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# FULL WAVE CALCULATION OF ELF/VLF WAVE FIELDS RADIATED FROM A DIPOLE ANTENNA IN THE IONOSPHERE

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## 1. Introduction

High-power transmitters at high latitudes such as HIPAS and Tromsø radiate ELF/VLF-modulated high-power HF waves to investigate nonlinear modulation of Polar ElectroJet (PEJ) current. Such a modulated current can radiate ELF/VLF waves [*Wong and Brant*, 1990] and act as an artificial antenna in the high-latitude ionosphere, possibly for long-range communications. Therefore, it is important to investigate the propagation characteristics of the ELF/VLF waves radiated upward in the ionosphere and downward to the ground.

The wave propagation radiated by an ELF/VLF source immersed in the anisotropic ionosphere has been investigated by using a waveguide mode theory [*Pappert*, 1973; *Tripathi et al.*, 1982]. This theory solves for electromagnetic fields of several intrinsic modes in the earth-ionosphere waveguide and adds them up to calculate the ELF/VLF propagation characteristics between the ionosphere and the ground. However, it is difficult for this theory to solve for accurate wave fields at altitudes above the source.

Alternatively, a full wave technique was developed to calculate ionospheric propagation of a VLF wave radiated by a dipole antenna on the ground [*Price*, 1967; *Nagano et al.*, 1991]. The propagation of the spherical wave is expressed as the superposition of a number of plane waves in a horizontally stratified medium. This technique gives accurate wave fields if a sufficient number of plane waves are added to express the spherical wave.

We have extended the full wave technique to calculate wave field intensities at satellite heights (above the source) as well as on the ground (below the source) when a dipole VLF source is located inside the ionosphere. In this paper, we compare the calculated field intensities to data observed during the Tromsø/Akebono campaign [*Kimura et al.*, 1992].

## 2. Basic equations

Suppose a dipole with current $I$ of length $\ell$ is located at $(x, y, z) = (0, 0, z_0)$ in an anisotropic uniform plasma. The electric field $\boldsymbol{E}$ at a frequency of $\omega$ radiated from the dipole is expressed as the superposition of a number of plane waves propagating in the direction of the wavevector $\boldsymbol{k} = (k_0 S_x, k_0 S_y, k_0 q)$ [*Wait*, 1964]:

$$\boldsymbol{E}(x, y, z) = \frac{j\omega\mu_0 I\ell k_0}{(2\pi)^3} \iiint_{-\infty}^{\infty} [\Omega]^{-1} \boldsymbol{n} \exp[-jk_0(S_x x + S_y y + q(z - z_0))] dS_x dS_y dq, \tag{1}$$

where $\mu_0$ and $k_0$ are the magnetic permeability and wavenumber in free space, respectively, and the unit vector $\boldsymbol{n}$ represents the orientation of the dipole. The tensor $[\Omega]$ is a $3 \times 3$ tensor defined as

$$[\Omega] \equiv \frac{1}{k_0^2}([\boldsymbol{kk}] - k^2[U] + k_0^2[\kappa]), \tag{2}$$

where $[U]$ is a unit diadic tensor, $[\kappa]$ is the plamsa dielectric tensor and $k = |\boldsymbol{k}|$.

In (1) the integration with $q$ becomes

$$\tilde{\boldsymbol{E}}(S_x, S_y, z) = \frac{j\omega\mu_0 I\ell k_0}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{adj}[\Omega]\boldsymbol{n}}{\det[\Omega]} \exp[-jk_0 q(z - z_0)] dq. \tag{3}$$

It turns out $\det[\Omega] = 0$ is identical with the so-called booker quartic equation so we can write

$$\det[\Omega] = \alpha(q - q^{\mathrm{Rup}})(q - q^{\mathrm{Lup}})(q - q^{\mathrm{Rdown}})(q - q^{\mathrm{Ldown}}), \tag{4}$$

where $\alpha$ is the coefficient of $q^4$. The four roots $q^i$ ($i = \mathrm{Rup}, \mathrm{Lup}, \mathrm{Rdown}, \mathrm{Ldown}$) represent the $z$-components of the refractive indices of the up-going R and L modes and down-going R and L modes, which are characteristic modes for VLF propagation in the ionosphere. Hence (3) is evaluated as the sum of residues around the four poles.

Next we calculate the propagation of each plane wave $\tilde{\boldsymbol{E}}(S_x, S_y, z)$ with $S_x$ and $S_y$ fixed, in a horizontally stratified system consisting of the ionosphere, free space and the ground, as shown in Figure 1. The source dipole is assumed to be located at $z = z_0$ in the ionosphere, above which there are $M$ layers and below which there are $N$ layers.

In the stratified medium the wave field $\tilde{\boldsymbol{e}}(S_x, S_y, z) \equiv (E_x, -E_y, Z_0 H_x, Z_0 H_y)^t$ at boundaries are connected by a matrix $[K]$ determined by the medium parameters. Here $H_x$ and $H_y$ are the wave magnetic field calculated from a Maxwell equation and $Z_0$ is the wave impedance in free space. The wave field at the dipole altitude $z_0$ is related to the fields at the top altitude ($z = z_M$) and at the bottom altitude ($z = z_{-N}$),

$$\tilde{\boldsymbol{e}}(z_{0+}) = \prod_{j=0}^{M-1} [K_j]^{-1} \tilde{\boldsymbol{e}}(z_M), \quad \tilde{\boldsymbol{e}}(z_{0-}) = \prod_{j=-1}^{-N} [K_j] \tilde{\boldsymbol{e}}(z_{-N}), \tag{5}$$

respectively. At the source altitude there is a discontinuity between $\tilde{\boldsymbol{e}}(z_{0+})$ and $\tilde{\boldsymbol{e}}(z_{0-})$,

$$\tilde{\boldsymbol{e}}(z_{0+}) - \tilde{\boldsymbol{e}}(z_{0-}) = \tilde{\boldsymbol{e}}_s^{\mathrm{up}} - \tilde{\boldsymbol{e}}_s^{\mathrm{down}}, \tag{6}$$

where $\tilde{\boldsymbol{e}}_s^{\mathrm{up}}$ and $\tilde{\boldsymbol{e}}_s^{\mathrm{down}}$ denote up- and down-going wave fields at $z = z_0$ radiated by the source dipole.

Assuming there is no reflection at $z > z_M$ and $z < z_{-N}$, we can consider only up-going R and L modes above $z_M$ (up to the upper ionosphere) and down-going TE and TM modes below $z_{-N}$ (inside the earth ground) so that

$$\tilde{\boldsymbol{e}}(z_M) = a_1 \tilde{\boldsymbol{e}}^{\mathrm{Rup}}(z_M) + a_2 \tilde{\boldsymbol{e}}^{\mathrm{Lup}}(z_M), \quad \tilde{\boldsymbol{e}}(z_{-N}) = b_1 \tilde{\boldsymbol{e}}^{\mathrm{TEdown}}(z_{-N}) + b_2 \tilde{\boldsymbol{e}}^{\mathrm{TMdown}}(z_{-N}) \tag{7}$$

Using (5), (6) and (7), we determine the unknown factors $a_1$, $a_2$, $b_1$ and $b_2$, and wave fields in each layer. In calculating (5), we use scaling-down and orthogonalization techniques to avoid numerical swamping.

Then, using $\tilde{\boldsymbol{E}}(S_x, S_y, z)$ in (1),

$$\boldsymbol{e}(x, y, z) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \tilde{\boldsymbol{e}}(S_x, S_y, z) \exp[-jk_0(S_x x + S_y y)] dS_x dS_y, \tag{8}$$

we can obtain the wave field intensities $\boldsymbol{e}$ at an arbitrary altitude $z$ of the wave radiated by a dipole immersed in the ionosphere.

## 3. Numerical Calculation

To evaluate the integration in (8) efficiently by using FFT, we have to sample $\tilde{\boldsymbol{e}}(S_x, S_y, z)$ in a specified area on the $S$-plane. Since $\tilde{\boldsymbol{e}}(S_x, S_y, z)$ oscillates with increasing $S_x$ and $S_y$, the sampling intervals $\Delta S_x$ and $\Delta S_y$, which determine the area of wave field in the $x$-$y$ plane, are taken to be more than two points in one oscillation period. On the other hand, the integrating area ($|S_x|, |S_y| \leq S_{max}$) determines the spatial resolution of the wave fields in the $x$-$y$ plane. Since $\tilde{\boldsymbol{e}}(S_x, S_y, z)$ decreases exponentially with $S_x$ and $S_y$, we set $S_{max}$ where the value of $|\tilde{\boldsymbol{e}}|$ becomes $-30$dB of the maximum value. These values depend on the altitude at which the wave field distribution is obtained.

At the source altitude, the plane waves with $S_x^2 + S_y^2 > n^2$, where $n$ is the refractive index of the mode, are evanescent in the $z$-direction with $Im(q) = \sqrt{(S_x^2 + S_y^2) - n^2}$. However, the evanescent waves with

not so large $Im(q)$ affect the wave field intensities on the $x$-$y$ plane. For typical propagation of a VLF wave radiated from a dipole in the ionosphere, we integrate on the area of $|S_x|, |S_y| < 2$ on the ground and $|S_x|, |S_y| < 30 \sim 40$ in the upper ionosphere.

## 4. Calculated results and comparison with the Tromsφ/Akebono campaign

As an example of the calculated results, here we compare the calculated wave field intensities with those observed during the Tromsφ/Akebono campaign [*Kimura et al*, 1992].

Figure 2 shows the profiles of the electron density and collision frequency used in the calculation. The electron density was estimated from the observed values by the EISCAT radar (70km ~ 400km) and by the Akebono satellite at an altitude of 850km during the experiment.

For such an electron density profile in the lower ionosphere (< 100km) *Barr and Stubbe* [1984] showed that the Hall current perpendicular to the dc electric field around an altitude of 70km can be an ELF/VLF source. The dc electric field was observed also by the STARE radar to be ~5mV/m, corresponding to a current of 0.2A over a length of 30km (the region heated by the HF wave). So we assumed in the calculation a dipole antenna with a dipole moment of $I\ell = 6 \times 10^3$[Am], that is located at 70km altitude. We could not determine the orientation of the dipole since the orientation of the dc electric field observed was not stable during the experiment. Therefore, we have assumed that the dipole lies in the east-west direction, which results in the calculated wave field intensities consistent with the observed data.

Figure 3 shows the calculated field intensities ($H_x$ component) on the ground. The $y$-axis points to magnetic north and the heating facility is located at the origin. The pattern is slightly elongated along the north-south direction, consistent with the dipole radiation pattern. Secondary peaks north and south of the primary peak indicate multiple reflection between the ionosphere and the ground. The observed field intensities (east-west ($x$) and north-south ($y$) components) of the wave magnetic field at Lycksele, 554km south to the heating facility, are listed in Table 1, which also includes the calculated intensities at that distance. There is fairly good agreement between the observed and calculated values, indicating the assumption of an east-west dipole of $6 \times 10^3$ at 70km altitude is valid for this case. (In Table 1 we also listed the calculated values for the north-south dipole located at the same altitude; the results being different only in factor.)

Figure 4 shows the calculated Poynting flux distribution at 1000km above the source. The field distribution seems to rotate by about 90 degree relative to the dipole radiation pattern. This may be caused by the partial reflection of the wave in the lower ionosphere. The pattern totally deviates to the south, because of the whistler-mode beam propagation along the geomagnetic field. The Poynting flux value observed by the Akebono satellite (at 22E, 67N and 1094km altitude) are listed also in Table 1. In this case we can also see exact agreement between the observed data and the calculated values.

## 5. Conclusion

We developed a full wave technique to calculate the field intensities both at the satellite heights and on the ground. This technique was applied to examine one of the results of Tromsφ/Akebono campaign. We could see fairly good agreement between the calculated field intensities and the observed both by Akebono satellite and on the ground. Our calculation method can also be applied to further investigate the efficiency of the PEJ used as a huge artificial ELF/VLF antenna.

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Table 1. Wave field intensities observed during Tromsø/Akebono campaign

| | observed value | calculated (E-W dipole) | calculated (N-S dipole) |
|---|---|---|---|
| Lycksele | 3.6fT($H_x$) 5.0fT($H_y$) | 3.3fT($H_x$) 6.6fT($H_y$) | 2.6fT($H_x$) 1.9fT($H_y$) |
| Akebono (Poynting flux) | $7.9 \times 10^{-13}$[W/m$^2$] | $7.9 \times 10^{-13}$[W/m$^2$] | — |

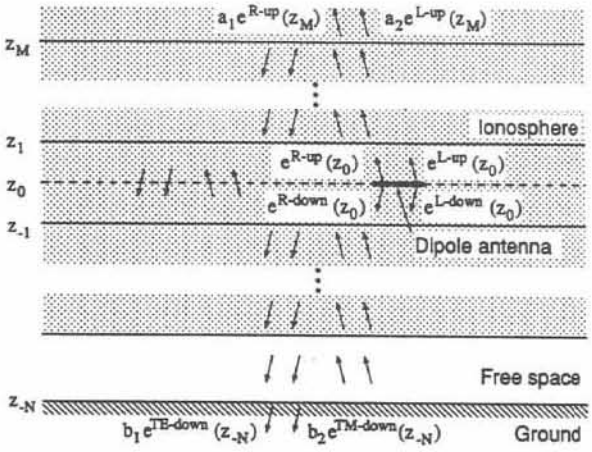


Figure 1. Calculation model

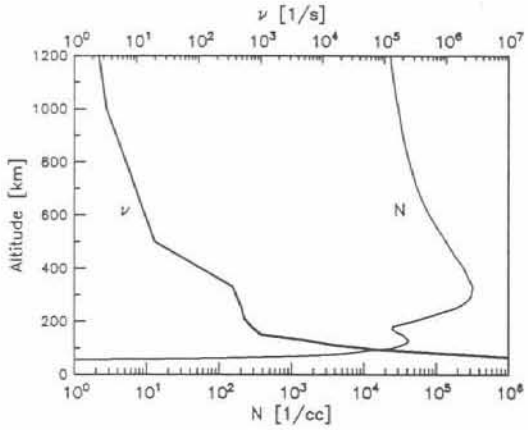


Figure 2. Ionospheric model

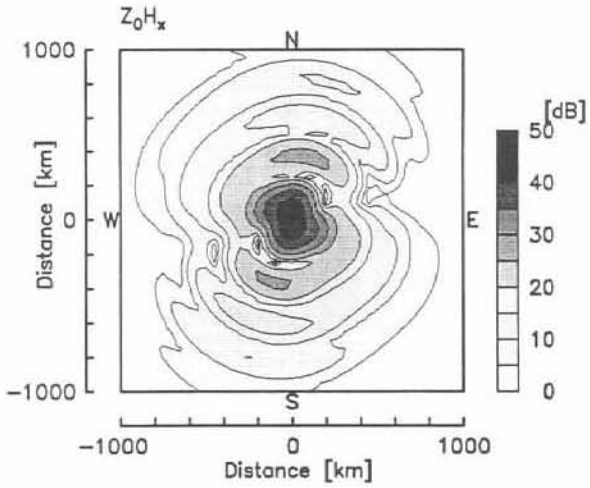


Figure 3. Calculated $H_x$ intensities on the ground, 0dB corresponding to 3.3fT

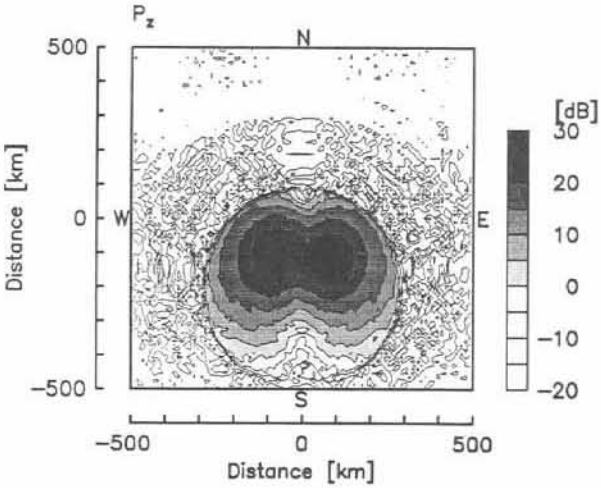


Figure 4. Calculated Poynting flux intensities at 1000km altitude. 0dB corresponding to 1pW/m$^2$