HIGH FREQUENCY RADIATION IN THE PRESENCE OF A PERFECTLY CONDUCTING SMOOTH CONVEX-CONCAVE BOUNDARY

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High-frequency asymptotic field solutions for perfectly conducting smooth boundaries with variable radius of curvature large compared to the wavelength are of interest for a variety of applications involving propagation along, and (or) scattering from, the surface contour. The simplest phenomena occur for convex shapes, where geometrically reflected fields and creeping wave diffracted fields in illuminated and shadow regions, as well as transition fields near shadow boundaries, are modeled from the rigorously solvable canonical problem of the circular cylinder. Also well understood, but more intricate, is the field behavior in the presence of totally illuminated concave shapes. Focusing can now occur, with the need for modifying geometric ray theory in transition regions near the caustics of the reflected ray system. When the source is located near or on the boundary (in the latter instance, this may express the effect of an equivalent source presented, for example, by an edge discontinuity), rays with many reflections are confined near the surface, with a consequent piling up of caustics in that vicinity. It may then no longer be possible to correct for each caustic individually; instead, the high order reflected rays must be treated collectively, either in integral form [1] or in terms of a selected number of whispering gallery modes that are guided along the surface [2]. This results in general in a hybrid formulation comprising in unique proportion a number of legitimate ray fields with few reflections, a thereby determined number of whispering gallery modes, plus a remainder integral that is either negligible or reducible to a Fresnel function [2],[3]. These conclusions are based on generalizations of the rigorous solution for the concave circular shape [4].

The most difficult problem is posed by a surface whose curvature changes from concave to convex. The focusing concave, and the defocusing convex, portions are connected by a transition region surrounding the inflection point. Unfortunately, there exists no canonical prototype configuration that yields a tractable rigorous solution, from which the inflection point transition can be inferred. Therefore, it is necessary to rely on direct asymptotic methods. When the source is far from the boundary, the ray method can be employed to approximate the surface currents on the illuminated portion, and asymptotic treatment of the physical optics integral containing these currents can then provide information about the inflection-induced transition behavior [5]. This does not account, however, for (trapped) whispering gallery (W.G.) modes and their conversion to (radiating) creeping waves, and vice versa, which is relevant when the source is located near the surface on the concave or convex sides, respectively, or when a creeping wave provides excitation for a shadowed convex-concave portion. The characteristics of an initially well confined W.G. mode propagating from the concave toward the convex side have been explored by two methods. The first, based on the boundary layer near the surface, leads to a parabolic equation that is solved numerically [6]. The second assumes the surface currents to be approximated by the analytic continuation of the W.G. surface fields from positive (concave) to negative (convex) radius of curvature, and then treats the resulting physical optics integral asymptotically [7]. The two methods have been compared numerically [8] and appear to yield comparable predictions.

The present paper addresses the concave-convex problem in some generality. The source may be placed arbitrarily, with particular emphasis on locations near the surface because this gives rise to the most intricate phenomena (see Figs. 1,2). In general, it is appropriate to employ a hybrid formulation that blends legitimate ray fields with whispering gallery modal fields. However, by tracking modal ray congruences, ray methods can be employed even for modal fields [9]. This permits an insight into the evolution of a W.G. mode as it approaches and passes the inflection point, and is converted into a creeping wave (Fig. 3). Predictions from the modal ray tracing will be compared with numerical solutions obtained from the parabolic equation algorithm for a model surface, and will be used to interpret the numerical results. The information thus developed for a single W.G. mode is then incorporated into the hybrid format:

$$G = \sum(\text{legitimate rays}) + \sum(\text{corresponding W.G. modes}) + R_N \qquad (1)$$

To assess the importance of the remainder integral R_N, its evolution along the surface, starting from known initial conditions on the concave side, will be computed with the parabolic equation algorithm. It is anticipated, based on experience from the concave surface problem [2], that R_N can be neglected, at least in certain parameter regimes, thereby simplifying the hybrid form in (1) substantially (see Fig. 4).

The presentation will emphasize the physical content of the wave phenomena associated with the various situations depicted in Figs. 1-4, the interplay of ray fields and W.G. mode fields in the hybrid representation, supported by numerical comparisons based on ray asymptotics, physical optics and the parabolic equation. At the time of submission, the parabolic equation algorithm is functioning and has produced substantial data for the W.G. modal field transition. The other numerical phases are being implemented.

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References

[1] V.M. Babich and V.S. Buldyrev, Asymptotic Methods of Short Wave Diffraction, Moscow: Nauka, 1972, Ch. 4.

[2] T. Ishihara, L.B. Felsen, and A. Green, "High-frequency Fields Excited by a Line Source Located on a Perfectly Conducting Concave Cylindrical Surface", IEEE Trans. Antennas Propagat., vol. AP-27, pp. 172-179, 1978.

[3] E. Topuz, E. Niver and L.B. Felsen, "Electromagnetic Fields Near a Concave Perfectly Conducting Cylindrical Surface", IEEE Trans. Antennas Propagat., vol. AP-30, pp. 1107-1120, 1982.

[4] T. Ishihara, "High Frequency Behavior of Source Excited Concave Surfaces", Ph.D. dissertation, Polytechnic Institute of New York, June 1978.

[5] Mohammad H. Rahnavard and Willard V.T. Rusch, "Surface-Curvature-Induced Microwave Shadows", IEEE Trans. Antennas Propagat., vol. AP-30, pp. 83-88, 1982.

[6] M.M. Popov and I. Pshenchik, "Whispering Gallery Waves Inflection Point of Boundary", Sov. Phys. Dokl. vol. 21, No. 10, pp. 822-825, 1976.

[7] V.S. Buldyrev, "Propagation of Whispering Gallery Waves Along a Concave-Convex Boundary", Proceedings of VII All-Union Symposium on Diffraction and Propagation of Waves, vol. I, pp. 33-36, Rostov on Don, 1977.

[8] A.I. Lanin and M.M. Popov, "Comparisons of Kirchoff's Method and Parabolic Equation Method in the Problem of a Whispering Gallery Mode in the Neighborhood of an Inflection Point", 1977 (in Russian).

[9] S.J. Maurer and L.B. Felsen, "Ray-Optical Techniques for Guided Waves", Proceedings of the IEEE, vol. 55, pp. 1718-1728, 1967.

Figures

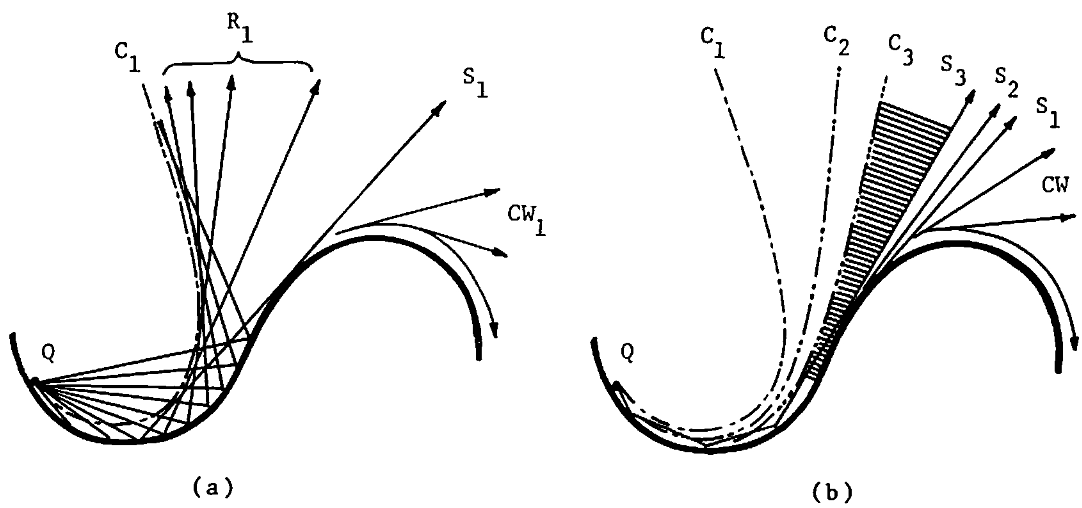

Fig. 1 - Source Q on concave side. Multiple reflected rays generate caustics. Tangent rays establish shadow boundaries and excite creeping rays on the convex side. Rays with many reflections must be treated collectively, for example, by whispering gallery (WG) modes. C_n: caustic: R_n: reflected rays; S_n: shadow boundary; CW_n: creeping rays; n: index denoting number of reflections.

a) Singly reflected rays.
b) Multiple reflected rays. Caustics C_n, $n > 3$, lie in shaded region. Creeping rays CW_n are not shown separately.

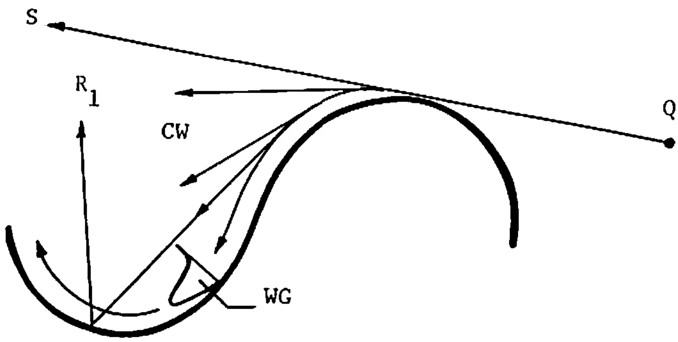

Fig. 2 - Source Q on convex side. Creeping rays CW are multiply reflected on concave side. The sketch shows collective treatment of CW_n, with large n, by W.G. mode.

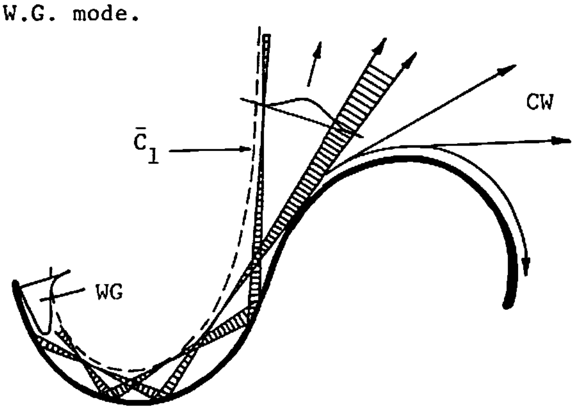

Fig. 3 - Whispering gallery mode incident on concave side. Modal ray congruences for WG mode are confined by modal caustic. Modal ray tracing in ray tubes (shaded) schematizes evolution of WG mode through inflection point, and excitation of creeping rays on convex side. $\bar{C}_m$: modal caustic. m denotes the index of the WG mode.

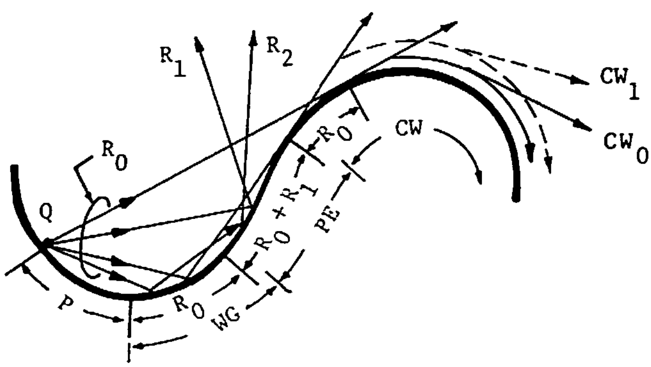

Fig. 4 - Hybrid formulation, with source Q on concave side. Possible options for surface field, based on Eq. (1), are shown. P: plane boundary perturbation; PE: parabolic equation. Other symbols as in Figs. 1-3. The WG transition through the inflection point region is treated by PE.