IMAGE CALCULATION OF FIELDS FROM ARBITRARY SOURCES TRANSMITTED INTO THE GROUND

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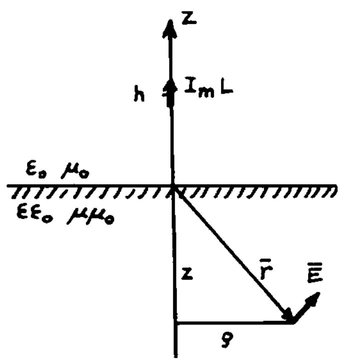
ABSTRACT

Exact image theory was recently developed by two of the present authors for the calculation of fields reflected from a planar interface between two homogenous media [1]-[3], and, subsequently applied for calculation of impedances for antennas above the earth [4],[5]. In this paper, an extension of that theory is given, with the aid of which fields transmitted through the interface can be effectively calculated. Being exact and very general, the theory is applicable to all kinds of problems involving transmission of information or electromagnetic energy through an interface, for example, submarine communication or microwave diathermy.

OUTLINE OF THE THEORY

Instead of considering the general source from the start, we study the simple vertical magnetic dipole (VMD) theory and make the necessary generalizations at the end. The source is the magnetic current function

(1) $J_m(r) = u_z I_m L \delta(\rho)\delta(z-h)$,



and the interface at z=0 separates the air (z>0) with the parameters ε_0, μ_0 from the ground with the parameters $\varepsilon\varepsilon_0, \mu\mu_0$. If the problem is Fourier transformed in the xy plane with the two-dimensional Fourier variable K, the reflected and transmitted fields can be solved in Fourier space and the solution for the transverse component of the electric field is

(2) $$e_1(z,K) = -j u \times K I_m L \left(\frac{2\mu\beta\beta_1}{\mu\beta+\beta_1} \frac{e^{-j\beta h}}{\beta}\right)\frac{e^{j\beta_1 z}}{2j\beta_1} \quad .$$

Here, β, β_1 are the propagation factors in the the air and ground media, respectively: $\beta=\sqrt{k^2-K\cdot K}$, $\beta_1=\sqrt{\varepsilon\mu k^2-K\cdot K}$, $-\pi<\arg(\beta,\beta_1)\leq 0$, with $k=\omega\sqrt{\mu_0\varepsilon_0}$. Inverse Fourier transformation of (2) with (3), (4) inserted, does not lead to an analytic expression, whence solution in physical space is seeked in terms of a magnetic image current function I_{mi} in the form

(3) $$E(r) = -\nabla x \int G_1(r-u_z z')u_z I_{mi}(z')dz',$$

where G_1 is the Green function in the ground, $G_1(r)=e^{-jk_1 D(r)}/4\pi D(r)$, and $D(r)=\sqrt{r\cdot r}$.
Fourier transformation of the transverse component of (3) gives rise to the following expression in the region z<0:

(4) $$e_1(z,K) = -ju_z x K \int I_m(z') \frac{e^{j\beta_1(z-z')}}{2j\beta_1} dz' .$$

It is sufficient to write the bracketed quantity in (2) as an integral of the type $\int I_m(z')\exp(-j\beta_1 z')dz'$. In comparison to the corresponding problem for the reflection image in [3], (2) possesses the complicating factor exp(-jβh). Analytic expansion is, however, possible when an integral identity is invoked from [6], which can be written in the form

(5) $$\left(\frac{B}{\beta+\beta_1}\right)^{\nu} \frac{e^{-j\beta h}}{\beta} = \int_0^{\infty} jH'(p)\left(\frac{h-H}{h+H}\right)^{\nu/2} J_\nu(p)e^{-j\beta_1 H} dp ,$$

where p is a real parameter, $B = k\sqrt{\mu\varepsilon-1}$ and H(p) is the function

(6) $$H(p) = \sqrt{(p/B)^2 + h^2} .$$

The branch of the square root should be such that $\exp(-j\beta_1 H(p))$ converges as $p \to \infty$. Because for large p, $H(p) \to p/B$, the condition can be written as $\mathrm{Im}\{\beta_1 p/k\sqrt{\mu\varepsilon-1}\} < 0$. The branch of H(p) will be considered later.
To be able to apply (5) in (2), the following decomposition must be made

(7) $$\frac{\beta}{\mu\beta+\beta_1} = \frac{1}{\mu+1} - \frac{2}{\mu^2-1} \sum_{m=1}^{\infty} \left(\frac{\mu-1}{\mu+1}\right)^m \left(\frac{B}{\beta+\beta_1}\right)^{2m}.$$

When (7) is substituted in (2) and (5) is applied, the result can be written in the form (4) and, hence, the image magnetic current function can be identified defining $I_{mi}(p)dp = I_{mi}(z')dz'$ for $z'=H(p)$:

(8) $$I_{mi}(p) = I_m L\left(\frac{2\mu}{\mu+1}\delta_+(p) + \frac{d}{dp} F_\mu(Bh,p)\right),$$

where we denote $\delta_+(p) = \lim \delta(p-\eta)$ for $\eta \to 0$ and

(9) $$F_\alpha(\tau,p) = \frac{2\alpha}{\alpha+1} J_0(p) - \frac{4\alpha}{\alpha^2-1} \sum_{m=1}^{\infty} \left(\frac{\alpha-1}{\alpha+1} \frac{1-\sqrt{(p/\tau)^2+1}}{1+\sqrt{(p/\tau)^2+1}}\right)^m J_{2m}(p) .$$

Because p is real, the individual terms converge as $1/\sqrt{p}$ for $p\to\infty$, whereas the image function can be shown to converge as $1/p\sqrt{p}$ if $h\neq\infty$. Also, the series always converges, because $J_n(p)$ drops off rapidly when n grows larger than p. $F_\alpha(\tau,p)$ is the counterpart of the function $f_\alpha(p)$ associated with the reflection image theory [3],[5] and (9) is a very effective means of calculating its values. The field can be obtained from (3) when the integration parameter is changed from z' to p via $z'(p)=H(p)$:

$$(10) \qquad E(r) = -\nabla\times\int G_1(r-u_zH(p))u_zI_{mi}(p)dp.$$

In the Green function definition, the branch of the complex distance function D must be so defined that we have for every ρ, z and p, $\mathrm{Im}\{k_1D\} = \mathrm{Im}\{k_1\sqrt{\rho^2+(z-H(p))^2}\} < 0$, whence the Green function never grows exponentially with ρ, z or p. Because the transmitting image is situated at points $z=z'=H(p)$, for $p=0\to\infty$, the branch of H(t) is determined from the fact that $\mathrm{Re}(H)>0$ so that no singularities due to the complex space image (see Ref. [1]) enter the half space $z<0$. It is easily shown that the image follows a hyperbolic curve with the asymptote $H_0(t) = p/B = p/k\sqrt{\mu\varepsilon-1}$. It is evident that the transmission image of a vertical electric dipole (VED) is obtained from (8) by just changing μ to ε and I_m to I.

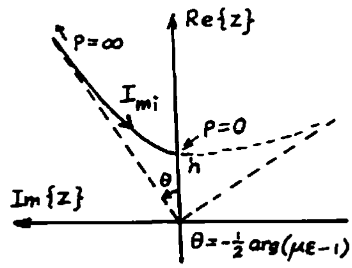


Image of the most general three-dimensional electric source J(r) can be derived after some further considerations to produce the expression [7]

$$(11) \quad J_i(r,p) = \left[uu\left(\frac{2\varepsilon}{\varepsilon+1}\delta_+(p)+F'_\varepsilon(Bz,p)\right) + \frac{1}{\mu}I_t\left(\frac{2\mu}{\mu+1}\delta_+(p)+F'_\mu(Bz,p)\right)\right]\cdot J(r)$$

$$+ uH'(p)\frac{\mu\varepsilon-1}{\varepsilon-\mu}(F_\varepsilon(Bz,p)-F_\mu(Bz,p))\nabla_t\cdot J(r) \ ,$$

where I_t is the unit dyadic in xy plane and $\nabla_t=\nabla_{xy}$. When the image source is known, the transmitted field is readily obtained from the integral expression

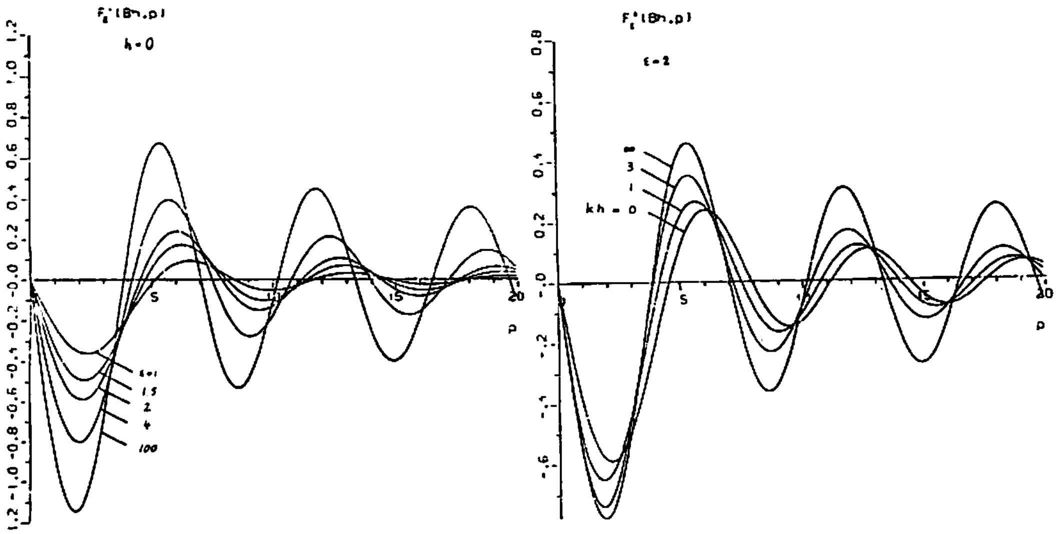
$$(12) \qquad E(r) = -j\omega\mu\mu_o\left(I + \frac{1}{k_1^2}\nabla\nabla\right)\cdot\int_{V}\int_{0}^{\infty}G_1(D)J_i(r',p)dpdV' \ ,$$

with D defined by $D = \sqrt{(\rho-\rho')^2+(z-H)^2}$ and $H = \sqrt{z'^2+(p/B)^2}$.
Corresponding expression for the image of an arbitrary three-dimensional magnetic current can be written from duality. The advantages of image calculation of fields are the same as for the reflection problem [1]-[5]

We made several tests for the new theory and found that

1. For $\varepsilon \to 1$, $\mu \to 1$ the fields produces by the image tend to fields from the original source in the air.
2. For $|\varepsilon| \to \infty$ the transmitted field vanishes.
3. For h=0, the transmitted image coincides with the original source plus the reflection image when ε is replaced by $1/\varepsilon$.
4. For $h \to \infty$, the transmitted field approaches that obtainable through plane-wave transmission coefficient calculation.
5. The images for equivalent electric and magnetic sources are equivalent.

Some calculated examples for the image function $\frac{d}{dp}F_\varepsilon(Bh,p)$ are given below:



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