COMPLEX RAY INTERPRETATION OF REFLECTION FROM TARGETS WITH INFLECTION POINTS

H.Ikuno* and L.B.Felsen**

* Department of Information Engineering
Kumamoto University, Kumamoto 860, Japan
** Department of Electrical Engineering and
Computer Science, Microwave Research Institute
Polytechnic Institute of New York
Route 110 Farmingdale, NY 11735, USA

1. Introduction

In a previous investigation, the high-frequency spectral contributions in numerical results for both transient and monochromatic plane wave backscattering from a perfectly conducting object with a convex-concave boundary shape have been studied by the physical optics method [1],[2]. In addition to the real stationary points at the conventional specular reflection points on the illuminated part of the surface, the physical optics integral is found to contain complex stationary points with real parts located near the non-specular inflection points [1],[2].

The problem is now being re-examined by regarding the complex stationary point contributions as reflections of complex incident rays from the analytic extension of the boundary into a complex coordinate space [3],[4]. It is verified that complex incident rays obtained by analytic extension of the real incident ray field are reflected from convex-to-concave transition points located near the inflection points on the illuminated part of the extended surface according to the laws of complex geometrical optics to yield a real contribution in the direction of the specular reflection of the incident ray.

2. Physical optics solution

We shall examine in terms of the physical optics approximation the non-specular reflections from two-dimensional perfectly conducting targets with inflection points. Here we treat only the scattering problem for the E-polarized field, because the far field has no polarization dependence in this approximation.

Suppressing a time dependence $\exp[-j\omega t)$, the plane wave

$$U_{inc}(r,\theta) = \exp[-jkr\cos(\theta-\alpha)],\ k=2\pi/\lambda \quad \lambda=\text{wavelength} \tag{1}$$

is incident on the scatterer at the angle α(see Fig.1). Then for the far-zone scattered field, we have

$$U(r,\theta) \simeq -(k/2\pi r)^{1/2}\exp[j(kr-\pi/4)] \int_{L_0} g(\phi)\exp[-jkh(\phi)]d\phi \tag{2}$$

where

$$g(\phi) = \rho\cos(\alpha-\eta(\phi))/\cos(\phi-\eta(\phi))$$
$$h(\phi) = \rho\cos(\phi-\alpha) + \rho\cos(\phi-\theta)\ .$$

In (2), the illuminated part of the scatterer surface is designated by L_0. In order to evaluate (2) by the method of stationary phase, we extend the real ϕ-space to the complex ϕ-space. Provided that $|\theta-\alpha|<\pi$ and the stationary points are isolated from each other, the method of stationary phase applied to (2) gives

$$U(r,\theta) \simeq -(1/r)^{1/2}\exp[jkr] \sum_{n=1}^{N} (1/jh''(\phi_n))^{1/2} g(\phi_n)\exp[-j(kh(\phi_n)+\pi/4)] \tag{3}$$

where the primes denote the derivative with respect to the argument and N is a total number of stationary points [5]. The stationary points are given by

$$\phi_n = (\theta+\alpha)/2 + \tan^{-1}[d\rho/\rho d\phi]_{\phi=\phi_n} \quad , \ n=1,2,\ldots,N \tag{4}$$

and

$$\eta(\phi_n) = (\theta+\alpha)/2 \ , \quad n=1,2,\ldots,N \quad . \tag{5}$$

In the case of periodically deformed cylinders [1] described by

$$\rho = a(\ 1 + \delta\cos(\nu\phi)),\ a > 0,\ 1 > \delta > 0,\ \nu=1,2,3,\ldots \quad , \tag{6}$$

Eq.(4) has complex solutions whose real parts are located on the illuminated part of the extended complex surface. For backscattering $\theta=\alpha$ in this case, Eq.(3) is a good approximation to (2) and yields an excellent estimate for high-frequency contributions [1],[6].

3. Complex ray tracing

Asymptotic treatment of a rigorous Green's function integration of the initially specified surface current distribution on the initial surface provides the foundation for the validity of the calculation of the field by complex rays since the contributions from isolated complex saddle points have been shown to correspond precisely to directly constructed complex ray fields [3],[7]. Referring to [3], let us evaluate the phase and the amplitude of the reflected ray from a complex stationary point with the aid of complex ray tracing.

We assume that the initial phase and the initial amplitude are represented as [3],

$$S_0(x_0(u),y_0(u)) = S_0(u),\ U_0(x_0(u),y_0(u)) = U_0(u) \tag{7}$$

where x_0 and y_0 are analytic function of u,

$$x_0 = x_0(u)\ ,\ y_0 = y_0(u). \tag{8}$$

The complex ray trajectories are straight lines emanating from the analytically continued initial surface in a complex coordinate space [3]:

$$x(v,u) = x_0(u) + v(\partial S_0(u)/\partial x_0),\ y(v,u) = y_0(u) + v(\partial S_0(u)/\partial y_0) \tag{9}$$

where

$$v = [(x-x_0)^2+(y-y_0)^2]^{1/2} \ . \tag{10}$$

The complex phase at (x,y) is given by

$$S(v,u) = S_0(u) + v \ . \tag{11}$$

In a similar manner, the complex ray amplitude U(v,u) is obtained from the variation of the complex ray tube cross section as

$$U(v,u) = U_0(u)[J(0,u)/J(v,u)]^{1/2},\ J(v,u) = \partial(x,y)/\partial(v,u) \tag{12}$$

with J(v,u) representing the Jacobian of the mapping [3].

These considerations are now applied to diffraction of a plane wave by a perfectly conducting periodically deformed cylinder with inflection points. The complex stationary points are located near the inflection points [1], [2]. The initial surface is described by

$$x_0(u) = \rho\cos\phi\ ,\ y_0(u) = \rho\sin\phi \tag{13}$$

with the initial phase distribution

$$S_0(u) = -\ \rho\cos(\phi-\alpha) \tag{14}$$

deduced from (1) by local field matching [3]. First we show the law of reflection of the complex ray at the complex stationary point. To do this, we need an outward normal vector at the stationary point (x_s,y_s). From a simple calculation together with (4), we have

$$n(x_s,y_s) = (|\cos(\phi_s-(\theta+\alpha)/2)|/\cos(\phi_s-(\theta+\alpha)/2)) \times (\boldsymbol{i}_x\cos((\theta+\alpha)/2)+\boldsymbol{i}_y\sin((\theta+\alpha)/2)) \quad (15)$$

where $\boldsymbol{i}_x$ and $\boldsymbol{i}_y$ are unit vectors along the x- and y-axes. Using (15), we get

$$n(x_s,y_s)\cdot(\boldsymbol{i}_x\cos\alpha + \boldsymbol{i}_y\sin\alpha) = n(x_s,y_s)\cdot(\boldsymbol{i}_x\cos\theta + \boldsymbol{i}_y\sin\theta) = \cos((\theta-\alpha)/2)|\cos(\phi_s-(\theta+\alpha)/2)|/\cos(\phi_s-(\theta+\alpha)/2) \quad (16)$$

where the dot denotes the inner product of vectors. Eq.(16) is the law of reflection of the complex ray at the complex stationary point. From (10), we have

$$v = r - \rho\cos(\phi-\theta) . \quad (17)$$

Eq.(11) together with (14) and (17) yields the phase of Eq.(3). So, the remaining problem is to evaluate the amplitude of the complex ray. It follows from (13) and (14) that on the initial surface, we have the relations as

$$\begin{aligned} \partial S_0/\partial x_0 &= (d\phi/d\tau)^2[(\rho^2-\rho'^2)\cos(2\phi-\alpha) + 2\rho\rho'\sin(2\phi-\alpha)] \\ \partial S_0/\partial y_0 &= (d\phi/d\tau)^2[(\rho^2-\rho'^2)\sin(2\phi-\alpha) - 2\rho\rho'\cos(2\phi-\alpha)] \end{aligned} \quad (18)$$

where τ is the length coordinate of the initial surface [3]. From (12) with (9) and (18), we have

$$J(v,u) = (d\phi/d\tau)[\rho\cos(\phi-\alpha)+\rho'\sin(\phi-\alpha)] + 2v(d\phi/d\tau)^3[\rho^2+2\rho'^2-\rho\rho'']. \quad (19)$$

At the stationary point, we get

$$J(v,u_s) = \cos((\theta-\alpha)/2) + 2v[\rho_s^2+2\rho_s'^2-\rho_s\rho_s'']/(\rho_s^2+\rho_s'^2)^{3/2} . \quad (20)$$

Therefore the amplitude of the complex ray from the complex stationary point becomes

$$U(v,u_s) = U(u_s)[\cos((\theta-\alpha)/2)/(\cos((\theta-\alpha)/2)+2v/a(\phi_s))]^{1/2} \quad (21)$$

where $a(\phi_s)$ is the radius of curvature at (x_s,y_s) defined by

$$a(\phi_s) = (\rho_s^2+\rho_s'^2)^{3/2}/[\rho_s^2+2\rho_s'^2-\rho_s\rho_s''] . \quad (22)$$

For the far field amplitude, we have

$$U(v,u_s) = U_0(u_s)[a(\phi_s)\cos((\theta-\alpha)/2)/2v]^{1/2} \quad (23)$$

since the first term of the right hand side of (20) can be neglected there. On the other hand, the second derivative of S(v,u) with respect to τ at the stationary point is given by

$$d^2S(v,u)/d\tau^2|_{\tau=\tau_s} = 2\cos((\theta-\alpha)/2)/a(\phi_s) . \quad (24)$$

The amplitude derived from (3) becomes

$$g(\phi_s)(-1/rh''(\phi_s))^{1/2} = (\rho_s^2+\rho_s'^2)^{1/2}[a(\phi_s)\cos((\theta-\alpha)/2)/2v]^{1/2}. \quad (25)$$

Eq.(23) coincides with (25) provided that $U_0(u_s) = (\rho_s^2+\rho_s'^2)^{1/2}$.

It is therefore verified that the complex ray solution agrees with the complex stationary point evaluation of the radiation integral (2). The non-specular reflection at the complex stationary point located near the inflection point in the real coordinate space is interpretable as specular reflection in the complex coordinate space.

4. Conclusion

Scattering data from smooth targets with inflection points contain features that can be correlated with the convex-to-concave transitions [1], [2]. These features imply non-specularity in the real coordinate space but cannot be diffraction, since the surface is analytic. In the illuminated region, they can be interpreted as specular reflections of complex rays from the complex extension of the surface.

A corresponding complex ray treatment may be applied to the incident and reflected creeping waves on the shadowed side [8]; local closure by complex ray tracing defines the properties of the local creeping wave modes that adapt continuously to the changing guiding environment [4]. Thus, scattering by smotth convex-concave surfaces should be analyzed by considering real as well as complex ray phenomena. Conclusions reached so far suggest that a systematic analysis of scattering by targets with inflection points should incorporate complex rays into the ensemble of reflected rays and reflected-diffracted (creeping) rays.

One of the authors(L.B.Felsen) acknowledges support from the U.S. Office of Naval Research, Electronics Branch.

5. References

(1) H.Ikuno, "Calculation of far-scattered fields by the method of stationary phase," IEEE Trans. on Antennas Propagat., AP-27, p.199, 1979.

(2) ——, "Transient scattering from periodic deformed cylinders," Radio Science, 19, p.1327, 1984.

(3) P.D.Einziger and L.B.Felsen, "Evanescent waves and complex rays," IEEE Trans. on Antennas Propagat.. AP-30, p.594, 1982.

(4) E.Heyman and L.B.Felsen, "Evanescent waves and complex rays for modal propagation in curved open waveguides," SIAM J. Appl. Math., 43, p.855, 1983.

(5) L.B.Felsen and N.Marcuvitz, Radiation and scattering of waves. Englewood Cliffs, NJ: Prentice Hall, 1973, pp.382-383.

(6) H.Ikuno and K.Takemoto, "Radar cross sections of periodic deformed cylinder," Trans. IECE of Japan, 61-B, p.866, 1978. (in Japanese)

(7) G.Ghione, I.Montrosset, and L.B.Felsen, "Complex ray analysis of radiation from large apertures with tapered illumination," IEEE Trans. on Antennas Propagat., AP-32, p.684, 1984.

(8) H.Ikuno and L.B.Felsen, "Complex ray interpretation of transient scattering from targets with inflection points," presented at the 1984 IEEE Antennas Propagat. Soc. and URSI Symp.(URSI/B-3-9), Boston, Mass., June, 1984.

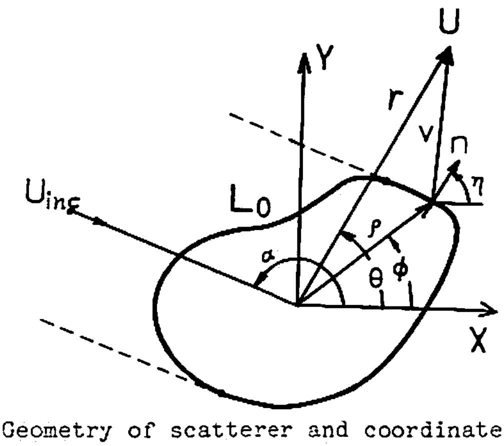

Fig.1 Geometry of scatterer and coordinate system