugate of X_{k+l}. But this definition is only valid if the sum of the frequencies of the three Fourier components is zero, which means

$$f_{k\pm l}=f_k\pm f_l \tag{5}$$

where $f_{k\pm l}$ is the frequency of the product wave in the case of coupling. The bicoherence spectrum is a normalized bispectrum:

$$b^2(k,l)=\frac{|E\{X_kX_lX_{k+l}^*\}|^2}{E\{|X_kX_l|^2\}E\{|X_{k+l}|^2\}} \tag{6}$$

where B is the modulus of B; and $0\leq b(k,l)\leq 1$.

The bispectrum and bicoherence are important tools to investigate the phase coherence between three Fourier components of frequencies f_k, f_l, and f_{k+l}. If the third wave at the frequency $f_{k+l}=f_k+f_l$ is spontaneously emitted, the phase of the Fourier component X_{k+l} will be independent of the phase of the two others, X_k and X_l, and the values of bispectrum (4) and bicoherence (6) will be zero. On the other hand, if the wave at the frequency f_{k+l} is phase coherent with the two other waves, the values of bispectrum and bicoherence will be nonzero.

3. Application of bispectrum to the satellite VLF data

The data used in this paper were based on the wide-band measurement observed onboard the ISIS satellite in which the Siple VLF transmitter was in operation on 28 December, 1987. Fig.1 illustrates an example of the part of the spectrogram for which we have performed the bispectral analysis.

Power spectral densities were computed by means of sampling the data record of the wideband signals at 1024 points with frequency resolution of 46.875Hz, and were averaged over 64 data records. Fig.2 indicates the power spectral density in a relative unit, which indicates that sideband structures are obviously seen around the Siple transmitter signal(f=2.34kHz). This kind of spectral broadening with sidebands and ELF emission is called Type 2 in Tanaka et al.(1987), while the spectral broadening without any association with ELF emission is called "Type 1"in Tanaka et al.(1987).

The bicoherence was computed by means of sampling the data record at 512 points with a frequency resolution of 97.6Hz, and was averaged over 64 data records of the transmitter pulse. The application of the bicoherence analysis to the spectral broadening of Type 1 has yielded that the computed value of bicoherence was always too small to identify any mode coupling, though Titova et al.(1984) have suggested that even Type 1 spectral broadening is due to the nonlinear wave-wave interaction. On the contrary, we have found a significant value for the bicoherence for the spectral broadening of Type 2 as in Fig.3. Fig.3 is the result of the bicoherence analy-

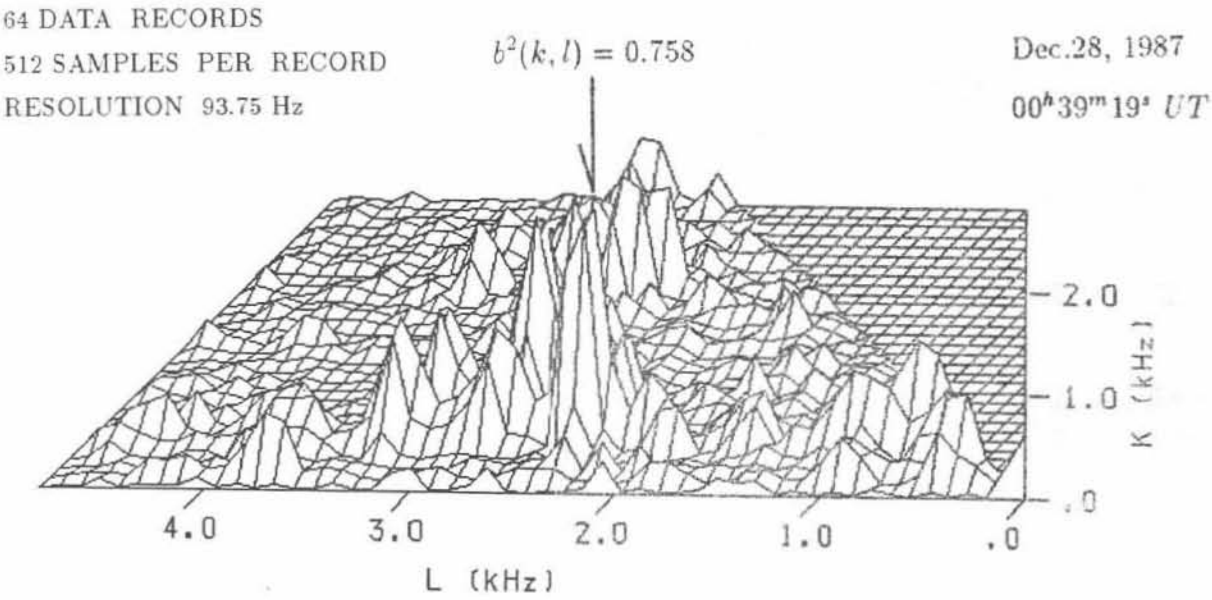

Fig.3 A typical result of the bicoherence computation for electric field data with a frequency resolution of 93.75Hz in the case of spectral broadening of Type 2. Bicoherence results to show the upper and lower sideband component being caused by a nonlinear process between the transmitter pulse (f_1=2.34kHz) and ELF emission(f_k=0.47kHz) because of a significant bicoherence value of 0.758.

sis for the VLF data in Fig.1 in which the sideband structure is noticeable. It is seen from the figure that the upper and lower sideband components are caused by a nonlinear interaction between the VLF transmitter pulse and an ELF emission on the basis of the obtained significant bicoherence value of 0.758.

4.Conclusion

The spectral broadening phenomena have been divided into the cases of Type 1 and Type 2. In this paper, we studied the generation mechanism of Type 2 by means of bicoherence analysis. The result of the bicoherence analysis indicates a large bicoherence value ($b^2(k,l)$=0.758) between the VLF transmitter signal and the ELF emission, which leads us to conclude that sidebands analyzed here were caused by the nonlinear wave-wave interaction.

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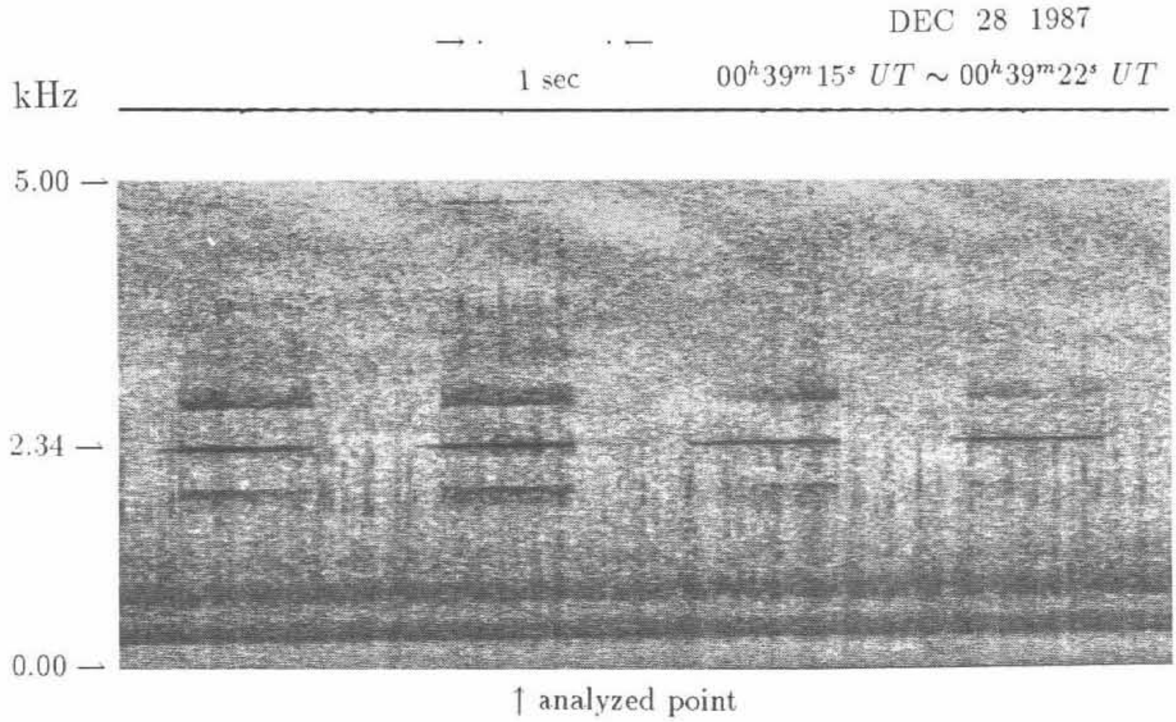

Fig.1 Spectrogram of the wideband electric field components on ISIS in December 28,1987, and we can identify the Siple transmitter pulses and their spectral broadening about the carriers.

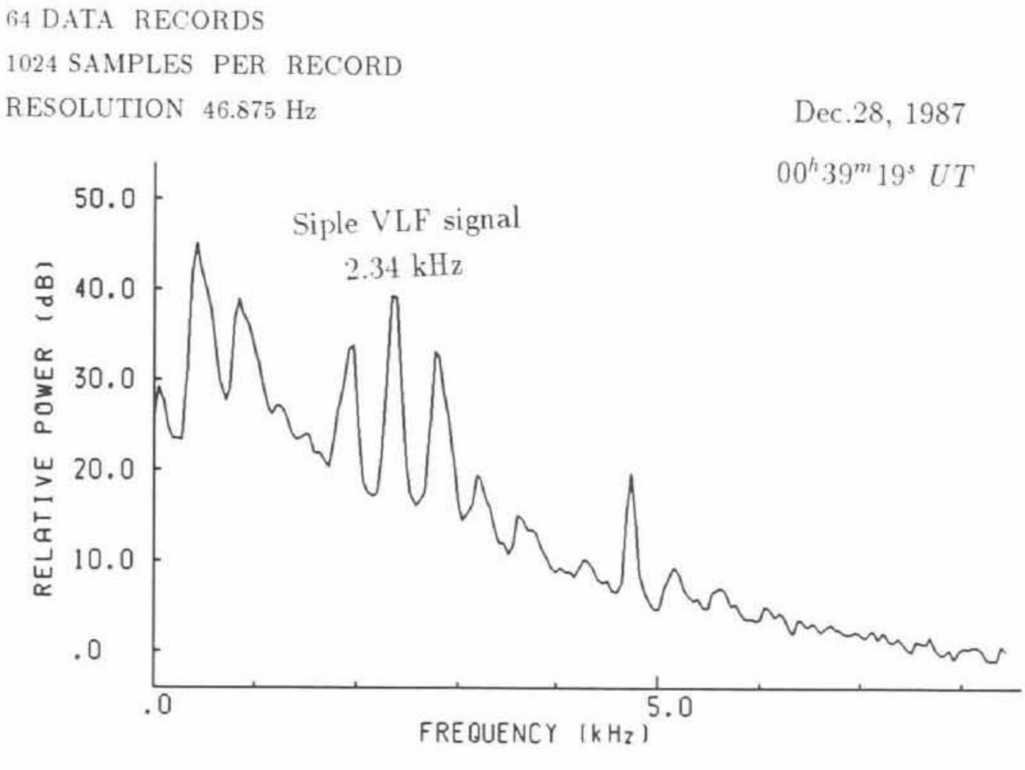

Fig.2 Power spectral density of the wideband signal in relative unit for a typical case of the spectral broadening of Type 2. We find the sideband structures.