A GENERAL RELATION OF SCATTERED WAVES BY A ROTATING OBJECT

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1. Introduction

The scattering of electromagnetic waves by a rotating object is one of the basic and interesting problem. The angular velocity of the rotating object is much smaller than that of the incident waves in the practical problems. Under this condition the scattered waves consist, at any time T, of the scattering patterns corresponding to the instantaneous position of the object, calculated as if the object was stationary.[1] The fields calculated in this way are called quasi-stationary (QS) fields.[2-4] We can use the QS fields in the practical problems.

General relations such as optical theorem and symmetric properties of the scattering matrix are known in the scattering by the stationary object.[5] These relations are based on the basic properties of the problem i.e., energy conservation law and time-reversal invariant properties of the problem.

A simple and interesting general relation is discovered in the scattering by a rotating two-dimensional object whose cross section has more than one reflection symmetry axis. This relation connects incident angle, scattered angle and angle of the reflection symmetry axis of the object at a given time with the phase differences between the amplitudes of the frequency components of the scattered wave. It is shown that this relation is due to energy conservation law and time-reversal invariance of the QS field.

2. Quasi-stationary field

The geometry of the problem is shown in Fig. 1. We first consider the observer's stationary system $K(X,Y,Z,T=R,\Theta,Z,T)$ and the rotating system $L(x,y,z,t=r,\theta,z,t)$ in which the object is stationary. The object is assumed to rotate with angular velocity Ω around the Z axis in the system K. It is also assumed that the cross section of the object has at least one reflection symmetry axis and this axis coincides with the x axis at as shown in Fig. 1. The coordinate transformations which relate the system L to the system K is given by

$$R=r,\ \Theta=\theta+\Omega t+\zeta,\ Z=z,\ T=t, \qquad (1)$$

where ζ is the angle of the symmetry axis at T=t=0 in the system K. The incident wave is assumed to be E-polarized in which incident E vector is parallel to the Z=z axis. The Z component of the incident wave in the

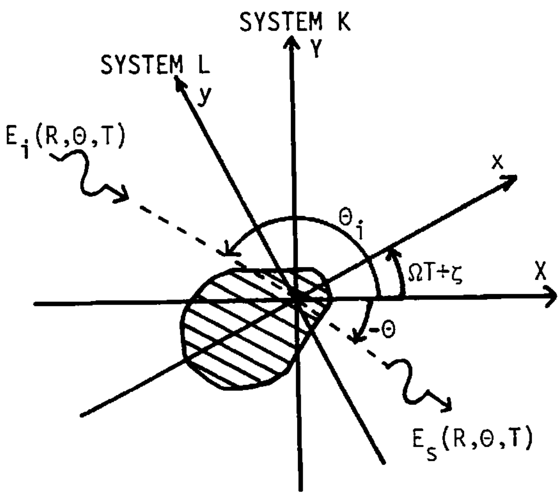

Fig. 1 Geometry of the problem

system K can be written

$$E_i(R,\Theta,T)=\exp[jk_0R\cos(\Theta-\Theta_i)+j\omega_0T], \tag{2}$$

where $k_0=\omega_0/c$, ω_0 is the angular frequency of the incident wave, Θ_i is the incident angle in the system K as shown in Fig. 1. The incident wave Eq. (2) can be expressed in the system L as

$$E_i(r,\theta,t)=\sum_{\nu} J_\nu(k_0r)\exp(j\nu\pi/2+j\nu\theta)\exp[-j(\Theta_i-\Omega t-\zeta)]\exp(j\omega_0t), \tag{3}$$

where $J_\nu(x)$ is the ν-th order Bessel function. Since $\omega_0 >> \Omega$, Ωt is assumed to be constant in the calculation of the QS field in Eq. (3). Hence, the scattered wave in the system L can be written

$$E_s(r,\theta,t)=\sum_{\nu}\sum_{m} C_{\nu,m} H_m^{(2)}(k_0r)\exp(jm\theta)\exp(j\omega_0t), \tag{4}$$

where $H_m^{(2)}(x)$ is the m-th order Hankel function of the second kind and $C_{\nu,m}$ are unknown coefficients which are determined by the boundary conditions. If the scattered wave in the system L is obtained, the scattered wave in the system K can be written

$$E_s(R,\Theta,T)=\sum_{\alpha} F_\alpha(\Theta,R)\exp[j(\omega_0+\alpha\Omega)T], \tag{5}$$

where $F_\alpha(\Theta,R)$ represent complex amplitudes of frequency components $\omega_0+\alpha\Omega$ of the scattered wave and are expressed as

$$F_\alpha(\Theta,R)=\sum_{\nu} S_{\nu,\nu-\alpha}\exp(j\nu\pi/2)H_{\nu-\alpha}^{(2)}(k_0R)\exp[j(\nu-\alpha)(\Theta-\zeta)-j\nu(\Theta_i-\zeta)], \tag{6}$$

where $$S_{\nu,m}=C_{\nu,m}\exp[j\nu(\Theta_i-\Omega t-\zeta-\pi/2)]. \tag{7}$$

The scattering cross section varies by the rotation of the object for the stationary observer at a given angle Θ in the systecm K. Therefore, the scattered wave at a given point Θ is modulated by the rotation of the object.

3. Properties of coefficients $S_{\nu,m}$

The frequency components of the scattered wave can be obtained by calculating the coefficients $S_{\nu,m}$. We consider general properties which $S_{\nu,m}$ must satisfy. Since the shape of cross section of the object is invariant under the transformation of $\theta=-\theta$ in the system L, we obtain the following relation from Eqs. (3) and (4) immediately:

$$S_{-\nu,-m}=(-1)^{\nu-m}S_{\nu,m}. \tag{8}$$

Another relation is

$$S_{\nu,m} = S_{m,\nu}. \tag{9}$$

The proof of Eq. (9) is given as follows: Introducing the matrix $\bar{S}$ having entries coeffecients $S_{\nu,m}$, we can express the total fields of $\exp(j\omega_0t)$ time dependence in the system L as

$$E(r,\theta)=E_i(r,\theta)+E_s(r,\theta)=1/2\vec{a}.\vec{h} +1/2\vec{a}.(\bar{I}+2\bar{S}).\vec{h} , \tag{10}$$

where $\vec{a}$ is a row vector whose components are given by

$$\vec{a}=\{\ldots,\ \exp[-jm(\Theta_i-\Omega t-\zeta-\pi/2)],\ \exp[-j(m+1)(\Theta_i-\Omega t-\zeta-\pi/2)],\ \ldots\}, \qquad (11)$$

and $\vec{h}_i$ and $\vec{h}_2$ are column vectors having entries

$$\vec{h}_n^T=\{\ldots,\ H_m^{(n)}(k_0r)\exp(jm\theta),\ H_{m+1}^{(n)}(k_0r)\exp[j(m+1)\theta],\ldots\},\ (n=1,2), \qquad (12)$$

where ^{T} denotes transpose and $\bar{I}$ denotes identity matrix. Under the QS field approximation the energy coservation law can be written

$$\vec{a}^*\cdot\vec{a}\ -\ \vec{a}^*\cdot(\bar{I}+2\bar{S}^*)\cdot(\bar{I}+2\bar{S})^T\cdot\vec{a}^T=0, \qquad (13)$$

where * denotes complex conjugate. From Eq. (13) we obtain the following relation:

$$(\bar{I}+2\bar{S}^*)\cdot(\bar{I}+2\bar{S}^T)\ =\bar{I}. \qquad (14)$$

In addition, the total QS field $E(r,\theta)$ must remain a solution upon time reversal. This corresponds to taking the complex conjugate of $E(r,\theta)$ giving

$$E^*(r,\theta)=1/2\vec{a}^*\cdot(\bar{I}+2\bar{S}^*)\cdot\vec{h}_1+1/2\vec{a}^*\cdot\vec{h}_2. \qquad (15)$$

We used relation $H_m^{(1)*}(x)=H_m^{(2)}(x)$ and invariant properties of the problem under the transformation $\theta=-\theta$. From Eqs. (12) and (15) we obtain

$$(\bar{I}+2\bar{S}^*)\cdot(\bar{I}+2\bar{S})\ =\bar{I}. \qquad (16)$$

From Eqs. (14) and (16) the matrix $\bar{S}$ must be symmetric i.e., Eq. (9) can be obtained.

5. Frequency components of the scattered wave

We first consider the case where α is an even number. The summation in Eq. (6) can be rewritten

$$F_{\pm\alpha}^e(\Theta,R)=\ (\nu=\alpha/2)+[(\nu=\alpha/2-1)+(\nu=\alpha/2+1)]+[(\nu=\alpha/2-2)+(\nu=\alpha/2+2)]+\ldots\ldots\ldots=$$
$$S_{\alpha/2,-\alpha/2}\exp[j\alpha/2(\pi/2)]H_{-\alpha/2}^{(2)}(k_0R)\exp[j(-\alpha/2)(\Theta-\zeta)-j(\alpha/2)(\Theta_i-\zeta)]+$$
$$\{S_{\alpha/2+1,-\alpha/2+1}\exp[j(\alpha/2+1)(\pi/2)]H_{-\alpha/2+1}^{(2)}(k_0R)\exp[\mp j(\alpha/2-1)(\Theta-\zeta)\mp j(\alpha/2+1)(\Theta_i-\zeta)]+$$
$$S_{\alpha/2-1,-\alpha/2-1}\exp[j(\alpha/2-1)(\pi/2)]H_{-\alpha/2-1}^{(2)}(k_0R)\exp[\mp j(\alpha/2+1)(\Theta-\zeta)\mp j(\alpha/2-1)(\Theta_i-\zeta)]\}-$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (17)$$

We can obtain following relations from Eqs. (8) and (9):

$$S_{\alpha/2+n,-\alpha/2+n}=(-1)^{\alpha}S_{-\alpha/2-n,\alpha/2-n}=S_{\alpha/2-n,-\alpha/2-n},\ (n=1,2,\ldots). \qquad (18)$$

Substituting relations (18) into (17), $F_\alpha^e(\Theta,R)$ can be written

$$F_{\pm\alpha}^e(\Theta,R)=\exp[j\alpha/2(\pi/2)]\exp[\mp j(\alpha/2)(\Theta+\Theta_i-2\zeta)]V_{\pm\alpha}^e, \qquad (19)$$

where

$$V_{\pm\alpha}^e\ =S_{\alpha/2,-\alpha/2}H_{-\alpha/2}^{(2)}(k_0R)+\sum_{n=1,2,3,\ldots}S_{\alpha/2+n,-\alpha/2+n}\times$$

$$\{\exp(j\pi n/2)H^{(2)}_{-\alpha/2+n}(k_0R)\exp[\pm jn(\Theta-\Theta_i)]+$$
$$\exp(-j\pi n/2)H^{(2)}_{-\alpha/2-n}(k_0R)\exp[\mp jn(\Theta-\Theta_i)]\}. \tag{20}$$

For the case where α is an odd number the following expression can be obtained, similarly:

$$F^{o}_{\pm\alpha}(\Theta,R)=\exp[j\alpha/2(\pi/2)]\exp[\mp j\alpha/2(\Theta+\Theta_i-2\zeta)]V^{e}_{\pm\alpha}, \tag{21}$$

where

$$V^{o}_{\pm\alpha}=\sum_{n=1,3,5,\ldots} S_{(\alpha+n)/2,(-\alpha+n)/2}\ \times$$
$$\{\exp[j(\pi/2)(n/2)]H^{(2)}_{(-\alpha+n)/2}(k_0R)\exp[\pm jn/2(\Theta-\Theta_i)]-$$
$$\exp[-j(\pi/2)(n/2)]H^{(2)}_{-(\alpha+n)/2}(k_0R)\exp[\mp jn/2(\Theta-\Theta_i)]\}. \tag{22}$$

Substituting the asymptotic form of the Hankel function into Eq. (20) and (22), we find that

$$V^{e}_{-\alpha}/V^{e}_{+\alpha}\simeq 1, \qquad V^{o}_{-\alpha}/V^{o}_{+\alpha}\simeq 1,\ (k_0R\gg 1). \tag{23}$$

Therefore, the following relation can be obtained in the far field:

$$F^{\substack{o\\e}}_{-\alpha}(\Theta,R)/F^{\substack{o\\e}}_{+\alpha}(\Theta,R)=\exp[j\alpha(\Theta+\Theta_i-2\zeta)]. \tag{24}$$

Relation (24) does not depend on whether the incident wave is E-polarized or H-polarized and does not depend on whether the object is dielectric or perfect conductor. It does not depend on the shape of cross section of the object under the condition that it has more than one reflection symmetry axis. Relation (24) shows the interesting application. If we know Θ and ζ, we can calculate the incident angle Θ_i by measuring the frequency components of the scattering waves at a given point Θ. We can calculate Θ_i and ζ by measuring the scattering waves at different two points of Θ.

6. Conclusions

An interesting general relation has been discovered in the scattering by a rotating two-dimensional objects. It has been shown this relation is due to the basic properties of the problem i.e., energy conservation law, time-reversal invariance and symmetry of the cross section of the object. We must notice that this relation is not simply due to the configuration of the measuring systems because the relation does not hold in the near fields.

References

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