COMPLEX SPECTRA IN HIGH FREQUENCY PROPAGATION AND DIFFRACTION

L.B. Felsen
Department of Electrical Engineering and Computer Science
Polytechnic Institute of New York, Farmingdale, New York 11735 USA

The modeling of electromagnetic wave propagation and diffraction in a complicated environment requires use of judicious approximations because rigorous formulations usually cannot be translated into practically useful results over the broad ranges of observables that may be of interest. For idealized "canonical" problems, which incorporate some particular feature of the environment, rigorous methods do yield success and can point the way toward approximations that can then be extended to the more general non-canonical case. Spectral representations of the wave fields are among the most powerful of such rigorous methods. At high frequencies, and with $\underset{\sim}{k}$ and ω denoting the vector spatial and temporal spectral wavenumbers, respectively, the plane wave spectral elements can be compacted by constructive interference to small spectral intervals around a source and receiver dependent central spectral value $(\underset{\sim}{k}_s,\omega)$ which is the appropriate one for the wave group that establishes the maximum field at the observer [1,2]. The trajectory of this wave group is the ray path (direct, reflected, refracted or diffracted) from the source at $\underset{\sim}{r}$ to the receiver at $\underset{\sim}{r}'$. By this shrinkage, the original global spectrum becomes localized around the central value $[\underset{\sim}{k}_s(\underset{\sim}{r},\underset{\sim}{r}'),\omega]$. This truncated spectrum can now be tracked through more general environments, which deviate from the canonical one by changes that occur over scales large compared to the local wavelength. This localization applies to propagation as well as diffraction phenomena. If the wave process is dispersive, the spectral wavenumbers $\underset{\sim}{k}_s$ and ω will also be time dependent.

The canonical $(\underset{\sim}{k},\omega)$ spectrum is real. Its constructive interference therefore synthesizes wave processes that involve propagating, undamped local plane waves. However, for wave processes undergoing focusing, there will be spatial "shadow" regions where the wavefields are weak, and similar phenomena are associated with wavefields that experience leakage and(or) radiation damping in a lossless environment. Such fields are characterized by evanescent plane waves with complex $\bar{\underset{\sim}{k}}$, and their constructive interference occurs around a central value $\bar{\underset{\sim}{k}}_s(\underset{\sim}{r},\underset{\sim}{r}')$. Because $\bar{\underset{\sim}{k}}_s$ is complex, the evanescent wave group that retains this central spectrum intact follows a "complex ray" trajectory in a complex coordinate space. Source regions as well as spatial features of media and scatterers must then be extended analytically into this complex space in order to ensure that a relevant complex ray reaches the observer at the real point $\underset{\sim}{r}$.

Tracking constructively interfering evanescent waves along complex rays has the same advantages as tracking non-evanescent wave groups along real rays. Such tracking localizes wave processes in complex space which would otherwise be smeared out in real space; i.e., a compact complex spectral object $\bar{\underset{\sim}{k}}_s$ requires a distribution of real spectral objects $\underset{\sim}{k}$. One of the most striking and familiar examples is a collimated field in the form of a Gaussian beam, which can be generated either by a continuum of real plane waves or by a single point source at a complex location. Tracking of such beam fields through complicated environments comprised of non-planar media, boundaries or scatterers is virtually impossible by repeated global real plane wave analysis and synthesis but can be accomplished by complex ray tracing because of the localization of the evanescent spectra [3-6].

In the formulation of high-frequency propagation and scattering problems either by local plane wave spectral synthesis or by the physical optics method, constructively interfering evanescent wave groups are made evident by complex stationary phase points. In the former, the stationary value $\tilde{k}_s$ identifies the central spectral value of the wave group reaching the observer whereas in the latter, the complex value $\tilde{r}_s$ of the space coordinate in the physical optics integration identifies the point on the complex extension of the surface, from which the complex ray originates. When both formulations are applied to the same problem, they yield consistent values $\tilde{k}_s$ and $\tilde{r}_s$. While complex spectral contributions are clearly of importance when they represent the entire field in a weak-field region, they may be significant even in the presence of non-evanescent constituents. Examples have shown that evanescent tunneling along complex rays produces clearly discernible features in ducted propagation [7] and in scattering from target shapes with concave-convex transitions [8].

Numerical tracing of strongly evanescent fields from source to observer through the environment is complicated by the fact that the corresponding complex rays penetrate deeply into the complex space. However, interest is frequently confined to less rapidly decaying weakly evanescent fields whose complex ray paths remain near real space. For beam-type excitation, one may then examine paraxial approximations that express the field at an off-axis observer by perturbation about their on-axis (maximum) values. The on-axis complex ray field can be approximated in turn by partially real ray fields that further simplify the tracking and evaluation process. Exploration of the quality of these approximations is important because, if applicable, they lead to substantial savings in computer time. It should be pointed out that the beam fields under consideration here are allowed to be strongly affected by the propagation or scattering process; i.e., complex ray paraxial theory accounts for complex perturbations of divergence, reflection and diffraction coefficients and therefore goes beyond conventional paraxial beam optics [9].

Superposition ("shooting") of paraxial Gaussian beams has recently been employed to model propagation and scattering of non-collimated wavefields [10, 11]. The resulting complex extension eliminates the need for uniformization of ordinary paraxial ray fields in regions near caustics, shadow boundaries, etc., because these transitional domains are now displaced into the complex coordinate space. Thereby, numerical computation is simplified substantially. However, the smoothing resulting from this beam-shooting eliminates features in the response that can only be assessed by a critical examination of the paraxial assumption.

In the presentation, the general ideas pertaining to complex spectra are discussed and illustrated by examples. Attention is then give to Gaussian beams, their complex ray tracing, and their paraxial approximation, especially as it pertains to the beam-shooting method. To assess the quality of paraxial assumptions, beam transmission and reflection at strongly curved dielectric interfaces and layers is examined, as is the use in the latter of multiple internally reflected ordinary, as well as compact "collective", complex rays [12]. Complete complex ray tracing is employed to generate the reference solution in these cases [4]. The results provide further confirmation of the utility of complex ray tracing per se, and of its simplification, when valid, via paraxial approximations.

Acknowledgement

This work was sponsored by the U.S. Army Research Office under Contract No. DAAG 29-82-K-0097, the National Science Foundation under Grant NO. EAR-8213147 and the Joint Services Electronics Program under Contract No. F-49620-82-C-0084.

References

[1] J.M. Arnold, "Oscillatory Integral Theory for Uniform Approximation of Wave Functions", Radio Sci. 17(1982), 1181-1191.

[2] L.B. Felsen, "Novel Ways for Tracking Rays", to be published in J. Opt. Soc. Am., Sec. A, 1985.

[3] W.Y. Wang and G.A. Deschamps", Application of Complex Ray Tracing to Scattering Problems", Proc. IEEE, 62(1974), 1541-1551.

[4] X.J. Gao and L.B. Felsen, "Complex Ray Analysis of Beam Transmission Through Two-Dimensional Radomes", to be published in IEEE Trans. Antennas Propagat.

[5] G. Ghione, I. Montrosset and R. Orta, "Complex Rays", in Hybrid Formulation of Wave Propagation and Scattering (L.B. Felsen, editor), Nijhoff, The Hague, 1984.

[6] J. Jacob and H.G. Unger, "Gaussian Beam Transformation Through Lenses and by Curved Gratings", in Hybrid Formulation of Wave Propagation and Scattering (L.B. Felsen, editor), Nijhoff, The Hague, 1984.

[7] E. Niver, A. Kamel and L.B. Felsen, "Modes to Replace Transitional Ray Fields in a Vertically Inhomogeneous Earth Model", to be published in Geophys. J. Roy. Astron. Soc.

[8] H. Ikuno and L.B. Felsen, "Complex Ray Interpretation of Transient Scattering from Targets with Inflection Points", presented at URSI/AP/S Meeting, Boston, Mass., USA, June 1984.

[9] L.B. Felsen, "Geometrical Theory of Diffraction, Evanescent Waves, Complex Rays and Gaussian Beams", Geophys. J. Roy. Astron. Soc. 79 (1984), 77-88.

[10] V. Cerveny, "Synthetic Body Wave Seismograms for Laterally Varying Structures by the Gaussian Beam Method", Geophys. J. Roy. Astron. Soc. 73 (1983), 389-426.

[11] R. Novack and K. Aki, "The 2-D Gaussian Beam Synthetic Method : Testing and Application", to be published in J. Geophys. Res.

[12] Y.Z. Ruan and L.B. Felsen, "Beam Fields in the Presence of Strongly Curved Interfaces and Layers--Complex Ray Paraxial Approximations". In preparation.